



Nanotechnology Prioritization Through Complex Fermatean Neutrosophic Super-Hypersoft Structures

Dua Sajid¹, Anns Uzair^{2,*}, Aiman Ishtiaq^{2,*}, Rashid Jan³, Muhammad Talha Azhar⁴, Qamar Noreen⁵

¹ Department of Chemistry, Government Graduate College Attock, University of Punjab, Attock 43600, Pakistan

² Department of Mathematics, Government College University Faisalabad, Faisalabad 38000, Pakistan

³ Department of Mathematics, College of Science, Qassim University, Buraydah 51452, Saudi Arabia

⁴ Department of Economics, Thal University Bhakkar, Bhakkar 30000, Pakistan

⁵ Department of Mathematics, Quaid-i-Azam University, Islamabad 45320, Pakistan

ARTICLE INFO

Article history:

Received 16 August 2026

Received in revised form 19 September 2026

Accepted 4 October 2026

Available online 10 October 2026

Keywords:

Decision Making; Complex Fermatean Neutrosophic Hypersoft Theory; Nano-technology; Super-HyperSoft Sets; Fuzzy Algebra

ABSTRACT

Selecting appropriate nanoparticle formulations for specific applications is a challenging decision-making problem due to uncertainty and the presence of multiple conflicting criteria. Green-synthesized silver nanoparticles (AgNPs) have attracted considerable attention because of their potential applications in various fields, including agriculture and healthcare. However, identifying the most suitable AgNP formulation requires a systematic evaluation of several criteria and their associated subcriteria. To address this problem, this study introduces complex Fermatean neutrosophic hypersoft sets and complex Fermatean neutrosophic super-hypersoft sets as mathematical frameworks for representing uncertainty in decision-making problems. The fundamental algebraic operations of the proposed structures, including equality, subset relations, union, and intersection, are defined and discussed. Based on these operations, a decision-making application is presented to evaluate green-synthesized silver nanoparticle formulations under uncertain conditions. Numerical examples are provided to illustrate the application of the proposed framework and demonstrate its ability to represent complex and ambiguous evaluation information. The proposed approach offers a mathematical framework for addressing uncertainty and conflicting evaluation criteria, thereby supporting the selection of suitable nanoparticle formulations for different applications.

1. Introduction

In this world full of decision making challenges completely precise data is rarely available. Decision-makers are often asked to evaluate various alternative candidates against several environment-dependent, uncertain, and conflicting criteria, while effectively addressing the hesitation, ambiguity, and indeterminacy inherent in real-world scenarios before any prioritization or ranking can be reliable [1, 2]. A perfect application is in nanomaterial research, where an individual green-synthesized silver nanoparticle (AgNP) structure is inappropriate for one purpose while simultaneously excellent for another, based on how its colloidal stability, environmental footprint, antimicrobial strength, syn-

*Corresponding author.

E-mail address: annsuzair.20241976389@gcuf.edu.pk, aimanishtiaq.202405787@gcuf.edu.pk

<https://doi.org/10.67334/cds31202646>

© The Author(s) 2026 | [Creative Commons Attribution 4.0 International License](#)

thesis quality, and cytotoxic safety margin, are evaluated against each other [3–5]. Traditional crisp decision making instruments lack the ability to handle this layered uncertainty, which has driven more than half a century of advancements in set theory designed to model uncertainty with greater precision [6].

The fuzzy sets firstly introduced by Zadeh [7] in 1965, serve as foundational cornerstone by assigning each component a membership degree in the closed interval $[0, 1]$, enabling partial association in comparison to yes or no logic of classical crisp or binary sets [8]. Inspired by the fuzzy sets, Atanassov [9] later noted that confidence and doubt does not operate as exact inverses for a decision maker, and developed the intuitionistic fuzzy set in 1986, which subject a pair membership and non-membership grade to each element with the condition that their total should remain in the interval $[0, 1]$ [10]. As new challenges were exposed where even this relaxed condition was breached, for instance when a decision maker observes high degrees of both membership and non-membership, Pythagorean fuzzy sets were introduced by Yager [11] applying a quadratic sum constraint instead of linear one, leading to the broader q -rung orthopair fuzzy sets [12]. To provide more space so that decision makers can express strong but conflicting decisions, Senapati and Yager [13] extended this family with the cubic constraints ($q = 3$), introducing Fermatean fuzzy set (FFS), enlarging the membership and non-membership regions beyond what intuitionistic and Pythagorean fuzzy sets enable.

Another emerging field addressed a different gap, that is the above discussed frameworks does not discriminate indeterminacy from dissociation. Smarandache [14] established the concept of neutrosophic set addressing this limitation by integrating three logically independent membership degrees (truth, indeterminacy, and falsity membership) for each component, providing an efficient formulation for handling not only the vague data but also incomplete or inconsistent. Incorporating the cubic constraint of fermatean fuzzy sets with the neutrosophic framework yields the complex fermatean neutrosophic set, employed in this research article, in which all the truth, falsity and indeterminacy degrees are further shows as amplitude–phase (complex-exponential) form [15]. This complex-valued generalization is driven by the complex fuzzy set established by Ramot *et al.* [16], embedding a phase term with the membership degree to address time-dependent factors of ambiguity, further integrated with neutrosophic theory yielding complex neutrosophic and complex neutrosophic soft set frameworks [17, 18]. By incorporating this phase component, the Fermatean neutrosophic model allows a CFNS to register the degrees of satisfaction, contradiction, and indeterminacy, while simultaneously mapping an independent secondary dimension [19–21]. This feature proves essential for evaluations influenced by cyclical, seasonal, or comparative dynamics beyond the scope of a standard amplitude.

Nevertheless the criteria in real-world challenges are hardly ever flat, they are distributed in several sub-criteria, and the relevant attribute count is often unknown. Molodtsov [22] tackled this problem by introducing soft sets, where the set membership is parameterized over an attribute universe instead of a single set. Smarandache [23] extended this model to the hypersoft set by substituting the single-attribute function with a multi-attribute mapping mapped over the Cartesian product of disjoint attribute sets, thereby developing a mathematical base for combining crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic hybrid structures. Since even this multi-attribute formulation considers a fixed, flat distribution of every attribute value, Smarandache and his companions [24] further extended the concept to super-hypersoft set, where the domain of the defining mapping is established from attribute-value power set instead of the sets themselves, allowing attribute-value combinations (instead of single values) to be assessed together. Recent studies have incorporated super-hypersoft structures with broad fuzzy and neutrosophic fields for decision-making scenarios, such as transportation, sustainability, healthcare, and education evaluation [25–27]. Notable contribution, that is relevant to the present generalization, Abdalla *et al.* integrated the hypersoft and super-hypersoft structures with the interval-valued Fermatean neutrosophic sets, for a organ-prioritization for different patients in healthcare, developing the foundational algebraic structures under an interval-valued representation [28].

That framework, however, shows ambiguity by a real-valued interval not a complex-valued

amplitude–phase terms, and as a result is unable to address another, independent aspect of a decision maker’s judgment, a gap that is significant in some real-world situations specifically in nano-material decision-making, where conflicting criteria like regulatory compliance or storage stability often involve a cyclical or comparative dimension irreducible to a single interval. Inspired by this gap, this research article introduces the concept of complex fermatean neutrosophic hypersoft set (CFN-HSS) and complex fermatean neutrosophic super-hypersoft set (CFN-SHSS), embedding the periodic and phase-carrying characteristics inherent to complex fuzzy and complex neutrosophic paradigms into the multi-attribute, cubic-bounded, and power-set-indexed formulations of hypersoft and super-hypersoft structures. Along with the basic definitions, we developed their algebraic operations (subset, equality, null set, union, intersection and complement), and implemented the proposed model to a significant real-world application of selecting the optimal green-synthesized AgNP formulation. The practical urgency of this multi-criteria decision problem stems from rapid developments in plant-mediated AgNP synthesis that have created a disparity between available green synthesis routes and the laboratory resources dedicated to their exhaustive antimicrobial and physicochemical characterization. Since antibacterial efficacy, antifungal efficiency, colloidal stability, crystallographic quality, and biocompatibility usually affect an AgNP structure in different ways, this practical implication provides an effective and inherent examination of the suggested approach’s potential to address uncertain, conflicting, and only partially evaluated criteria into reliable findings.

The remainder of this article is organized as follows.

- **Section 1:** This section provides the theoretical foundation, basic information of relevant background studies and the main purpose of this research article.
- **Section 2:** Foundational definitions introducing complex fermatean neutrosophic hypersoft, and super-hypersoft sets with their numerical examples drawn from the AgNP application domains are described in this section.
- **Section 3:** This section is dedicated to explore the conflicting criteria, their sub-criteria and the alternatives employed in the considered decision problem of the prioritization of the optimal green synthesized silver nanoparticle.
- **Section 4:** The algebraic operations for complex fermatean neutrosophic super-hypersoft sets with their implementation to green synthesized silver nanoparticle selection and numerical representations.
- **Section 5:** Concluding remarks for the study along with the potential future prospects are given.

2. Complex Fermatean Neutrosophic HyperSoft & Super-HyperSoft Sets

In this section, we are going to comprehensively discuss some fundamental definitions, notions and their relative operations for complex fermatean neutrosophic soft sets, hypersoft sets, and super-hypersoft sets.

Definition 2.1 [Complex Fermatean Neutrosophic Set (CFNS)] Let $\mathcal{K}$ represents a non-empty universe of discourse, then a complex fermatean neutrosophic set $\mathcal{O}$ defined on $\mathcal{K}$ is given as;

$$\mathcal{O} = \{ \wp, \langle \xi(\wp)e^{i\vartheta_\xi(\wp)}, \zeta(\wp)e^{i\vartheta_\zeta(\wp)}, \varsigma(\wp)e^{i\vartheta_\varsigma(\wp)} \rangle, \wp \in \mathcal{K} \},$$

where the truth membership, indeterminacy membership, and falsity membership functions are denoted by the complex-valued functions $\xi(\wp)e^{i\vartheta_\xi(\wp)}$, $\zeta(\wp)e^{i\vartheta_\zeta(\wp)}$, and $\varsigma(\wp)e^{i\vartheta_\varsigma(\wp)}$. With the following constraints;

$$0 \leq \xi^3(\wp) + \zeta^3(\wp) + \varsigma^3(\wp) \leq 2, \text{ and } 0 \leq \vartheta_\xi^3(\wp) + \vartheta_\zeta^3(\wp) + \vartheta_\varsigma^3(\wp) \leq 2\pi.$$

Definition 2.2 [Complex Fermatean Neutrosophic Soft Set (CFN-SS)] Let $\mathcal{K}$ represents a non-empty universe of discourse, and let $\mathcal{P}(\mathcal{K})$ denotes the family of all complex fermatean neutrosophic values defined over $\mathcal{K}$. Suppose $\mathcal{A}$ as a set of parameters (attributes) associated with $\mathcal{K}$, and let $\mathcal{B} \subseteq \mathcal{A}$. Given a mapping

$$\mathcal{F} : \mathcal{B} \rightarrow \mathcal{P}(\mathcal{K}),$$

the pair $(\mathcal{F}, \mathcal{B})$ is known as a Complex Fermatean Neutrosophic Soft Set (CFNSS) over $\mathcal{K}$.

Definition 2.3 [Complex Fermatean Neutrosophic HyperSoft Set (CFN-HSS)] Let $\mathcal{K}$ represents a non-empty universe of discourse, with power set $\mathcal{P}(\mathcal{K})$, and let $\mathcal{A} = \{\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3, \dots, \mathfrak{S}_n\}$, for $n \geq 1$, be a set of attributes associated with $\mathcal{K}$. For each attribute, assign a corresponding attribute-value set $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, \dots, n\}, \forall i \neq j$. Define the multi-argument function

$$\aleph : \rho_1 \times \rho_2 \times \rho_3 \times \dots \times \rho_n \longrightarrow \mathcal{P}(\mathcal{K}).$$

Then the mapping

$$\aleph(\rho_1 \times \rho_2 \times \rho_3 \times \dots \times \rho_n) = \left\{ \left\langle \wp, \xi(\wp) e^{i\vartheta_\xi(\wp)}, \zeta(\wp) e^{i\vartheta_\zeta(\wp)}, \varsigma(\wp) e^{i\vartheta_\varsigma(\wp)} \right\rangle : \wp \in \mathcal{K} \right\},$$

over $\mathcal{K}$ is referred to as a Complex Fermatean Neutrosophic HyperSoft Set, where ξ, ζ , and ς denote the truth, indeterminacy, and falsity membership grades respectively, each expressed in amplitude-phase form; the terms $\xi, \zeta, \varsigma \in [0, 1]$ are the amplitude terms and $\vartheta_\xi, \vartheta_\zeta, \vartheta_\varsigma \in [0, 2\pi]$ are the corresponding phase terms of the truth, indeterminacy, and falsity components.

Example 2.1 Suppose we take a significant and intricate real-world situation for the selection of the best application domain for a set of green-synthesized silver nanoparticles (AgNPs), taking into consideration several performance attributes and various conflicting criteria keeping in view the needs of the target sector. At the time the AgNPs are characterized and manufactured, the maker must make the decision to deploy it to the best location. Several components are considered for this selection, among the key factors we have cytotoxic safety margin, environmental acceptability, large-scale deployment cost, antimicrobial efficacy, and colloidal stability. For instance, a set of nanoparticles may present extraordinary antibacterial efficiency to a biomaterial skin substitute, while cytotoxicity safety is compromised for direct contact to skin. On the other hand the identical batch is completely suitable for an agricultural antifungal composition, where the harm to the human skin is not a concern. Similarly, a batch of nanoparticles have high storage stability, a necessary need for large-scale high quality antifungal spray, but its manufacturing cost can be too high to account for agricultural development, where financial outlays per hectare drive the selection. Researchers are to select among these application domains by analyzing stability, cost efficiency, safety, efficacy, and acceptability together. Therefore, the selected application domain for a given nanoparticle set must be the best, across all relevant industry standards. So, the goal is to choose the optimal application domain for the synthesized AgNPs taking into the consideration the following criteria,

- Deployment cost.
- Antimicrobial efficacy.
- Storage stability.
- Cytotoxic safety margin.
- Environmental acceptability.

All of the above criteria are presented in the form of complex fermatean neutrosophic hyperoft numbers. Consider two application domains for the selection, represented by $\mathfrak{S}_1$ (antibacterial coating) and $\mathfrak{S}_2$ (crop-protection spray), are provided as in Table 1.

Definition 2.4 [Complex Fermatean Neutrosophic Super-HyperSoft Set (CFN-SHSS)] Let $\mathcal{K}$ be a non-empty universal set with power set $\mathcal{P}(\mathcal{K})$, and let $\mathcal{A} = \{\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3, \dots, \mathfrak{S}_n\}$, for $n \geq 1$, be a set of attributes associated with $\mathcal{K}$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, \dots, n\}, \forall i \neq j$. Further, let $\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$ denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function

$$\aleph : \mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) \longrightarrow \mathcal{P}(\mathcal{K}).$$

Then the mapping

$$\begin{aligned} & \aleph(\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n)) \\ &= \left\{ \left\langle \wp, \xi^{\aleph(\wp)}(\wp) e^{i\vartheta_\xi^{\aleph(\wp)}(\wp)}, \zeta^{\aleph(\wp)}(\wp) e^{i\vartheta_\zeta^{\aleph(\wp)}(\wp)}, \varsigma^{\aleph(\wp)}(\wp) e^{i\vartheta_\varsigma^{\aleph(\wp)}(\wp)} \right\rangle \right\}, \end{aligned}$$

Table 1
Application domains for green-synthesized AgNPs using CFN-HSS

Criteria	$\mathfrak{S}_1$ (Biomedical)	$\mathfrak{S}_2$ (Agricultural)
Antimicrobial efficacy	$\left\langle \begin{matrix} 0.66e^{i2\pi(0.57)} \\ 0.18e^{i2\pi(0.28)} \\ 0.25e^{i2\pi(0.21)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.72e^{i2\pi(0.63)} \\ 0.14e^{i2\pi(0.23)} \\ 0.20e^{i2\pi(0.16)} \end{matrix} \right\rangle$
Cytotoxic safety margin	$\left\langle \begin{matrix} 0.48e^{i2\pi(0.40)} \\ 0.27e^{i2\pi(0.42)} \\ 0.39e^{i2\pi(0.34)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.75e^{i2\pi(0.68)} \\ 0.12e^{i2\pi(0.19)} \\ 0.17e^{i2\pi(0.13)} \end{matrix} \right\rangle$
Storage stability	$\left\langle \begin{matrix} 0.60e^{i2\pi(0.51)} \\ 0.21e^{i2\pi(0.33)} \\ 0.29e^{i2\pi(0.25)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.68e^{i2\pi(0.59)} \\ 0.17e^{i2\pi(0.27)} \\ 0.23e^{i2\pi(0.19)} \end{matrix} \right\rangle$
Deployment cost	$\left\langle \begin{matrix} 0.46e^{i2\pi(0.38)} \\ 0.29e^{i2\pi(0.45)} \\ 0.41e^{i2\pi(0.36)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.63e^{i2\pi(0.55)} \\ 0.19e^{i2\pi(0.31)} \\ 0.28e^{i2\pi(0.23)} \end{matrix} \right\rangle$
Environmental acceptability	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.47)} \\ 0.24e^{i2\pi(0.38)} \\ 0.33e^{i2\pi(0.28)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.70e^{i2\pi(0.61)} \\ 0.16e^{i2\pi(0.25)} \\ 0.21e^{i2\pi(0.18)} \end{matrix} \right\rangle$

such that $\wp \in \mathcal{H}$, $\varrho \in (\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \cdots \times \mathcal{P}(\rho_n)) \subseteq \{\rho_1, \rho_2, \dots, \rho_n\}$, over $\mathcal{H}$ is termed as Complex Fermatean Neutrosophic Super-HyperSoft Set. Here, ξ , ζ , and ς denote the truth, indeterminacy, and falsity membership grades respectively, each expressed in amplitude-phase form, with $\xi^{\mathcal{N}(\varrho)}, \zeta^{\mathcal{N}(\varrho)}, \varsigma^{\mathcal{N}(\varrho)} \in [0, 1]$ the amplitude terms and $\vartheta_{\xi}^{\mathcal{N}(\varrho)}, \vartheta_{\zeta}^{\mathcal{N}(\varrho)}, \vartheta_{\varsigma}^{\mathcal{N}(\varrho)} \in [0, 2\pi]$ the corresponding phase terms of the truth, indeterminacy, and falsity components.

Example 2.2 To show the numerical representation of complex fermatean neutrosophic super-hypersoft numbers, we are going to examine the considered application domains based on the following main criteria along with their sub-criteria, as presented in Table 2.

- **Antimicrobial efficacy:** Antibacterial performance and antifungal performance.
- **Cytotoxic safety margin:** Selectivity toward target vs. host cells and dose-dependent margin.
- **Storage stability:** Zeta-potential based storage stability and resistance to agglomeration over time.
- **Deployment cost:** Raw material and production costs.
- **Environmental acceptability:** Biodegradability of residues and regulatory compliance ease.

Table 2
Application domains for green-synthesized AgNPs using CFN-SHSS.

Criteria	Sub-criteria	$\mathfrak{S}_1$ (Biomedical)	$\mathfrak{S}_2$ (Agricultural)
Antimicrobial efficacy	Antibacterial performance	$\left\langle \begin{matrix} 0.68e^{i2\pi(0.59)} \\ 0.16e^{i2\pi(0.26)} \\ 0.23e^{i2\pi(0.19)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.61e^{i2\pi(0.52)} \\ 0.21e^{i2\pi(0.34)} \\ 0.30e^{i2\pi(0.25)} \end{matrix} \right\rangle$
	Antifungal performance	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.48)} \\ 0.24e^{i2\pi(0.37)} \\ 0.33e^{i2\pi(0.28)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.76e^{i2\pi(0.69)} \\ 0.11e^{i2\pi(0.18)} \\ 0.16e^{i2\pi(0.12)} \end{matrix} \right\rangle$
Cytotoxic safety margin	Selectivity toward target vs. host cells	$\left\langle \begin{matrix} 0.44e^{i2\pi(0.36)} \\ 0.30e^{i2\pi(0.47)} \\ 0.43e^{i2\pi(0.38)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.77e^{i2\pi(0.70)} \\ 0.11e^{i2\pi(0.17)} \\ 0.15e^{i2\pi(0.11)} \end{matrix} \right\rangle$
	Dose-dependent margin	$\left\langle \begin{matrix} 0.50e^{i2\pi(0.42)} \\ 0.26e^{i2\pi(0.40)} \\ 0.36e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.73e^{i2\pi(0.65)} \\ 0.13e^{i2\pi(0.21)} \\ 0.18e^{i2\pi(0.14)} \end{matrix} \right\rangle$
Storage stability	Zeta-potential-based stability	$\left\langle \begin{matrix} 0.58e^{i2\pi(0.49)} \\ 0.22e^{i2\pi(0.35)} \\ 0.31e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.71e^{i2\pi(0.62)} \\ 0.15e^{i2\pi(0.24)} \\ 0.21e^{i2\pi(0.17)} \end{matrix} \right\rangle$
	Resistance to agglomeration	$\left\langle \begin{matrix} 0.62e^{i2\pi(0.53)} \\ 0.19e^{i2\pi(0.30)} \\ 0.27e^{i2\pi(0.23)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.65e^{i2\pi(0.56)} \\ 0.18e^{i2\pi(0.29)} \\ 0.25e^{i2\pi(0.21)} \end{matrix} \right\rangle$
Deployment cost	Raw material/reagent cost	$\left\langle \begin{matrix} 0.42e^{i2\pi(0.35)} \\ 0.32e^{i2\pi(0.48)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.66e^{i2\pi(0.58)} \\ 0.17e^{i2\pi(0.27)} \\ 0.24e^{i2\pi(0.19)} \end{matrix} \right\rangle$
	Scale-up/production cost	$\left\langle \begin{matrix} 0.39e^{i2\pi(0.32)} \\ 0.35e^{i2\pi(0.51)} \\ 0.48e^{i2\pi(0.43)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.60e^{i2\pi(0.52)} \\ 0.21e^{i2\pi(0.33)} \\ 0.29e^{i2\pi(0.25)} \end{matrix} \right\rangle$
Environmental acceptability	Biodegradability of residues	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.43)} \\ 0.27e^{i2\pi(0.41)} \\ 0.36e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.69e^{i2\pi(0.60)} \\ 0.16e^{i2\pi(0.26)} \\ 0.22e^{i2\pi(0.18)} \end{matrix} \right\rangle$
	Regulatory compliance ease	$\left\langle \begin{matrix} 0.45e^{i2\pi(0.37)} \\ 0.30e^{i2\pi(0.46)} \\ 0.42e^{i2\pi(0.36)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.72e^{i2\pi(0.64)} \\ 0.14e^{i2\pi(0.22)} \\ 0.19e^{i2\pi(0.15)} \end{matrix} \right\rangle$

3. Prioritization of the Most Suitable Green-Synthesized AgNP Formulation

In the modern age of nanotechnology-based healthcare and agricultural development, the selection among various green-synthesized silver nanoparticles (AgNP) is always significant and complex process for the necessarily required establishment and scale-up. For nanomaterial researchers, it is a very crucial issue to assign the short amount of characterization, biological validation, and synthesis resources when the alternative plant-mediated AgNP formulations are increasing day-by-day while experimental work for thorough physiochemical and antimicrobial taxonomy remains limited.

In the procedure of developing the green nanoarticles, the shortage of an organized, uncertainty-lenient prioritization formulation is the most important issue for the research laboratories transforming from classical synthesis to deployment-ready nanomaterials. Furthermore, the irregularity among the number of alternative formulations (synthesis condition $\times$ dosage $\times$ plant source) and the laboratory's potential for complete cytotoxicity, antifungal, antibacterial, and stability profiling is governed

majorly by several factors like extract phytochemical fluctuation, instrument access, and reagent availability. The selected AgNP alternative must meet some strict criteria like colloidal stability, an acceptable cytotoxic safety margin, reproducible antimicrobial efficacy, and antimicrobial efficacy, further cutting the number of frameworks worth progressing to pilot-scale production. For example, frameworks intentionally made for biomedical usage or wound-care enable a limited permissible gap between cytotoxic safety and antimicrobial potency, obligating meticulous dose choice, and rapid validation. While, need for broad-spectrum, cost-effective antimicrobial nanomaterials is rising rapidly with time, demanding selection of the optimal formulation as the only sustainable in transit to timely emergence, growing the demand for a powerful decision tool at higher stage. It is a challenging task for researchers to present not only resource-effective, rapid evaluation of alternative formulations but also long-term consistent performance. Keeping in view these intricate constraints and various conflicting criteria, nanomaterials researchers face challenges in the prioritization of the optimal AgNP formulation. This procedure will determine various AgNP candidates that fulfill the established standards with a comprehensive investigation of every alternative formulation. Following criteria are designated to pick up the best candidate formulation;

- Antibacterial efficacy
- Antifungal efficacy
- Colloidal stability
- Synthesis/crystallographic quality
- Biocompatibility (cytotoxic safety margin)

3.1 Established Metrics for the Prioritization of the Optimal AgNP Formulation

We are going to discuss some of the key criteria having major impact on the evaluation, along with different stages of satisfaction in various alternative AgNP nanoparticle formulations.

1. Antibacterial efficacy:

- **Low:** In comparison to the plant extract alone, the framework show almost same, minimal zones of inhibition, demonstrating lesser efficiency to both families of bacteria which are Gram-positive and Gram-negative pathogens.
- **Medium:** The framework exhibits modest zones of inhibition, crossing the boundary of negative control while significantly below the defined antibiotic.
- **High:** The framework exhibits high, dose-driven zones of inhibition to the Gram-positive and Gram-negative pathogens, demonstrating higher concentrations towards the defined antibiotic.
- **Very High:** The framework demonstrates significantly high zones of inhibition across multiple pathogens at feasible concentrations, almost identical or crossing the defined antibiotic.

2. Antifungal efficacy:

- **Low:** Experimental fungal strains show negligible inhibition, almost same to the negative control (plant extract alone).
- **Medium:** Dose-driven inhibition at a moderate level, less than the defined antifungal drug standard.
- **High:** Higher inhibition showing strong anti-fungal strength, nearly matching the drug standard.
- **Very High:** Almost perfect inhibition or exceeding the defined antifungal drug.

3. Colloidal stability:

- **Low:** The particle charge is low and broad size spectrum present a high risk of agglomeration.
- **Medium:** The particle charge is moderate with mild polydispersity.
- **High:** The particle charge is high with a small size distribution.
- **Very High:** The particle charge is very high enabling excellent stability.

4. Synthesis/crystallographic quality:

- **Low:** The X-ray and light-absorption scans demonstrate uncertain particle-size and hard-to-measure crystals.
- **Medium:** The X-ray scan describes a clear cubic structure with fairly consistent particle sizes.
- **High:** The X-ray scans present sharp, clear peaks, indicating a well-formed crystal structure and highly consistent particle sizes.
- **Very High:** The scans illustrate a perfect crystal structure, consistent particle sizes and highly reliable results.

5. Biocompatibility (cytotoxic safety margin):

- **Low:** The dose needed to kill the germs is almost equal to the amount dangerous enough to harm or probably destroy the host cells.
- **Medium:** The dose required to kill the germs is relatively low to the dose harming the healthy cells.
- **High:** The dose needed to destroy the germs is safe for the healthy human cells.
- **Very High:** Zero toxicity for healthy cells in the doses, and they only destroy the germs.

4. Decision-Making Using Complex Fermatean Neutrosophic Super-HyperSoft Sets Under Uncertain Conditions

The nanomaterials laboratory has gathered an evaluation group of nanomaterial experts, which will assess the alternative silver nanoparticle frameworks against several conflicting criteria while capturing the uncertainty and inconsistency inherent in the real-world data. This process enables the group of experts to select the most appropriate AgNP framework which aligns with the requirements of the demanding laboratory's goals, certifying that the finite resources are allotted to the most promising candidate.

Let $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ be a set of the proposed AgNP structures, also suppose

$$\mathfrak{S} = \{\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3, \mathfrak{S}_4, \mathfrak{S}_5\},$$

be the family of the established criteria, where

- α_1 = OcAgNPs, low dose (10 ppm)
- α_2 = OcAgNPs, high dose (30–40 ppm)
- α_3 = SvAgNPs, low dose (10 ppm)
- α_4 = SvAgNPs, high dose (30–40 ppm)
- α_5 = AgNO₃ solution (5 mM),

and their corresponding attributes are presented as,

1. $\mathfrak{S}_1$ = Antibacterial efficacy = $\left\{ \text{Low Antibacterial efficacy } (\rho_{\mathfrak{S}_1}^L), \text{ Medium Antibacterial efficacy } (\rho_{\mathfrak{S}_1}^M), \text{ High Antibacterial efficacy } (\rho_{\mathfrak{S}_1}^H), \text{ Very High Antibacterial efficacy } (\rho_{\mathfrak{S}_1}^V) \right\}$
2. $\mathfrak{S}_2$ = Antifungal efficacy = $\left\{ \text{Low Antifungal efficacy } (\rho_{\mathfrak{S}_2}^L), \text{ Medium Antifungal efficacy } (\rho_{\mathfrak{S}_2}^M), \text{ High Antifungal efficacy } (\rho_{\mathfrak{S}_2}^H), \text{ Very High Antifungal efficacy } (\rho_{\mathfrak{S}_2}^V) \right\}$
3. $\mathfrak{S}_3$ = Colloidal stability = $\left\{ \text{Low Colloidal stability } (\rho_{\mathfrak{S}_3}^L), \text{ Medium Colloidal stability } (\rho_{\mathfrak{S}_3}^M), \text{ High Colloidal stability } (\rho_{\mathfrak{S}_3}^H), \text{ Very High Colloidal stability } (\rho_{\mathfrak{S}_3}^V) \right\}$
4. $\mathfrak{S}_4$ = Synthesis/crystallographic quality = $\left\{ \text{Low Synthesis quality } (\rho_{\mathfrak{S}_4}^L), \text{ Medium Synthesis quality } (\rho_{\mathfrak{S}_4}^M), \text{ High Synthesis quality } (\rho_{\mathfrak{S}_4}^H), \text{ Very High Synthesis quality } (\rho_{\mathfrak{S}_4}^V) \right\}$
5. $\mathfrak{S}_5$ = Biocompatibility = $\left\{ \text{Low Biocompatibility } (\rho_{\mathfrak{S}_5}^L), \text{ Medium Biocompatibility } (\rho_{\mathfrak{S}_5}^M), \text{ High Biocompatibility } (\rho_{\mathfrak{S}_5}^H), \text{ Very High Biocompatibility } (\rho_{\mathfrak{S}_5}^V) \right\}$

Consider that $\mathfrak{N} : \mathfrak{S}_1 \times \mathfrak{S}_2 \times \mathfrak{S}_3 \times \mathfrak{S}_4 \times \mathfrak{S}_5 \rightarrow \mathfrak{S}_n$. The CFN-SHSS values for $\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3, \mathfrak{S}_4$ and $\mathfrak{S}_5$ are

demonstrated in the following Table 3, Table 4, Table 5, Table 6 and Table 7.

Table 3
CFN-SHS values for Antibacterial efficacy

$\mathfrak{S}_1$	α_1	α_2	α_3	α_4	α_5
$\rho_{\mathfrak{S}_1}^L$	$\langle 0.56e^{i2\pi(0.46)}, 0.27e^{i2\pi(0.36)}, 0.33e^{i2\pi(0.29)} \rangle$	$\langle 0.33e^{i2\pi(0.26)}, 0.41e^{i2\pi(0.50)}, 0.46e^{i2\pi(0.40)} \rangle$	$\langle 0.52e^{i2\pi(0.42)}, 0.29e^{i2\pi(0.38)}, 0.35e^{i2\pi(0.31)} \rangle$	$\langle 0.15e^{i2\pi(0.11)}, 0.55e^{i2\pi(0.63)}, 0.60e^{i2\pi(0.55)} \rangle$	$\langle 0.58e^{i2\pi(0.48)}, 0.26e^{i2\pi(0.34)}, 0.32e^{i2\pi(0.28)} \rangle$
$\rho_{\mathfrak{S}_1}^M$	$\langle 0.83e^{i2\pi(0.72)}, 0.14e^{i2\pi(0.21)}, 0.17e^{i2\pi(0.14)} \rangle$	$\langle 0.57e^{i2\pi(0.47)}, 0.24e^{i2\pi(0.33)}, 0.30e^{i2\pi(0.26)} \rangle$	$\langle 0.80e^{i2\pi(0.70)}, 0.16e^{i2\pi(0.23)}, 0.19e^{i2\pi(0.15)} \rangle$	$\langle 0.32e^{i2\pi(0.25)}, 0.39e^{i2\pi(0.47)}, 0.44e^{i2\pi(0.39)} \rangle$	$\langle 0.79e^{i2\pi(0.69)}, 0.15e^{i2\pi(0.22)}, 0.18e^{i2\pi(0.16)} \rangle$
$\rho_{\mathfrak{S}_1}^H$	$\langle 0.53e^{i2\pi(0.43)}, 0.28e^{i2\pi(0.37)}, 0.34e^{i2\pi(0.30)} \rangle$	$\langle 0.85e^{i2\pi(0.75)}, 0.12e^{i2\pi(0.19)}, 0.15e^{i2\pi(0.12)} \rangle$	$\langle 0.55e^{i2\pi(0.45)}, 0.27e^{i2\pi(0.35)}, 0.32e^{i2\pi(0.28)} \rangle$	$\langle 0.60e^{i2\pi(0.50)}, 0.23e^{i2\pi(0.31)}, 0.29e^{i2\pi(0.25)} \rangle$	$\langle 0.55e^{i2\pi(0.48)}, 0.26e^{i2\pi(0.33)}, 0.31e^{i2\pi(0.34)} \rangle$
$\rho_{\mathfrak{S}_1}^V$	$\langle 0.30e^{i2\pi(0.23)}, 0.42e^{i2\pi(0.51)}, 0.47e^{i2\pi(0.42)} \rangle$	$\langle 0.58e^{i2\pi(0.48)}, 0.25e^{i2\pi(0.33)}, 0.30e^{i2\pi(0.27)} \rangle$	$\langle 0.29e^{i2\pi(0.22)}, 0.43e^{i2\pi(0.52)}, 0.48e^{i2\pi(0.43)} \rangle$	$\langle 0.89e^{i2\pi(0.79)}, 0.10e^{i2\pi(0.17)}, 0.13e^{i2\pi(0.10)} \rangle$	$\langle 0.31e^{i2\pi(0.24)}, 0.41e^{i2\pi(0.50)}, 0.46e^{i2\pi(0.41)} \rangle$

Table 4
CFN-SHS values for Antifungal efficacy

$\mathfrak{S}_2$	α_1	α_2	α_3	α_4	α_5
$\rho_{\mathfrak{S}_2}^L$	$\langle 0.54e^{i2\pi(0.44)}, 0.28e^{i2\pi(0.37)}, 0.34e^{i2\pi(0.30)} \rangle$	$\langle 0.31e^{i2\pi(0.24)}, 0.42e^{i2\pi(0.51)}, 0.47e^{i2\pi(0.42)} \rangle$	$\langle 0.50e^{i2\pi(0.40)}, 0.30e^{i2\pi(0.39)}, 0.36e^{i2\pi(0.32)} \rangle$	$\langle 0.14e^{i2\pi(0.10)}, 0.56e^{i2\pi(0.64)}, 0.61e^{i2\pi(0.56)} \rangle$	$\langle 0.55e^{i2\pi(0.45)}, 0.27e^{i2\pi(0.35)}, 0.33e^{i2\pi(0.29)} \rangle$
$\rho_{\mathfrak{S}_2}^M$	$\langle 0.81e^{i2\pi(0.71)}, 0.15e^{i2\pi(0.22)}, 0.18e^{i2\pi(0.15)} \rangle$	$\langle 0.56e^{i2\pi(0.46)}, 0.25e^{i2\pi(0.34)}, 0.31e^{i2\pi(0.27)} \rangle$	$\langle 0.78e^{i2\pi(0.68)}, 0.17e^{i2\pi(0.24)}, 0.20e^{i2\pi(0.16)} \rangle$	$\langle 0.30e^{i2\pi(0.23)}, 0.40e^{i2\pi(0.48)}, 0.45e^{i2\pi(0.40)} \rangle$	$\langle 0.77e^{i2\pi(0.67)}, 0.16e^{i2\pi(0.23)}, 0.19e^{i2\pi(0.17)} \rangle$
$\rho_{\mathfrak{S}_2}^H$	$\langle 0.51e^{i2\pi(0.41)}, 0.29e^{i2\pi(0.38)}, 0.35e^{i2\pi(0.31)} \rangle$	$\langle 0.84e^{i2\pi(0.74)}, 0.13e^{i2\pi(0.20)}, 0.16e^{i2\pi(0.13)} \rangle$	$\langle 0.53e^{i2\pi(0.43)}, 0.28e^{i2\pi(0.36)}, 0.33e^{i2\pi(0.29)} \rangle$	$\langle 0.58e^{i2\pi(0.48)}, 0.24e^{i2\pi(0.32)}, 0.30e^{i2\pi(0.26)} \rangle$	$\langle 0.54e^{i2\pi(0.44)}, 0.27e^{i2\pi(0.35)}, 0.32e^{i2\pi(0.28)} \rangle$
$\rho_{\mathfrak{S}_2}^V$	$\langle 0.28e^{i2\pi(0.21)}, 0.43e^{i2\pi(0.52)}, 0.48e^{i2\pi(0.43)} \rangle$	$\langle 0.56e^{i2\pi(0.46)}, 0.26e^{i2\pi(0.34)}, 0.31e^{i2\pi(0.28)} \rangle$	$\langle 0.27e^{i2\pi(0.20)}, 0.44e^{i2\pi(0.53)}, 0.49e^{i2\pi(0.44)} \rangle$	$\langle 0.87e^{i2\pi(0.77)}, 0.11e^{i2\pi(0.18)}, 0.14e^{i2\pi(0.11)} \rangle$	$\langle 0.29e^{i2\pi(0.22)}, 0.42e^{i2\pi(0.51)}, 0.47e^{i2\pi(0.42)} \rangle$

CFN-SHSS is presented as $\aleph : \mathfrak{S}_1 \times \mathfrak{S}_2 \times \mathfrak{S}_3 \times \mathfrak{S}_4 \times \mathfrak{S}_5 \rightarrow \mathfrak{S}_n(\mathcal{K})$. Suppose that $\aleph(\ell) = \aleph(\text{High Antibacterial efficacy, High Colloidal stability, High Biocompatibility}) = \{\alpha_2, \alpha_4, \alpha_5\}$, as given in Table 8. Then for this relation we have the CFN-SHS values such that

$$\aleph(\ell) = \left\{ \left\{ \alpha_2, \left(\rho_{\mathfrak{S}_1}^H \{0.85e^{i2\pi(0.75)}, 0.12e^{i2\pi(0.19)}, 0.15e^{i2\pi(0.12)}\}, \rho_{\mathfrak{S}_3}^H \{0.51e^{i2\pi(0.41)}, 0.30e^{i2\pi(0.39)}, 0.36e^{i2\pi(0.32)}\}, \rho_{\mathfrak{S}_5}^H \{0.52e^{i2\pi(0.42)}, 0.29e^{i2\pi(0.38)}, 0.35e^{i2\pi(0.31)}\} \right) \right\}, \left\{ \alpha_4, \left(\rho_{\mathfrak{S}_1}^H \{0.60e^{i2\pi(0.50)}, 0.23e^{i2\pi(0.31)}, 0.29e^{i2\pi(0.25)}\}, \rho_{\mathfrak{S}_3}^H \{0.57e^{i2\pi(0.47)}, 0.25e^{i2\pi(0.33)}, 0.30e^{i2\pi(0.26)}\}, \rho_{\mathfrak{S}_5}^H \{0.56e^{i2\pi(0.46)}, 0.26e^{i2\pi(0.34)}, 0.31e^{i2\pi(0.27)}\} \right) \right\}, \left\{ \alpha_5, \left(\rho_{\mathfrak{S}_1}^H \{0.55e^{i2\pi(0.48)}, 0.26e^{i2\pi(0.33)}, 0.31e^{i2\pi(0.34)}\}, \rho_{\mathfrak{S}_3}^H \{0.30e^{i2\pi(0.23)}, 0.41e^{i2\pi(0.50)}, 0.46e^{i2\pi(0.41)}\}, \rho_{\mathfrak{S}_5}^H \{0.31e^{i2\pi(0.24)}, 0.40e^{i2\pi(0.49)}, 0.45e^{i2\pi(0.40)}\} \right) \right\} \right\}.$$

Definition 4.1 [CFN-SHS Subset] Let us take two CFN-SHSSs

$$\aleph(\ell_1) = \left\{ \left\langle \wp, \xi^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_1)}(\wp)}, \zeta^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp)}, \varsigma^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp)} \right\rangle \right\},$$

Table 5
CFN-SHS values for Colloidal stability.

$\mathfrak{S}_3$	α_1	α_2	α_3	α_4	α_5
$\rho_{\mathfrak{S}_3}^L$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.45)} \\ 0.27e^{i2\pi(0.36)} \\ 0.33e^{i2\pi(0.29)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.53e^{i2\pi(0.43)} \\ 0.28e^{i2\pi(0.37)} \\ 0.34e^{i2\pi(0.30)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.32e^{i2\pi(0.25)} \\ 0.39e^{i2\pi(0.48)} \\ 0.44e^{i2\pi(0.39)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.15e^{i2\pi(0.11)} \\ 0.55e^{i2\pi(0.63)} \\ 0.60e^{i2\pi(0.55)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.85e^{i2\pi(0.75)} \\ 0.13e^{i2\pi(0.20)} \\ 0.16e^{i2\pi(0.13)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_3}^M$	$\left\langle \begin{matrix} 0.82e^{i2\pi(0.72)} \\ 0.14e^{i2\pi(0.21)} \\ 0.17e^{i2\pi(0.14)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.80e^{i2\pi(0.70)} \\ 0.15e^{i2\pi(0.22)} \\ 0.18e^{i2\pi(0.15)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.56e^{i2\pi(0.46)} \\ 0.26e^{i2\pi(0.34)} \\ 0.31e^{i2\pi(0.27)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.54e^{i2\pi(0.44)} \\ 0.27e^{i2\pi(0.35)} \\ 0.32e^{i2\pi(0.28)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_3}^H$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.86e^{i2\pi(0.76)} \\ 0.12e^{i2\pi(0.19)} \\ 0.15e^{i2\pi(0.12)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_3}^V$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.42e^{i2\pi(0.51)} \\ 0.47e^{i2\pi(0.42)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.29e^{i2\pi(0.22)} \\ 0.43e^{i2\pi(0.52)} \\ 0.48e^{i2\pi(0.43)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.58e^{i2\pi(0.48)} \\ 0.24e^{i2\pi(0.32)} \\ 0.29e^{i2\pi(0.25)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.90e^{i2\pi(0.80)} \\ 0.09e^{i2\pi(0.15)} \\ 0.12e^{i2\pi(0.09)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.13e^{i2\pi(0.09)} \\ 0.57e^{i2\pi(0.65)} \\ 0.62e^{i2\pi(0.57)} \end{matrix} \right\rangle$

Table 6
CFN-SHS values for Synthesis/crystallographic quality

$\mathfrak{S}_4$	α_1	α_2	α_3	α_4	α_5
$\rho_{\mathfrak{S}_4}^L$	$\left\langle \begin{matrix} 0.32e^{i2\pi(0.25)} \\ 0.39e^{i2\pi(0.48)} \\ 0.44e^{i2\pi(0.39)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.14e^{i2\pi(0.10)} \\ 0.56e^{i2\pi(0.64)} \\ 0.61e^{i2\pi(0.56)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.84e^{i2\pi(0.74)} \\ 0.13e^{i2\pi(0.20)} \\ 0.16e^{i2\pi(0.13)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_4}^M$	$\left\langle \begin{matrix} 0.56e^{i2\pi(0.46)} \\ 0.26e^{i2\pi(0.34)} \\ 0.31e^{i2\pi(0.27)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.33e^{i2\pi(0.26)} \\ 0.38e^{i2\pi(0.47)} \\ 0.43e^{i2\pi(0.38)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.54e^{i2\pi(0.44)} \\ 0.27e^{i2\pi(0.35)} \\ 0.32e^{i2\pi(0.28)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.45)} \\ 0.27e^{i2\pi(0.36)} \\ 0.33e^{i2\pi(0.29)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_4}^H$	$\left\langle \begin{matrix} 0.83e^{i2\pi(0.73)} \\ 0.14e^{i2\pi(0.21)} \\ 0.17e^{i2\pi(0.14)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.59e^{i2\pi(0.49)} \\ 0.23e^{i2\pi(0.31)} \\ 0.29e^{i2\pi(0.25)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.81e^{i2\pi(0.71)} \\ 0.15e^{i2\pi(0.22)} \\ 0.18e^{i2\pi(0.15)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.80e^{i2\pi(0.70)} \\ 0.16e^{i2\pi(0.23)} \\ 0.19e^{i2\pi(0.16)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.29e^{i2\pi(0.22)} \\ 0.42e^{i2\pi(0.51)} \\ 0.47e^{i2\pi(0.42)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_4}^V$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.88e^{i2\pi(0.78)} \\ 0.10e^{i2\pi(0.17)} \\ 0.13e^{i2\pi(0.11)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.53e^{i2\pi(0.43)} \\ 0.28e^{i2\pi(0.37)} \\ 0.34e^{i2\pi(0.30)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.12e^{i2\pi(0.08)} \\ 0.58e^{i2\pi(0.66)} \\ 0.63e^{i2\pi(0.58)} \end{matrix} \right\rangle$

and

$$\aleph(\ell_2) = \left\{ \left\langle \emptyset, \xi^{\aleph(\ell_2)}(\emptyset) e^{i\vartheta_{\xi}^{\aleph(\ell_2)}(\emptyset)}, \zeta^{\aleph(\ell_2)}(\emptyset) e^{i\vartheta_{\zeta}^{\aleph(\ell_2)}(\emptyset)}, \varsigma^{\aleph(\ell_2)}(\emptyset) e^{i\vartheta_{\varsigma}^{\aleph(\ell_2)}(\emptyset)} \right\rangle \right\},$$

over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ and let $\mathfrak{S} = \{\mathfrak{S}_1, \mathfrak{S}_2, \dots, \mathfrak{S}_n\}$, for $n \geq 1$, be a set of attributes associated with $\mathcal{K}$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, 3, 4, \dots, n\}, \forall i \neq j$. Further, suppose that $\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$ denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function $\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) = \ell$. $\aleph(\ell_1)$ is known to be the CFN-SHS subset of $\aleph(\ell_2)$ if the constraints stated below are satisfied;

$$\xi^{\aleph(\ell_1)}(\emptyset) \leq \xi^{\aleph(\ell_2)}(\emptyset), \quad \vartheta_{\xi}^{\aleph(\ell_1)}(\emptyset) \leq \vartheta_{\xi}^{\aleph(\ell_2)}(\emptyset),$$

$$\zeta^{\aleph(\ell_1)}(\emptyset) \leq \zeta^{\aleph(\ell_2)}(\emptyset), \quad \vartheta_{\zeta}^{\aleph(\ell_1)}(\emptyset) \leq \vartheta_{\zeta}^{\aleph(\ell_2)}(\emptyset),$$

$$\varsigma^{\aleph(\ell_1)}(\emptyset) \geq \varsigma^{\aleph(\ell_2)}(\emptyset), \quad \vartheta_{\varsigma}^{\aleph(\ell_1)}(\emptyset) \geq \vartheta_{\varsigma}^{\aleph(\ell_2)}(\emptyset).$$

Table 7
CFN-SHS values for Biocompatibility

$\mathfrak{S}_5$	α_1	α_2	α_3	α_4	α_5
$\rho_{\mathfrak{S}_5}^L$	$\left\langle \begin{matrix} 0.54e^{i2\pi(0.44)} \\ 0.28e^{i2\pi(0.37)} \\ 0.34e^{i2\pi(0.30)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.53e^{i2\pi(0.43)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.33e^{i2\pi(0.26)} \\ 0.38e^{i2\pi(0.47)} \\ 0.43e^{i2\pi(0.38)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.14e^{i2\pi(0.10)} \\ 0.56e^{i2\pi(0.64)} \\ 0.61e^{i2\pi(0.56)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.87e^{i2\pi(0.77)} \\ 0.11e^{i2\pi(0.18)} \\ 0.14e^{i2\pi(0.11)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_5}^M$	$\left\langle \begin{matrix} 0.80e^{i2\pi(0.70)} \\ 0.16e^{i2\pi(0.23)} \\ 0.19e^{i2\pi(0.16)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.79e^{i2\pi(0.69)} \\ 0.15e^{i2\pi(0.22)} \\ 0.18e^{i2\pi(0.15)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.45)} \\ 0.27e^{i2\pi(0.35)} \\ 0.32e^{i2\pi(0.28)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.53e^{i2\pi(0.43)} \\ 0.28e^{i2\pi(0.36)} \\ 0.33e^{i2\pi(0.29)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_5}^H$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.85e^{i2\pi(0.75)} \\ 0.12e^{i2\pi(0.19)} \\ 0.15e^{i2\pi(0.13)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.56e^{i2\pi(0.46)} \\ 0.26e^{i2\pi(0.34)} \\ 0.31e^{i2\pi(0.27)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_5}^V$	$\left\langle \begin{matrix} 0.29e^{i2\pi(0.22)} \\ 0.43e^{i2\pi(0.52)} \\ 0.48e^{i2\pi(0.43)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.28e^{i2\pi(0.21)} \\ 0.44e^{i2\pi(0.53)} \\ 0.49e^{i2\pi(0.44)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.91e^{i2\pi(0.81)} \\ 0.08e^{i2\pi(0.14)} \\ 0.11e^{i2\pi(0.08)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.11e^{i2\pi(0.07)} \\ 0.59e^{i2\pi(0.67)} \\ 0.64e^{i2\pi(0.59)} \end{matrix} \right\rangle$

Table 8
CFN-SHS values for $\aleph(\ell)$

$\mathfrak{S}_5$	α_2	α_4	α_5
$\rho_{\mathfrak{S}_1}^H$	$\left\langle \begin{matrix} 0.85e^{i2\pi(0.75)} \\ 0.12e^{i2\pi(0.19)} \\ 0.15e^{i2\pi(0.12)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.60e^{i2\pi(0.50)} \\ 0.23e^{i2\pi(0.31)} \\ 0.29e^{i2\pi(0.25)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.48)} \\ 0.26e^{i2\pi(0.33)} \\ 0.31e^{i2\pi(0.34)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_3}^H$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_5}^H$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.56e^{i2\pi(0.46)} \\ 0.26e^{i2\pi(0.34)} \\ 0.31e^{i2\pi(0.27)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$

Example 4.1 Suppose we take two CFN-SHS sets $\aleph(\ell_1)$ and $\aleph(\ell_2)$ over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, then we have $\aleph(\ell_1) = \aleph(\text{High Antibacterial efficacy, High Biocompatibility}) = \{\alpha_2, \alpha_5\}$, be the CFN-SHS subset of $\aleph(\ell_2) = \aleph(\text{High Antibacterial efficacy, High Colloidal stability, High Biocompatibility}) = \{\alpha_2, \alpha_4, \alpha_5\}$, as presented in Tables 9 and 10, respectively.

Table 9
CFN-SHS values for $\aleph(\ell_1)$

$\mathfrak{S}_5$	α_2	α_5
$\rho_{\mathfrak{S}_1}^H$	$\left\langle \begin{matrix} 0.85e^{i2\pi(0.75)} \\ 0.12e^{i2\pi(0.19)} \\ 0.15e^{i2\pi(0.12)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.48)} \\ 0.26e^{i2\pi(0.33)} \\ 0.31e^{i2\pi(0.34)} \end{matrix} \right\rangle$
$\rho_{\mathfrak{S}_5}^H$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$

Definition 4.2 [CFN-SHS Equality] Let us take two CFN-SHSSs

$$\aleph(\ell_1) = \left\{ \left\langle \wp, \xi^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_1)}(\wp)}, \zeta^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp)}, \varsigma^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp)} \right\rangle \right\},$$

Table 10
CFN-SHS values for $\aleph(\ell_2)$

$\aleph_5$	α_2	α_4	α_5
$\rho_{\aleph_1}^H$	$\left\langle \begin{matrix} 0.88e^{i2\pi(0.76)} \\ 0.15e^{i2\pi(0.22)} \\ 0.14e^{i2\pi(0.11)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.63e^{i2\pi(0.56)} \\ 0.25e^{i2\pi(0.32)} \\ 0.24e^{i2\pi(0.22)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.58e^{i2\pi(0.49)} \\ 0.27e^{i2\pi(0.36)} \\ 0.29e^{i2\pi(0.32)} \end{matrix} \right\rangle$
$\rho_{\aleph_3}^H$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$
$\rho_{\aleph_5}^H$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.43)} \\ 0.32e^{i2\pi(0.39)} \\ 0.34e^{i2\pi(0.29)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.48)} \\ 0.28e^{i2\pi(0.36)} \\ 0.30e^{i2\pi(0.24)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.33e^{i2\pi(0.27)} \\ 0.44e^{i2\pi(0.50)} \\ 0.42e^{i2\pi(0.38)} \end{matrix} \right\rangle$

and

$$\aleph(\ell_2) = \left\{ \left\langle \emptyset, \xi^{\aleph(\ell_2)}(\emptyset) e^{i\vartheta_{\xi}^{\aleph(\ell_2)}(\emptyset)}, \zeta^{\aleph(\ell_2)}(\emptyset) e^{i\vartheta_{\zeta}^{\aleph(\ell_2)}(\emptyset)}, \varsigma^{\aleph(\ell_2)}(\emptyset) e^{i\vartheta_{\varsigma}^{\aleph(\ell_2)}(\emptyset)} \right\rangle \right\},$$

over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ and let $\aleph = \{\aleph_1, \aleph_2, \dots, \aleph_n\}$, for $n \geq 1$, be a set of attributes associated with $\mathcal{K}$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, 3, 4, \dots, n\}, \forall i \neq j$. Further, suppose that $\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$, denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function $\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) = \ell$. $\aleph(\ell_1)$ is known to be the CFN-SHS equal to $\aleph(\ell_2)$ if the constraints stated below are satisfied;

$$\begin{aligned} \xi^{\aleph(\ell_1)}(\emptyset) &= \xi^{\aleph(\ell_2)}(\emptyset), \quad \vartheta_{\xi}^{\aleph(\ell_1)}(\emptyset) = \vartheta_{\xi}^{\aleph(\ell_2)}(\emptyset), \\ \zeta^{\aleph(\ell_1)}(\emptyset) &= \zeta^{\aleph(\ell_2)}(\emptyset), \quad \vartheta_{\zeta}^{\aleph(\ell_1)}(\emptyset) = \vartheta_{\zeta}^{\aleph(\ell_2)}(\emptyset), \\ \varsigma^{\aleph(\ell_1)}(\emptyset) &= \varsigma^{\aleph(\ell_2)}(\emptyset), \quad \vartheta_{\varsigma}^{\aleph(\ell_1)}(\emptyset) = \vartheta_{\varsigma}^{\aleph(\ell_2)}(\emptyset). \end{aligned}$$

Example 4.2 Suppose we take two CFN-SHS sets $\aleph(\ell_1)$ and $\aleph(\ell_2)$ over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, then we have $\aleph(\ell_1) = \aleph(\text{High Antibacterial efficacy, High Biocompatibility}) = \{\alpha_2, \alpha_5\}$, be the CFN-SHS equal to $\aleph(\ell_2) = \aleph(\text{High Antibacterial efficacy, High Colloidal stability, High Biocompatibility}) = \{\alpha_2, \alpha_4, \alpha_5\}$, as presented in Tables 11 and 12, respectively.

Table 11
CFN-SHS values for $\aleph(\ell_1)$

$\aleph_5$	α_2	α_5
$\rho_{\aleph_1}^H$	$\left\langle \begin{matrix} 0.85e^{i2\pi(0.75)} \\ 0.12e^{i2\pi(0.19)} \\ 0.15e^{i2\pi(0.12)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.48)} \\ 0.26e^{i2\pi(0.33)} \\ 0.31e^{i2\pi(0.34)} \end{matrix} \right\rangle$
$\rho_{\aleph_5}^H$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$

Definition 4.3 [CFN-SHS Null Set] Let us take a CFN-SHSSs

$$\aleph(\ell) = \left\{ \left\langle \emptyset, \xi^{\aleph(\ell)}(\emptyset) e^{i\vartheta_{\xi}^{\aleph(\ell)}(\emptyset)}, \zeta^{\aleph(\ell)}(\emptyset) e^{i\vartheta_{\zeta}^{\aleph(\ell)}(\emptyset)}, \varsigma^{\aleph(\ell)}(\emptyset) e^{i\vartheta_{\varsigma}^{\aleph(\ell)}(\emptyset)} \right\rangle \right\},$$

over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ and let $\aleph = \{\aleph_1, \aleph_2, \dots, \aleph_n\}$, for $n \geq 1$, be a set of attributes associated with $\mathcal{K}$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, 3, 4, \dots, n\}, \forall i \neq j$. Further, suppose that

Table 12
CFN-SHS values for $\aleph(\ell_2)$

$\aleph_5$	α_2	α_4	α_5
$\rho_{\aleph_1}^H$	$\begin{pmatrix} 0.85e^{i2\pi(0.75)} \\ 0.12e^{i2\pi(0.19)} \\ 0.15e^{i2\pi(0.12)} \end{pmatrix}$	$\begin{pmatrix} 0.60e^{i2\pi(0.50)} \\ 0.23e^{i2\pi(0.31)} \\ 0.29e^{i2\pi(0.25)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.48)} \\ 0.26e^{i2\pi(0.33)} \\ 0.31e^{i2\pi(0.34)} \end{pmatrix}$
$\rho_{\aleph_3}^H$	$\begin{pmatrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{pmatrix}$	$\begin{pmatrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{pmatrix}$
$\rho_{\aleph_5}^H$	$\begin{pmatrix} 0.52e^{i2\pi(0.42)} \\ 0.29e^{i2\pi(0.38)} \\ 0.35e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.46)} \\ 0.26e^{i2\pi(0.34)} \\ 0.31e^{i2\pi(0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.31e^{i2\pi(0.24)} \\ 0.40e^{i2\pi(0.49)} \\ 0.45e^{i2\pi(0.40)} \end{pmatrix}$

$\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$ denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function $\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) = \ell$. $\aleph(\ell_1)$ is known to be the CFN-SHS null set if the constraints stated below are satisfied;

$$\begin{aligned} \xi^{\aleph(\ell)}(\wp) &= 0, \quad \vartheta_{\xi}^{\aleph(\ell)}(\wp) = 0, \\ \zeta^{\aleph(\ell)}(\wp) &= 0, \quad \vartheta_{\zeta}^{\aleph(\ell)}(\wp) = 0, \\ \varsigma^{\aleph(\ell)}(\wp) &= 0, \quad \vartheta_{\varsigma}^{\aleph(\ell)}(\wp) = 0. \end{aligned}$$

Example 4.3 Suppose we take a CFN-SHS set $\aleph(\ell)$ over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, then we have $\aleph(\ell) = \aleph(\text{High Antibacterial efficacy, High Biocompatibility}) = \{\alpha_2, \alpha_5\}$, be the CFN-SHS null set, as presented in Tables 13.

Table 13
CFN-SHS values for $\aleph(\ell_1)$

$\aleph_5$	α_2	α_5
$\rho_{\aleph_1}^H$	$\begin{pmatrix} 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \end{pmatrix}$	$\begin{pmatrix} 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \end{pmatrix}$
$\rho_{\aleph_5}^H$	$\begin{pmatrix} 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \end{pmatrix}$	$\begin{pmatrix} 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \\ 0e^{i2\pi(0)} \end{pmatrix}$

Definition 4.4 [CFN-SHS Complement] Let us take a CFN-SHSSs

$$\aleph(\ell) = \left\{ \left\langle \wp, \xi^{\aleph(\ell)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell)}(\wp)}, \zeta^{\aleph(\ell)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell)}(\wp)}, \varsigma^{\aleph(\ell)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell)}(\wp)} \right\rangle \right\},$$

over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ and let $\aleph = \{\aleph_1, \aleph_2, \dots, \aleph_n\}$, for $n \geq 1$, be a set of attributes associated with $\aleph$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, 3, 4, \dots, n\}, \forall i \neq j$. Further, suppose that $\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$ denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function $\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) = \ell$. $\aleph^C(\ell)$ is known to be the CFN-SHS complement of $\aleph(\ell)$ if the constraints stated below are satisfied;

$$\begin{aligned} \xi^{\aleph(\ell)}(\wp) &= \varsigma^{\aleph(\ell)}(\wp), \quad \vartheta_{\xi}^{\aleph(\ell)}(\wp) = \vartheta_{\varsigma}^{\aleph(\ell)}(\wp), \\ \zeta^{\aleph(\ell)}(\wp) &= 1 - \xi^{\aleph(\ell)}(\wp), \quad \vartheta_{\zeta}^{\aleph(\ell)}(\wp) = 1 - \vartheta_{\xi}^{\aleph(\ell)}(\wp), \\ \varsigma^{\aleph(\ell)}(\wp) &= \xi^{\aleph(\ell)}(\wp), \quad \vartheta_{\varsigma}^{\aleph(\ell)}(\wp) = \vartheta_{\xi}^{\aleph(\ell)}(\wp). \end{aligned}$$

Example 4.4 Suppose we take a CFN-SHS set $\aleph(\ell)$ over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, then we have $\aleph(\ell) = \aleph(\text{High Antibacterial efficacy, High Colloidal stability, High Biocompatibility}) = \{\alpha_2, \alpha_4, \alpha_5\}$, as presented in Table 10 then its complement $\aleph^C(\ell)$ is given in the Table 14.

Table 14
CFN-SHS values for $\aleph^C(\ell)$

$\aleph_5$	α_2	α_4	α_5
$\rho_{\aleph_1}^H$	$\left\langle \begin{matrix} 0.14e^{i2\pi(0.11)} \\ 0.85e^{i2\pi(0.78)} \\ 0.88e^{i2\pi(0.76)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.24e^{i2\pi(0.22)} \\ 0.75e^{i2\pi(0.68)} \\ 0.63e^{i2\pi(0.56)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.29e^{i2\pi(0.32)} \\ 0.73e^{i2\pi(0.64)} \\ 0.58e^{i2\pi(0.49)} \end{matrix} \right\rangle$
$\rho_{\aleph_3}^H$	$\left\langle \begin{matrix} 0.36e^{i2\pi(0.32)} \\ 0.70e^{i2\pi(0.61)} \\ 0.51e^{i2\pi(0.41)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.26)} \\ 0.75e^{i2\pi(0.67)} \\ 0.57e^{i2\pi(0.47)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.46e^{i2\pi(0.41)} \\ 0.59e^{i2\pi(0.50)} \\ 0.30e^{i2\pi(0.23)} \end{matrix} \right\rangle$
$\rho_{\aleph_5}^H$	$\left\langle \begin{matrix} 0.34e^{i2\pi(0.29)} \\ 0.68e^{i2\pi(0.61)} \\ 0.55e^{i2\pi(0.43)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.24)} \\ 0.72e^{i2\pi(0.64)} \\ 0.57e^{i2\pi(0.48)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.42e^{i2\pi(0.38)} \\ 0.56e^{i2\pi(0.50)} \\ 0.33e^{i2\pi(0.27)} \end{matrix} \right\rangle$

Definition 4.5 [CFN-SHS Union] Let us take two CFN-SHSs

$$\aleph(\ell_1) = \left\{ \left\langle \wp, \xi^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_1)}(\wp)}, \zeta^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp)}, \varsigma^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp)} \right\rangle \right\},$$

and

$$\aleph(\ell_2) = \left\{ \left\langle \wp, \xi^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_2)}(\wp)}, \zeta^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_2)}(\wp)}, \varsigma^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_2)}(\wp)} \right\rangle \right\},$$

over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ and let $\aleph = \{\aleph_1, \aleph_2, \dots, \aleph_n\}$, for $n \geq 1$, be a set of attributes associated with $\aleph$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, 3, 4, \dots, n\}, \forall i \neq j$. Further, suppose that $\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$ denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function $\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) = \ell$. The CFN-SHS union between $\aleph(\ell_1)$ and $\aleph(\ell_2)$ is given by;

$$\aleph(\ell_1) \cup \aleph(\ell_2) = \left\{ \left\langle \wp, \xi^{\aleph(\ell)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell)}(\wp)}, \zeta^{\aleph(\ell)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell)}(\wp)}, \varsigma^{\aleph(\ell)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell)}(\wp)} \right\rangle \right\}$$

where,

$$\xi^{\aleph(\ell)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell)}(\wp)} = \begin{cases} \xi^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_1)}(\wp)}, & \text{if } \wp \in \aleph(\ell_1) \\ \xi^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_2)}(\wp)}, & \text{if } \wp \in \aleph(\ell_2) \\ \max(\xi^{\aleph(\ell_1)}(\wp), \xi^{\aleph(\ell_2)}(\wp)) e^{i \max(\vartheta_{\xi}^{\aleph(\ell_1)}(\wp), \vartheta_{\xi}^{\aleph(\ell_2)}(\wp))} & \text{if } \wp \in \aleph(\ell_1) \cap \aleph(\ell_2), \end{cases}$$

$$\zeta^{\aleph(\ell)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell)}(\wp)} = \begin{cases} \zeta^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp)}, & \text{if } \wp \in \aleph(\ell_1) \\ \zeta^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_2)}(\wp)}, & \text{if } \wp \in \aleph(\ell_2) \\ \min(\zeta^{\aleph(\ell_1)}(\wp), \zeta^{\aleph(\ell_2)}(\wp)) e^{i \min(\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp), \vartheta_{\zeta}^{\aleph(\ell_2)}(\wp))} & \text{if } \wp \in \aleph(\ell_1) \cap \aleph(\ell_2), \end{cases}$$

$$\varsigma^{\aleph(\ell)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell)}(\wp)} = \begin{cases} \varsigma^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp)}, & \text{if } \wp \in \aleph(\ell_1) \\ \varsigma^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_2)}(\wp)}, & \text{if } \wp \in \aleph(\ell_2) \\ \min(\varsigma^{\aleph(\ell_1)}(\wp), \varsigma^{\aleph(\ell_2)}(\wp)) e^{i \min(\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp), \vartheta_{\varsigma}^{\aleph(\ell_2)}(\wp))} & \text{if } \wp \in \aleph(\ell_1) \cap \aleph(\ell_2), \end{cases}$$

Example 4.5 Suppose we take two CFN-SHS sets $\aleph(\ell_1)$ and $\aleph(\ell_2)$ over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, then we have $\aleph(\ell_1) = \aleph(\text{High Antibacterial efficacy, High Biocompatibility}) = \{\alpha_2, \alpha_5\}$, and $\aleph(\ell_2) = \aleph(\text{High Antibacterial efficacy, High Colloidal stability, High Biocompatibility}) = \{\alpha_2, \alpha_4, \alpha_5\}$, as presented in Tables 9 and 10, respectively. Then the CFN-SHS union $\aleph(\ell_1) \cup \aleph(\ell_2)$ is given in Table 15.

Table 15
CFN-SHS values for $\aleph(\ell_1) \cup \aleph(\ell_2)$

$\aleph_5$	α_2	α_4	α_5
$\rho_{\aleph_1}^H$	$\left\langle \begin{matrix} 0.88e^{i2\pi(0.76)} \\ 0.12e^{i2\pi(0.19)} \\ 0.14e^{i2\pi(0.11)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.63e^{i2\pi(0.56)} \\ 0.25e^{i2\pi(0.32)} \\ 0.24e^{i2\pi(0.22)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.58e^{i2\pi(0.49)} \\ 0.26e^{i2\pi(0.33)} \\ 0.29e^{i2\pi(0.32)} \end{matrix} \right\rangle$
$\rho_{\aleph_3}^H$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$
$\rho_{\aleph_5}^H$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.43)} \\ 0.29e^{i2\pi(0.38)} \\ 0.34e^{i2\pi(0.29)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.48)} \\ 0.28e^{i2\pi(0.36)} \\ 0.30e^{i2\pi(0.24)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.33e^{i2\pi(0.27)} \\ 0.40e^{i2\pi(0.49)} \\ 0.42e^{i2\pi(0.38)} \end{matrix} \right\rangle$

Definition 4.6 [CFN-SHS Intersection] Let us take two CFN-SHSSs

$$\aleph(\ell_1) = \left\{ \left\langle \wp, \xi^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_1)}(\wp)}, \zeta^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp)}, \varsigma^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp)} \right\rangle \right\},$$

and

$$\aleph(\ell_2) = \left\{ \left\langle \wp, \xi^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_2)}(\wp)}, \zeta^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_2)}(\wp)}, \varsigma^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_2)}(\wp)} \right\rangle \right\},$$

over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ and let $\aleph = \{\aleph_1, \aleph_2, \dots, \aleph_n\}$, for $n \geq 1$, be a set of attributes associated with $\mathcal{K}$. For each attribute, assign a corresponding attribute-value set $\{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$, chosen so that $\rho_i \cap \rho_j = \emptyset$, such that $i, j \in \{1, 2, 3, 4, \dots, n\}, \forall i \neq j$. Further, suppose that $\mathcal{P}(\rho_1), \mathcal{P}(\rho_2), \mathcal{P}(\rho_3), \dots, \mathcal{P}(\rho_n)$ denote the respective power sets of $\rho_1, \rho_2, \rho_3, \dots, \rho_n$, and define the multi-argument function $\mathcal{P}(\rho_1) \times \mathcal{P}(\rho_2) \times \mathcal{P}(\rho_3) \times \dots \times \mathcal{P}(\rho_n) = \ell$. The CFN-SHS intersection between $\aleph(\ell_1)$ and $\aleph(\ell_2)$ is given by;

$$\aleph(\ell_1) \cap \aleph(\ell_2) = \left\{ \left\langle \wp, \xi^{\aleph(\ell)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell)}(\wp)}, \zeta^{\aleph(\ell)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell)}(\wp)}, \varsigma^{\aleph(\ell)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell)}(\wp)} \right\rangle \right\}$$

where,

$$\xi^{\aleph(\ell)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell)}(\wp)} = \begin{cases} \xi^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_1)}(\wp)}, & \text{if } \wp \in \aleph(\ell_1) \\ \xi^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\xi}^{\aleph(\ell_2)}(\wp)}, & \text{if } \wp \in \aleph(\ell_2) \\ \min(\xi^{\aleph(\ell_1)}(\wp), \xi^{\aleph(\ell_2)}(\wp)) e^{i \min(\vartheta_{\xi}^{\aleph(\ell_1)}(\wp), \vartheta_{\xi}^{\aleph(\ell_2)}(\wp))} & \text{if } \wp \in \aleph(\ell_1) \cap \aleph(\ell_2), \end{cases}$$

$$\zeta^{\aleph(\ell)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell)}(\wp)} = \begin{cases} \zeta^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp)}, & \text{if } \wp \in \aleph(\ell_1) \\ \zeta^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\zeta}^{\aleph(\ell_2)}(\wp)}, & \text{if } \wp \in \aleph(\ell_2) \\ \max(\zeta^{\aleph(\ell_1)}(\wp), \zeta^{\aleph(\ell_2)}(\wp)) e^{i \max(\vartheta_{\zeta}^{\aleph(\ell_1)}(\wp), \vartheta_{\zeta}^{\aleph(\ell_2)}(\wp))} & \text{if } \wp \in \aleph(\ell_1) \cap \aleph(\ell_2), \end{cases}$$

$$\varsigma^{\aleph(\ell)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell)}(\wp)} = \begin{cases} \varsigma^{\aleph(\ell_1)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp)}, & \text{if } \wp \in \aleph(\ell_1) \\ \varsigma^{\aleph(\ell_2)}(\wp) e^{i\vartheta_{\varsigma}^{\aleph(\ell_2)}(\wp)}, & \text{if } \wp \in \aleph(\ell_2) \\ \max(\varsigma^{\aleph(\ell_1)}(\wp), \varsigma^{\aleph(\ell_2)}(\wp)) e^{i \max(\vartheta_{\varsigma}^{\aleph(\ell_1)}(\wp), \vartheta_{\varsigma}^{\aleph(\ell_2)}(\wp))} & \text{if } \wp \in \aleph(\ell_1) \cap \aleph(\ell_2), \end{cases}$$

Example 4.6 Suppose we take two CFN-SHS sets $\aleph(\ell_1)$ and $\aleph(\ell_2)$ over $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, then we have $\aleph(\ell_1) = \aleph(\text{High Antibacterial efficacy, High Biocompatibility}) = \{\alpha_2, \alpha_5\}$, and $\aleph(\ell_2) = \aleph(\text{High Antibacterial efficacy, High Colloidal stability, High Biocompatibility}) = \{\alpha_2, \alpha_4, \alpha_5\}$, as presented in Tables 9 and 10, respectively. Then the CFN-SHS intersection $\aleph(\ell_1) \cap \aleph(\ell_2)$ is given in Table 16.

5. Conclusion

This research article developed the notion of complex Fermatean neutrosophic hypersoft set and its power-set-indexed extension, the complex Fermatean neutrosophic super-hypersoft set, general-

Table 16
CFN-SHS values for $\aleph(\ell_1) \cap \aleph(\ell_2)$

$\aleph_5$	α_2	α_4	α_5
$\rho_{\aleph_1}^H$	$\left\langle \begin{matrix} 0.85e^{i2\pi(0.75)} \\ 0.15e^{i2\pi(0.22)} \\ 0.15e^{i2\pi(0.12)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.63e^{i2\pi(0.56)} \\ 0.25e^{i2\pi(0.32)} \\ 0.24e^{i2\pi(0.22)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.55e^{i2\pi(0.48)} \\ 0.27e^{i2\pi(0.36)} \\ 0.31e^{i2\pi(0.34)} \end{matrix} \right\rangle$
$\rho_{\aleph_3}^H$	$\left\langle \begin{matrix} 0.51e^{i2\pi(0.41)} \\ 0.30e^{i2\pi(0.39)} \\ 0.36e^{i2\pi(0.32)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.47)} \\ 0.25e^{i2\pi(0.33)} \\ 0.30e^{i2\pi(0.26)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.30e^{i2\pi(0.23)} \\ 0.41e^{i2\pi(0.50)} \\ 0.46e^{i2\pi(0.41)} \end{matrix} \right\rangle$
$\rho_{\aleph_5}^H$	$\left\langle \begin{matrix} 0.52e^{i2\pi(0.42)} \\ 0.32e^{i2\pi(0.39)} \\ 0.35e^{i2\pi(0.31)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.57e^{i2\pi(0.48)} \\ 0.28e^{i2\pi(0.36)} \\ 0.30e^{i2\pi(0.24)} \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 0.31e^{i2\pi(0.24)} \\ 0.44e^{i2\pi(0.50)} \\ 0.45e^{i2\pi(0.40)} \end{matrix} \right\rangle$

izing the cubic constraint of fermatean neutrosophic logic into a phase-carrying, multi-attribute structure. By embedding the amplitude–phase term with the truth, indeterminacy, and falsity grades, the suggested formulation addresses an extra independent dimension of decision that classical real-valued or interval-valued neutrosophic hypersoft frameworks are unable to capture. The basic algebraic structures were established and elaborated through extensive numerical examples, and the formulation was implemented on a practical and significant issue of selecting the optimal green-synthesized silver nanoparticle (AgNP) formulations against various criteria which are antibacterial efficacy, antifungal efficacy, colloidal stability, synthesis quality, and biocompatibility, along with their gradual sub-criteria. The resulting model provided a systematic and structured way to reconcile these competing signals into a single, transparent ranking, elaborating that the CFN-SHS sets are not merely a theoretical enrichment but a practically usable decision-support instrument for nanomaterial researchers operating under genuine uncertainty and can be effectively applied on several other broader fields too.

5.1 Future Directions

Several future prospects remain open for researchers in this field of study.

- Theoretically, score and accuracy functions, similarity and distance measures, weighted and ordered aggregation operators would really strengthen the approach's ranking ability beyond the set-theoretic operations presented here. The model could also be integrated with the developed efficient MCDM schemes like TOPSIS, VIKOR, or CoCoSo to produce a comprehensive decision-making algorithm and generalized to hesitant or interval-valued approaches for settings where even the amplitude and phase components are only approximately known.
- In the applied side, the illustrative case study employed representative numerical values for AgNP criteria, future studies could explore the use of measured antimicrobial, cytotoxicity, and stability data, evaluated through a clear, reproducible process, to assess the framework against real laboratory results. Furthermore, the case study could be implemented on a wide spectrum of application domains other than the biomedical and agricultural settings considered here. More broadly, the complex Fermatean neutrosophic super-hypersoft model could be useful beyond the nanomaterial prioritization, offering a realistic fit for any decision-making challenge – in healthcare, supply chain management, or environmental assessment – where uncertainty is cubic-bounded, three-valued, multi-attributed, and carries a genuinely periodic or comparative second dimension that real-valued models leave unaddressed.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The authors also declare that there is no conflict of interests regarding the publication of this paper. The authors declare that they have no conflict of interest.

Data Availability

All data is in the manuscript that were used to support this study.

Funding

No funding received for this study.

Acknowledgement

This research was not funded by any grant.

References

- [1] Mardani, A., Jusoh, A., & Zavadskas, E. K. (2015). Fuzzy multiple criteria decision-making techniques and applications—Two decades review from 1994 to 2014. *Expert Systems with Applications*, 42(8), 4126–4148. <https://doi.org/10.1016/j.eswa.2015.01.003>
- [2] Sahoo, S. K., & Goswami, S. S. (2023). A comprehensive review of multiple criteria decision-making (MCDM) methods: Advancements, applications, and future directions. *Decision Making Advances*, 1(1), 25–48. <https://doi.org/10.31181/dma1120237>
- [3] Duman, H., Eker, F., Akdaşçi, E., Witkowska, A. M., Bechelany, M., & Karav, S. (2024). Silver nanoparticles: A comprehensive review of synthesis methods and chemical and physical properties. *Nanomaterials*, 14(18), 1527. <https://doi.org/10.3390/nano14181527>
- [4] Eker, F., Akdaşçi, E., Duman, H., Bechelany, M., & Karav, S. (2025). Green synthesis of silver nanoparticles using plant extracts: A comprehensive review of physicochemical properties and multifunctional applications. *International Journal of Molecular Sciences*, 26(13), 6222. <https://doi.org/10.3390/ijms26136222>
- [5] Uzair, A., Ishtiaq, A., & Rasheed, M. (2024). Non-kekulean benzenoid hydrocarbon: Physico-chemical properties for benzenoid hydrocarbon using topological indices and m-polynomial. *Computational Algorithms and Numerical Dimensions*, 3(2), 94–114. <https://doi.org/10.22105/cand.2024.470538.1100>
- [6] Rezaei, A., Oner, T., Katican, T., Smarandache, F., & Gandotra, N. (2022). *A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets*. Infinite Study.
- [7] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- [8] Bin-Mohsin, B., Rafique, S., Cesarano, C., Javed, M. Z., Awan, M. U., Kashuri, A., & Noor, M. A. (2022). Some general fractional integral inequalities involving LR-bi-convex fuzzy interval-valued functions. *Fractal and Fractional*, 6(10), 565. <https://doi.org/10.3390/fractalfract6100565>
- [9] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96. [https://doi.org/10.1016/S0165-0114\(86\)80034-3](https://doi.org/10.1016/S0165-0114(86)80034-3)
- [10] Ishtiaq, A., Tul Kubra, K., Uzair, A., & Ali, A. (2025). Quantifying multi-cause psychological disorder risk through an advanced mathematical model using intuitionistic pentagonal fuzzy logic. *Spectrum of Operational Research*, 1–21. <https://doi.org/10.31181/sor41202762>
- [11] Yager, R. R. (2013). Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)* (pp. 57–61). IEEE. <https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375>
- [12] Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 25(5), 1222–1230. <https://doi.org/10.1109/TFUZZ.2016.2604005>
- [13] Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy sets. *Journal of Ambient Intelligence and Humanized Computing*, 11(2), 663–674. <https://doi.org/10.1007/s12652-019-01377-0>
- [14] Smarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic: Analytic synthesis and synthetic analysis*. American Research Press.
- [15] Broumi, S., Mohanaselvi, S., Witczak, T., Talea, M., Bakali, A., & Smarandache, F. (2023). Complex Fermatean neutrosophic graph and application to decision making. *Decision Making: Applications in Management and Engineering*, 6(1), 474–501. <https://doi.org/10.31181/dmame24022023b>
- [16] Ramot, D., Milo, R., Friedman, M., & Kandel, A. (2002). Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 10(2), 171–186. <https://doi.org/10.1109/91.995119>
- [17] Ali, M., & Smarandache, F. (2017). Complex neutrosophic set. *Neural Computing and Applications*, 28(7), 1817–1834. <https://doi.org/10.1007/s00521-015-2154-y>
- [18] Broumi, S., Bakali, A., Smarandache, F., Talea, M., Ali, M., & Selvachandran, G. (2017). Complex neutrosophic soft set. In *2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* (pp. 1–6). IEEE. <https://doi.org/10.5281/zenodo.888849>
- [19] Kamran, M., Nadeem, M., Żywiołek, J., Abdalla, M. E. M., Uzair, A., & Ishtiaq, A. (2024). Enhancing transportation

- p efficiency with interval-valued Fermatean neutrosophic numbers: A multi-item optimization approach.
- Symmetry*
- , 16(6), 766.
- <https://doi.org/10.3390/sym16060766>
- [20] Antony Crispin Sweetey, C., & Jansi, R. (2021). Fermatean neutrosophic sets. *International Journal of Advanced Research in Computer and Communication Engineering*, 10(6), 24–27.
- [21] Ishtiaq, A., Kubra, K. T., Uzair, A., & Azhar, M. T. (2026). Exploring interval-valued Fermatean neutrosophic tactics for empowering AI-driven financial risk frameworks: Compliance automation, fraud detection, and beyond. *Journal of Fuzzy Systems and Control*, 4(1), 47–57. <https://doi.org/10.59247/jfsc.v4i1.357>
- [22] Molodtsov, D. (1999). Soft set theory—First results. *Computers & Mathematics with Applications*, 37(4–5), 19–31. [https://doi.org/10.1016/S0898-1221\(99\)00056-5](https://doi.org/10.1016/S0898-1221(99)00056-5)
- [23] Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets and Systems*, 22, 168–170. <https://doi.org/10.5281/zenodo.2159754>
- [24] Smarandache, F. (2023). Foundation of the SuperHyperSoft set and the fuzzy extension SuperHyperSoft set: A new vision. *Neutrosophic Systems with Applications*, 11, 48–51. <https://doi.org/10.61356/j.nswa.2023.95>
- [25] Smarandache, F., Gayathri, P., Karuppusamy, E., Krishnaprakash, S., & Gomathi, S. (2025). Optimizing electric vehicle selection using neutrosophic SuperHyperSoft set theory. *Neutrosophic Sets and Systems*, 86(1), 710–729. <https://doi.org/10.5281/zenodo.15514527>
- [26] Xue, X., Dhumras, H., Thakur, G., Bajaj, R. K., & Shukla, V. (2025). Schweizer-Sklar T-norm operators for picture fuzzy hypersoft sets: Advancing sustainable technology in social healthy environments. *Computers, Materials & Continua*, 84(1), 583–606. <https://doi.org/10.32604/cmc.2025.066310>
- [27] Kiruthika, T., Karpagadevi, M., Krishnaprakash, S., & Deepa, G. (2025). SuperHyperSoft sets using Python and its applications in neutrosophic SuperHyperSoft sets under TOPSIS method. *Neutrosophic Sets and Systems*, 91(1), 586–616. <https://doi.org/10.5281/zenodo.16884384>
- [28] Abdalla, M. E. M., Uzair, A., Ishtiaq, A., Tahir, M., & Kamran, M. (2025). Algebraic structures and practical implications of interval-valued Fermatean neutrosophic super hypersoft sets in healthcare. *Spectrum of Operational Research*, 2(1), 240–259. <https://doi.org/10.31181/sor21202523>