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Evaluating E-Commerce Websites under Uncertainty Using a Hybrid Fuzzy Knowledge–Entropy–TODIM Framework

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ABSTRACT

Assessing the reliability of e-commerce platforms is a challenging task due to the large number of interrelated evaluation criteria and the uncertainty inherent in decision-making data. To address this issue, this study proposes a novel decision-support framework that integrates Triangular Fuzzy Numbers (TFNs), the entropy weighting method, and the TODIM approach. TFNs are employed to model uncertainty in expert assessments, the entropy method determines the objective importance of the evaluation criteria, and TODIM is used to rank the e-commerce platforms. A case study involving five international e-commerce platforms evaluated against eleven criteria demonstrates the applicability of the proposed framework. Its performance is further validated through comparisons with several well-established multi-criteria decision-making (MCDM) methods. In addition, sensitivity analysis using two different approaches confirms the robustness and reliability of the obtained rankings. The results demonstrate that the proposed framework provides consistent and reliable rankings, making it an effective decision-support tool for evaluating the reliability of e-commerce platforms.

1. Introduction

Zadeh[1] was the first to provide a mathematical framework for representing uncertainty, vagueness, and imprecision through the theory of fuzzy sets. In classical set theory, an element either belongs to a set or does not belong to it. However, such a binary representation is often too restrictive for describing real-world problems, where uncertainty and ambiguity are inherent. To overcome this

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limitation, Zadeh introduced the concept of fuzzy sets, in which each element is assigned a membership degree that expresses the extent to which it belongs to a given set. Since then, fuzzy set theory has found widespread applications in numerous fields, including decision-making, pattern recognition, control systems, artificial intelligence, and computational intelligence. Its primary advantage lies in its ability to model human reasoning, handle incomplete and imprecise information, and support more reliable decision-making in complex real-world environments.

Zadeh[2] subsequently introduced the concept of the Fuzzy Information Measure (FIM) to quantify the uncertainty inherent in fuzzy information. Following this pioneering work, numerous researchers further investigated FIM and extended its application to a wide range of practical problems. Over time, various definitions of FIM have been proposed based on different theoretical viewpoints and application requirements. For instance, some researchers developed exponential forms of FIM[3], whereas others introduced parameterized measures that depend on one or more adjustable parameters [4], [5], [6], [7]. However, many of these measures do not satisfy the fundamental properties required for fuzzy information measures, while several parameterized approaches satisfy these properties only for specific parameter values. Consequently, their practical applicability is limited.

In contrast, the amount of useful information contained in fuzzy data can be quantified using the Fuzzy Knowledge Measure (FKM) [8], [9], [10], [11], [12]. In its normalized form, FKM is regarded as the dual of FIM. Numerous studies have shown that a reduction in uncertainty is accompanied by an increase in knowledge. Since FKM focuses on measuring useful information rather than uncertainty itself, it often provides more informative and meaningful results for analysis and decision-making. Consequently, a variety of fuzzy knowledge measures have been developed based on different theoretical foundations, including information theory, entropy theory, and other mathematical frameworks.

Motivated by these observations, this study proposes a novel Fuzzy Knowledge Measure (FKM) together with its fundamental mathematical properties. Several numerical examples are provided to demonstrate that the proposed FKM overcomes the limitations of existing fuzzy knowledge measures while satisfying the desirable theoretical properties of fuzzy information measures. Furthermore, based on the proposed FKM, a corresponding Fuzzy Ambiguity Measure (FAM) is derived to further extend its applicability in uncertainty modelling and decision-making.

Nowadays, e-commerce websites have transformed numerous sectors of the economy, including education [13], healthcare, retail, entertainment [14], business [14], e-government, e-banking, and financial services [15]. This rapid digital transformation has significantly changed consumer purchasing behaviour and improved the accessibility of products and services. The widespread adoption of e-commerce was further accelerated by the COVID-19 pandemic, which fundamentally changed the way consumers interact with online platforms [16]. Today, customers can purchase products ranging from everyday household items to industrial equipment through thousands of online platforms. This unprecedented growth has been enabled by the rapid expansion of Internet connectivity and the increasing use of smartphones [17].

Evaluating the quality of e-commerce websites is a complex multi-criteria decision-making problem because numerous interrelated factors simultaneously influence users' overall experience. Website performance depends on various criteria, including loading speed, security, information quality, usability, customer service, and user satisfaction [18], [19]. Consequently, several MCDM techniques, such as the Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS), have been widely employed to evaluate e-commerce platforms [20], [21], [22]. However, these methods often have difficulty modelling the uncertainty, ambiguity, and inconsistency inherent in expert judgments, particularly in rapidly evolving e-commerce environments [20].

Despite the considerable progress achieved in this area, several challenges remain. Existing website evaluation models often pay limited attention to customer satisfaction and subjective user perceptions. Furthermore, conventional MCDM methods cannot adequately represent uncertainty, decision-maker bias, and conflicting expert opinions. Although fuzzy MCDM approaches improve the modelling of uncertainty, they frequently involve high computational complexity and face difficulties in integrating subjective assessments with objective information. In addition, traditional fuzzy models are generally designed for static decision environments and therefore have limited capability to accommodate continuously changing and real-time information generated by modern e-commerce platforms.

Multi-criteria decision-making (MCDM) provides a systematic framework for selecting the most appropriate alternative while simultaneously considering multiple, often conflicting, evaluation criteria. A typical MCDM procedure consists of three stages. The first stage involves constructing the decision matrix by identifying the alternatives, evaluation criteria, and the corresponding assessment data. The second stage determines the relative importance of the evaluation criteria through an appropriate weighting method. Finally, a suitable ranking method is employed to identify the most preferable alternative. Since the reliability of the final ranking strongly depends on the quality of the criterion weights and the decision model, selecting appropriate weighting and ranking techniques is of paramount importance. However, the presence of uncertainty, vagueness, and conflicting expert opinions often reduces the effectiveness of conventional decision-making methods when solving complex real-world problems [20], [23].

Motivated by these challenges, this study proposes an integrated TFN–Entropy–TODIM framework for evaluating the reliability of e-commerce websites under uncertain decision environments. Triangular Fuzzy Numbers (TFNs) are employed to model uncertainty and linguistic assessments provided by experts. The entropy weighting method is used to determine objective criterion weights, while the TODIM method, developed on the principles of Prospect Theory, is adopted to rank the alternatives by simultaneously considering gains and losses. The proposed framework effectively models uncertain, incomplete, and conflicting evaluation information, providing a reliable and practical decision-support tool for assessing e-commerce websites.

1.1 Motivation and Contribution

Although numerous fuzzy information measures and fuzzy knowledge measures have been proposed in the literature, several important limitations remain unresolved. Existing fuzzy knowledge measures may assign identical knowledge values to different fuzzy sets, resulting in insufficient discrimination capability. Moreover, many existing fuzzy information and fuzzy knowledge measures fail to preserve the linguistic ordering of fuzzy information. Current fuzzy accuracy measures may also generate identical similarity values for different fuzzy patterns, leading to ambiguous pattern recognition results.

To overcome these limitations, this paper makes the following contributions:

- A novel Fuzzy Knowledge Measure (FKM) is proposed, and its fundamental mathematical properties are rigorously established (Theorems 1 and 2).
- The proposed FKM preserves the ordering of linguistic variables and provides improved discrimination among fuzzy sets (Examples 2–4).
- A new Fuzzy Ambiguity Measure (FAM) is derived from the proposed FKM, and its mathematical properties are formally proved (Theorem 3).
- Several numerical examples demonstrate that the proposed measures outperform existing approaches in pattern recognition problems (Examples 5 and 6).

- The practical applicability of the proposed framework is illustrated through a real-world case study involving the evaluation of e-commerce websites using an integrated TFN–Entropy–TODIM decision-making model. Comparative and sensitivity analyses further confirm the effectiveness and robustness of the proposed approach.

1.2 *Structure of the Research Process*

The remainder of this paper is organized as follows. Section 2 presents the basic concepts and operations related to fuzzy sets, the Fuzzy Information Measure (FIM), and the Fuzzy Knowledge Measure (FKM). Section 3 introduces the proposed FKM and investigates its theoretical properties. Section 4 provides several numerical examples to validate the proposed measure and compare it with existing approaches. Section 5 presents the proposed Fuzzy Ambiguity Measure (FAM) together with illustrative examples demonstrating its effectiveness. Section 6 describes the proposed TFN–Entropy–TODIM framework and its application to the evaluation of e-commerce websites. Finally, Section 7 concludes the paper and outlines directions for future research.

2. Basic Definitions

Definition 1: [1] Let $\breve{\mathbb{U}} = \left\{\bar{\Psi}_1, \bar{\Psi}_2, \ldots, \bar{\Psi}_z\right\}$ be a finite set,then $\bar{\mathscr{L}} = \left\{\left(\bar{\Psi}_t, \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right) : \bar{\Psi}_t \in \breve{\mathbb{U}}\right\}$ is called a FS on $\breve{\mathbb{U}}$, where $\mu_{\bar{\mathscr{L}}} : \breve{\mathbb{U}} \to [0, 1]$ denotes the membership function.
Definition 2: [1] A fuzzy set $\bar{\mathscr{L}}^*$ is characterized as a sharpned version of a fuzzy set $\bar{\mathscr{L}}$ if it meets subsequent necessities:

$$\begin{array}{l} \mu_{\bar{\mathscr{L}}^*}\left(\bar{\Psi}_t\right) \leq \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right), \text{ if } \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right) < 0.5 \quad \forall t, \\ \mu_{\bar{\mathscr{L}}*}\left(\bar{\Psi}_t\right) \geq \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right), \text{ if } \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right) \geq 0.5 \quad \forall t. \end{array}$$

Definition 3: The following are some elementary operations on FSs being proposed by Zadeh [1]:
Let $\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2 \in FSs(\breve{\mathbb{U}})$. Then,
(i) $\bar{\mathscr{L}}_1 \subseteq \bar{\mathscr{L}}_2$ if $\mu_{\bar{\mathscr{L}}_1}\left(\bar{\Psi}_t\right) \leq \mu_{\bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right), \forall \bar{\Psi}_t \in \breve{\mathbb{U}}$
(ii) $\bar{\mathscr{L}}_1 = \bar{\mathscr{L}}_2$ if $\bar{\mathscr{L}}_1 \subseteq \bar{\mathscr{L}}_2$ and $\bar{\mathscr{L}}_2 \subseteq \bar{\mathscr{L}}_1$
(iii) $\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2 = \left\{\left(\bar{\Psi}_t, \mu_{\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right)\right) : \bar{\Psi}_t \in \breve{\mathbb{U}}\right\}$ where $\mu_{\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right) = \max\left\{\mu_{\bar{\mathscr{L}}_1}\left(\bar{\Psi}_t\right), \mu_{\bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right)\right\}, \forall \bar{\Psi}_t \in \breve{\mathbb{U}}$
(iv) $\bar{\mathscr{L}}_1 \cap \bar{\mathscr{L}}_2 = \left\{\left(\bar{\Psi}_t, \mu_{\bar{\mathscr{L}}_1 \cap \bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right)\right) : \bar{\Psi}_t \in \breve{\mathbb{U}}\right\}$ where $\mu_{\bar{\mathscr{L}}_1 \cap \bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right) = \min\left\{\mu_{\bar{\mathscr{L}}_1}\left(\bar{\Psi}_t\right), \mu_{\bar{\mathscr{L}}_2}\left(\bar{\Psi}_t\right)\right\}, \forall \bar{\Psi}_t \in \breve{\mathbb{U}}$
(v) $\bar{\mathscr{L}}^c = \left\{\left(\bar{\Psi}_t, \mu_{\bar{\mathscr{L}}^c}\left(\bar{\Psi}_t\right)\right) : \bar{\Psi}_t \in \breve{\mathbb{U}}\right\}$ where $\mu_{\bar{\mathscr{L}}^c}\left(\bar{\Psi}_t\right) = 1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right), \forall \bar{\Psi}_t \in \breve{\mathbb{U}}$
Definition 4: [[24],[25]]A Triangular Fuzzy Number is denoted by $\tilde{K} = \langle (K_1, K_2, K_3); P_K(t) \rangle$, where the membership function $P_K(t)$ is defined as:

$$P_K(t) = \begin{cases} \tau_K\left(\frac{t-K_1}{K_2-K_1}\right) & \text{if } K_1 \leq t \leq K_2, \\ \tau_K & \text{if } t = K_2, \\ \tau_K\left(\frac{K_3-t}{K_3-K_2}\right) & \text{if } K_2 < t \leq K_3, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

where $\tau_K \in [0, 1]$, representing the maximum truth-membership degree.

Definition 5: Let $\breve{\jmath} = \langle (\breve{\jmath}_1, \breve{\jmath}_2, \breve{\jmath}_3); \tau_{\breve{\jmath}} \rangle$ and $\breve{\ell} = \left\langle \left(\breve{\ell}_1, \breve{\ell}_2, \breve{\ell}_3\right); \tau_{\breve{\ell}} \right\rangle$ be two triangular fuzzy numbers[25] and $\breve{\gamma} \neq 0$ be any real number. Then:

(i)- Adding two triangular fuzzy numbers

$$\breve{\jmath} + \breve{\ell} = \left\langle \left(\breve{\jmath}_1 + \breve{\ell}_1, \breve{\jmath}_2 + \breve{\ell}_2, \breve{\jmath}_3 + \breve{\ell}_3 \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle \tag{2}$$

(ii)- Subtracting two triangular fuzzy numbers

$$\breve{\jmath} - \breve{\ell} = \left\langle \left(\breve{\jmath}_1 - \breve{\ell}_3, \breve{\jmath}_2 - \breve{\ell}_2, \breve{\jmath}_3 - \breve{\ell}_1 \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle \tag{3}$$

(iii)- Inverse of a triangular fuzzy numbers

$$\breve{\jmath}^{-1} = \left\langle \left(\left(\frac{1}{\breve{\jmath}_3}, \frac{1}{\breve{\jmath}_2}, \frac{1}{\breve{\jmath}_1} \right) \right); \tau_{\breve{\jmath}} \right\rangle, \quad \text{Where } (\breve{\jmath} \neq 0) \tag{4}$$

(iv)- Multiplying a triangular fuzzy numbers by constant value

$$\breve{\gamma}\breve{\jmath} = \begin{cases} \langle (\breve{\gamma}\breve{\jmath}_1, \breve{\gamma}\breve{\jmath}_2, \breve{\gamma}\breve{\jmath}_3); \tau_{\breve{\jmath}} \rangle & \text{if } (\breve{\gamma} > 0) \\ \langle (\breve{\gamma}\breve{\jmath}_3, \breve{\gamma}\breve{\jmath}_2, \breve{\gamma}\breve{\jmath}_1); \tau_{\breve{\jmath}} \rangle & \text{if } (\breve{\gamma} < 0) \end{cases} \tag{5}$$

(v)- Diving a triangular fuzzy number by constant value

$$\frac{\breve{\jmath}}{\breve{\gamma}} = \begin{cases} \left\langle \left(\frac{\breve{\jmath}_1}{\breve{\gamma}}, \frac{\breve{\jmath}_2}{\breve{\gamma}}, \frac{\breve{\jmath}_3}{\breve{\gamma}} \right); \tau_{\breve{\jmath}} \right\rangle & \text{if } (\breve{\gamma} > 0) \\ \left\langle \left(\frac{\breve{\jmath}_3}{\breve{\gamma}}, \frac{\breve{\jmath}_2}{\breve{\gamma}}, \frac{\breve{\jmath}_1}{\breve{\gamma}} \right); \tau_{\breve{\jmath}} \right\rangle & \text{if } (\breve{\gamma} < 0) \end{cases} \tag{6}$$

(vi)- Multiplying two triangular fuzzy numbers

$$\breve{\jmath}\breve{\ell} = \begin{cases} \left\langle \left(\breve{\jmath}_1\breve{\ell}_1, \breve{\jmath}_2\breve{\ell}_2, \breve{\jmath}_3\breve{\ell}_3 \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle & \text{if } \left(\breve{\jmath}_3 > 0, \breve{\ell}_3 > 0 \right) \\ \left\langle \left(\breve{\jmath}_1\breve{\ell}_3, \breve{\jmath}_2\breve{\ell}_2, \breve{\jmath}_3\breve{\ell}_1 \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle & \text{if } \left(\breve{\jmath}_3 < 0, \breve{\ell}_3 > 0 \right) \\ \left\langle \left(\breve{\jmath}_3\breve{\ell}_3, \breve{\jmath}_2\breve{\ell}_2, \breve{\jmath}_1\breve{\ell}_1 \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle & \text{if } \left(\breve{\jmath}_3 < 0, \breve{\ell}_3 < 0 \right) \end{cases} \tag{7}$$

(vii)- Division of two triangular fuzzy numbers

$$\frac{\breve{\jmath}}{\breve{\ell}} = \begin{cases} \left\langle \left(\frac{\breve{\jmath}_1}{\breve{\ell}_3}, \frac{\breve{\jmath}_2}{\breve{\ell}_2}, \frac{\breve{\jmath}_3}{\breve{\ell}_1} \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle & \text{if } \left(\breve{\jmath}_3 > 0, \breve{\ell}_3 > 0 \right) \\ \left\langle \left(\frac{\breve{\jmath}_3}{\breve{\ell}_3}, \frac{\breve{\jmath}_2}{\breve{\ell}_2}, \frac{\breve{\jmath}_1}{\breve{\ell}_1} \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle & \text{if } \left(\breve{\jmath}_3 < 0, \breve{\ell}_3 > 0 \right) \\ \left\langle \left(\frac{\breve{\jmath}_3}{\breve{\ell}_1}, \frac{\breve{\jmath}_2}{\breve{\ell}_2}, \frac{\breve{\jmath}_1}{\breve{\ell}_3} \right); \tau_{\breve{\jmath}} \wedge \tau_{\breve{\ell}} \right\rangle & \text{if } \left(\breve{\jmath}_3 < 0, \breve{\ell}_3 < 0 \right) \end{cases} \tag{8}$$

Score function: A score function is employed to determine an ordering of triangular fuzzy numbers

$$S(A) = \left(\frac{1}{4} \right) (a + 2b + c) \tag{9}$$

2.1 *Comparison of Existing Measures*

Zadeh[1] became first to introduce the concept of fuzzy sets, laying the foundation for a new way to handle uncertainty and imprecision in data. Building on this, Zadeh[2] later proposed the idea of an information measure that combines probability with the membership function, allowing researchers to better analyze the ambiguity or indistinctness inherent in fuzzy collections. Subsequently, De Luca and Termini[26] proposed four fundamental principles that must be fulfilled by a function $\mathscr{E} : FSs(\breve{\mathbb{U}}) \to [0, 1]$ in order to qualify as a FIM, outlined as follows:

(i) Sharpness: $\mathscr{E}(\bar{\mathscr{L}})$ has minimum value iff $\bar{\mathscr{L}}$ is a crisp set, i.e. 0=$\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) = 1, \quad \forall t$.
(ii) Maximality: $\mathscr{E}(\bar{\mathscr{L}})$ has maximum value iff $\bar{\mathscr{L}}$ to be a most fuzzy set, i.e. $\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) = 0.5 \quad \forall t$.
(iii) Resolution: $\mathscr{E}(\bar{\mathscr{L}}) \geq \mathscr{E}\left(\bar{\mathscr{L}}^*\right)$ where $\bar{\mathscr{L}}^*$ is the sharpened version of $\bar{\mathscr{L}}$.
(iv) Symmetry: $\mathscr{E}(\bar{\mathscr{L}}) = \mathscr{E}\left(\bar{\mathscr{L}}^c\right)$ where $\bar{\mathscr{L}}^c$ is complement set of $\bar{\mathscr{L}}$.
Since FKM are supplementary to FIM, therefore Singh et al. [11] proposed four properties that a function $\mathscr{V}_m : FSs(\check{\mathbb{U}}) \rightarrow [0,1]$ must satisfy in order to be defined as a FKM, outlined essentially as follows:
N(1) Sharpness: $\mathscr{V}_m(\bar{\mathscr{L}}) = 0 \Leftrightarrow \mu_{\bar{\mathscr{L}}}(\bar{\Psi}) = 0.5 \forall \bar{\Psi} \in \check{\mathbb{U}}$.
N(2) Maximality: $\mathscr{V}_m(\bar{\mathscr{L}}) = 1 \Leftrightarrow \quad \mu_{\bar{\mathscr{L}}}(\bar{\Psi}) = 0 \quad$ or $\mu_{\bar{\mathscr{L}}}(\bar{\Psi}) = 1 \forall \bar{\Psi} \in \check{\mathbb{U}}$.
N(3) Resolution: $\mathscr{V}_m(\bar{\mathscr{L}}) \leq \mathscr{V}_m\left(\bar{\mathscr{L}}^*\right), \bar{\mathscr{L}}^*$ is sharpened version of $\bar{\mathscr{L}}$.
N(4) Symmetry : $\mathscr{V}_m(\bar{\mathscr{L}}) = \mathscr{V}_m\left(\bar{\mathscr{L}}^c\right), \bar{\mathscr{L}}^c$ is complement of $\bar{\mathscr{L}}$.
Various authors have provided the following fuzzy information measures.
1. De Luca and Termini (1972):
The FIM was initially given by De Luca and Termini[26] utilizing the principles of probability theory from [2], as detailed below:

$$\mathscr{E}(\bar{\mathscr{L}}) = -\frac{1}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) \log_2\left(\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)\right) + \left(1 - \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)\right) \log_2\left(1 - \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)\right)\right] \tag{10}$$

2. Bhandari and Pal[27]:
They focus on a single parameter as outlined below:

$$\mathscr{E}_{\check{\gamma}}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}(1-\check{\gamma})} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \log_2\left[\mu_{\bar{\mathscr{L}}}^{\check{\gamma}}\left(\bar{\Psi}_t\right) + \left(1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\check{\gamma}}\right], \check{\gamma} \neq 1. \tag{11}$$

3. Pal and Pal[3] introduced an entirely new exponential FIM for FSs, as detailed below:

$$\mathscr{E}_{PP}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right) e^{1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)} + \left(1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right) e^{\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}\right] \tag{12}$$

4. Kapur[7] presented a parametric FKM associated with the probability distribution outlined in [28], as detailed below.:

$$\mathscr{E}_{\check{\gamma}}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}\left(2^{1-\check{\gamma}} - 1\right)} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\mu_{\bar{\mathscr{L}}}^{\check{\gamma}}\left(\bar{\Psi}_t\right) + \left(1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\check{\gamma}} - 1\right], \check{\gamma} \neq 1. \tag{13}$$

5. Hooda[5] provided a comprehensive information metric, utilizing two parameters as:

$$\mathscr{E}_{\check{\gamma}}^{\check{\delta}}(\bar{\mathscr{L}}) = \frac{1}{1-\check{\delta}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\left(\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)^{\check{\gamma}} + \left(1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)^{\check{\gamma}}\right)\right)^{\frac{\check{\delta}-1}{\check{\gamma}-1}-1}\right], \check{\gamma} \neq \delta, \check{\gamma} > 0, \check{\delta} > 0, \check{\gamma} \neq 1. \tag{14}$$

6. Joshi and Kumar[6]: They offer an improved modified two parametric FKM, which is seen below.:

$$\mathscr{E}_{\check{\gamma}}^{\check{\delta}}(\bar{\mathscr{L}}) = \frac{\check{\gamma} \times \check{\delta}}{\mathfrak{s}(\check{\gamma} - \check{\delta})} \left[\sum_{\mathrm{t}=1}^{\mathfrak{s}} \left\{\left(\mu_{\bar{\mathscr{L}}}^{\check{\delta}}\left(\bar{\Psi}_t\right) + \left(1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\check{\delta}}\right)^{\frac{1}{\check{\delta}}} - \left(\mu_{\bar{\mathscr{L}}}^{\check{\gamma}}\left(\bar{\Psi}_t\right) + \left(1 - \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\check{\gamma}}\right)^{\frac{1}{\check{\gamma}}}\right\}\right], \check{\gamma} \neq \check{\delta}, \check{\gamma} > 0, \check{\delta} > 0. \tag{15}$$

7.Arya and Kumar[4] defined a fuzzy information measure using a single parameter, as described below.:

$$\mathscr{E}_{\breve{\gamma}}^{VS}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}\left(\breve{\gamma}-\breve{\gamma}^{-1}\right)} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\mu_{\bar{\mathscr{L}}}^{\breve{\gamma}^{-1}}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\breve{\gamma}^{-1}} - \left(\mu_{\bar{\mathscr{L}}}^{\breve{\gamma}}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\breve{\gamma}}\right)\right], \breve{\gamma} \neq 1. \tag{16}$$

Several important measures of fuzzy knowledge presented in various studies were as studies:
1.Singh et al.[11] defined FKM as stated below:

$$\mathscr{V}^{S}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[2\left\{\mu_{\bar{\mathscr{L}}}^{2}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{2}\right\} - 1\right] \tag{17}$$

2.Singh et al.[12] established a measure of knowledge grounded in parameters as outlined below.:

$$\mathscr{V}_{\breve{\gamma}}^{S}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[2\left\{\mu_{\bar{\mathscr{L}}}^{\breve{\gamma}}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\breve{\gamma}}\right\} - 1\right], \breve{\gamma} > 0. \tag{18}$$

3.Arya and Kumar[8] defined FKM utilising the logarithmic function as outlined below:

$$\mathscr{V}^{V}(\bar{\mathscr{L}}) = \log_2 \left[\frac{2}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left(\mu_{\bar{\mathscr{L}}}^{2}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{2}\right]\right. \tag{19}$$

4. Singh and Ganie[29] presented a categorised FKM using two parameters laid out below:

$$\mathscr{V}_{\breve{\gamma},\breve{\delta}}^{S}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[2\left\{\mu_{\bar{\mathscr{L}}}^{\breve{\gamma}}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{\breve{\gamma}}\right\}^{\frac{\breve{\gamma}-1}{\breve{\delta}-1}} - 1\right], \breve{\gamma} \in (1,2], \breve{\delta} \geq \breve{\gamma}. \tag{20}$$

5.Singh and Kumar[10]established a FKM as identified:

$$\mathscr{V}^{AS}(\bar{\mathscr{L}}) = \frac{(\sqrt[3]{4}-1)^{-1}}{\mathfrak{s}} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\sqrt[3]{4\left\{\mu_{\bar{\mathscr{L}}}^{3}\left(\bar{\Psi}_t\right) + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{3}\right.}\right\} - 1\right] \tag{21}$$

6.Dinesh and Kumar[9] presented the fuzzy knowledge measure, which is as follows:

$$\mathscr{V}^{DK}(\bar{\mathscr{L}}) = \frac{1}{\mathfrak{s}\left(1-e^{0.75}\right)} \sum_{\mathrm{t}=1}^{\mathfrak{s}} \left[\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right) e^{\left(1-\mu_{\bar{\mathscr{L}}}^{2}\left(\bar{\Psi}_t\right)\right)} + \left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right) e^{\left(1-\left(1-\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right)^{2}\right.} - e^{0.75}\right] \tag{22}$$

Nonetheless, these operators exhibit certain limitations as demonstrated in Example 1,Sect 4., as they produce inaccurate outcomes and also certain measures do not fulfill the order of linguistic variables(refer Example 4-5).Consequently, there is a need for an innovative FKM that addresses the shortcomings of the current measures and also which produces results with greater accuracy compared to existing methods.

3. Proposed Knowledge Measure

This section introduces a novel measure of knowledge and examines its alignment with the four axioms outlined in [11]. Subsequently, several theorems defining the characteristics of the proposed knowledge measure are reviewed. Let $\bar{\mathscr{L}} = \left\{\left(\bar{\Psi}_t, \mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right) : \bar{\Psi}_t \in \breve{\mathbb{U}}\right\} \in FSs(\breve{\mathbb{U}})$ We are delighted to present a new fuzzy knowledge measure defined as follows:

$$\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=\frac{1}{n}\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right| \tag{23}$$

Where $\bar{\mathscr{L}}\in FSs(\check{\mathbb{U}})$. The validity of $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ is established in the following theorem.
Theorem 1. Let $\bar{\mathscr{L}}=\left\{\left(\bar{\Psi}_t,\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right):\bar{\Psi}_t\in\check{\mathbb{U}}\right\}\in FSs(\check{\mathbb{U}})$ be a non-empty and finite set. Then $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ must satisfies all the four conditions:
N(1) $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=0$ iff $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=0.5\quad\forall\bar{\Psi}\in\check{\mathbb{U}}$
N(2) $\mathbb{D}^{\gtreqqless}(\bar{\mathscr{L}})=1$ iff $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=0$ or $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=1\quad\forall\bar{\Psi}\in\check{\mathbb{U}}$
N(3) $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})\leq\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{*}\right)$, $\bar{\mathscr{L}}^{*}$ is sharpened version of $\bar{\mathscr{L}}$.
N(4) $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{c}\right)$, $\bar{\mathscr{L}}^{c}$ is complement of $\bar{\mathscr{L}}$.
Figure 1 illustrates the behavior of the proposed fuzzy knowledge measure over the membership and non-membership domains, highlighting its symmetry and monotonic characteristics.
Proof: N(1) If $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=0$ then equation(23) becomes,
$\frac{1}{n}\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|=0$
$\Leftrightarrow\sum_{t=1}^{n}\left|\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}\right|=0$
$\Leftrightarrow\left|\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}\right|=0$
$\Leftrightarrow\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}=0$
$\Leftrightarrow\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}=\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}$
$\Leftrightarrow\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)=0.5,\quad\forall 1\leq t\leq n$
So, $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=0$ if and only if $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=0.5$.

N(2) Secondly, we will prove that,
$\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=1$ iff $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=0$ or $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=1\quad\forall\bar{\Psi}\in\check{\mathbb{U}}$
Taking $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=1$
$\Leftrightarrow\frac{1}{n}\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|=1$
$\Leftrightarrow\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\psi_t))}}{|\pi^{-1}-1|}\right|=n$
$\Leftrightarrow\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|=\sum_{t=1}^{n}1$
$\Leftrightarrow\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|=1$
Take $\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\psi_t))}}{\pi^{-1}-1}=1$
Then, we have$\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}=\pi^{-1}-1$
Now put,$\pi^{\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}=\kappa$
Therefore,we've
$\frac{1}{\kappa}-\frac{\kappa}{\pi}=\pi^{-1}-1$
On simplifying the above equation, we get
$\kappa^2+(1-\pi)\kappa-\pi=0\Rightarrow\kappa=(-1)or\kappa=\pi$
For $\kappa=-1\Rightarrow\pi^{\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}=-1$, which is not possible.
For $\kappa=\pi\Rightarrow\pi^{\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}=\pi$, we get $\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)=0$
Similarly, if we take $\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{\pi^{-1}-1}=-1$
Then, we will get $\mu_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)=1$
Therefore,we've proved that$\mathbb{D}^{\gtreqqless}(\bar{\mathscr{L}})=1$ iff $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=0$ or $\mu_{\bar{\mathscr{L}}}(\bar{\Psi})=1\quad\forall\bar{\Psi}\in\check{\mathbb{U}}$

N(3) Let $\bar{\mathscr{L}},\bar{\mathscr{L}}^{*}\in FSs(\check{\mathbb{U}})$, where $\bar{\mathscr{L}}^{*}$ is sharpened version of $\bar{\mathscr{L}}$. We will prove that $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})\leq\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{*}\right)$

It is noticed that

$$\left|\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)} - \pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}\right| = \begin{cases} \pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)} - \pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}, & \text{if } \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) < 0.5 \\ 0, & \text{if } \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) = 0.5 \\ \pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))} - \pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)}, & \text{if } \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) > 0.5 \end{cases}$$

Following this, we will determine whether or not
$\left|\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)} - \pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}\right|$ is an increasing or decreasing function.
Since, the denominator is positive, it will not effect the numinator.
Now, consider $z(\zeta) = \pi^{-\zeta} + \pi^{-(1-\zeta)}$. Then $\frac{dz}{d\zeta} = -\left(\pi^{-\zeta} + \pi^{-(1-\zeta)}\right) < 0$ for $\zeta < 0.5$
Now, if we take $z(\zeta) = -\pi^{-\zeta} + \pi^{-(1-\zeta)}$ Then, $\frac{dz}{d\zeta}\pi^{-\zeta} + \pi^{-(1-\zeta)} > 0$ for $\zeta > 0.5$
Clearly, $\left|\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)} - \pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}\right|$ is decreasing when $\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) < 0.5$ and is increasing when $\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) > 0.5$. Hence, $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ will also be decreasing when $\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) < 0.5$ and increasing when $\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) > 0.5$. Therefore, $\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^*\right) \geq \mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ if $\mu_{\bar{\mathscr{L}}^*}\left(\bar{\Psi}_t\right) \leq \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) < 0.5$ and$\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^*\right) \geq \mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ if $\mu_{\bar{\mathscr{L}}^*}\left(\bar{\Psi}_t\right) \geq \mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t) \geq 0.5$.

N(4) By definition given in eq.(14), it follows that $\mathbb{D}^{\varphi}(\bar{\mathscr{L}}) = \mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^c\right)$.

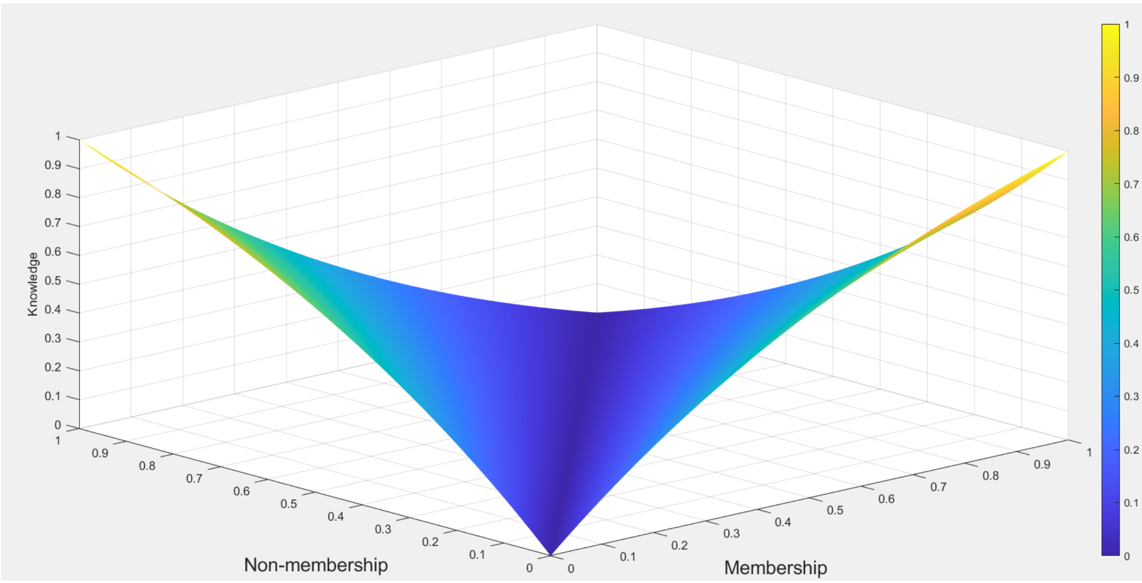

Fig. 1. Proposed knowledge measure

Theorem 2: Let $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ be a fuzzy knowledge measure then $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ must satisfies the properties:
(i) $\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2\right) + \mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}_1 \cap \bar{\mathscr{L}}_2\right) = \mathbb{D}^{\varphi}\left(\bar{\mathcal{T}}_1\right) + \mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}_2\right)$.
(ii) For any $\bar{\mathscr{L}} \in FSs(\breve{\mathbb{U}}), \mathbb{D}^{\varphi}(\bar{\mathscr{L}}) \in [0,1]$.
(iii)$\mathbb{D}^{\varphi}(\bar{\mathscr{L}}) = \mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^c\right)$, where $\bar{\mathscr{L}}^c$ is complement of fuzzy set $\bar{\mathscr{L}}$.
Proof **(i)** Let $\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2 \in FSs(\breve{\mathbb{U}})$, then by definition we have

$$\mathbb{D}^{\varphi}(\bar{\mathscr{L}}) = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t)} - \pi^{-(1-\mu_{\bar{\mathscr{L}}}(\bar{\Psi}_t))}}{|\pi^{-1} - 1|}\right|$$

Therefore,

$$\mathbb{D}^{\varphi}(\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2) = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2}(\bar{\Psi}_t)} - \pi^{-(1-\mu_{\bar{\mathscr{L}}_1 \cup \bar{\mathscr{L}}_2}(\bar{\Psi}_t))}}{|\pi^{-1} - 1|}\right|$$

Let us divide the set $\check{\mathbb{U}}$ into two parts:

$$\check{\mathbb{U}}_1=\{\bar{\Psi}_t\in\check{\mathbb{U}}:\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t)\le\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)\},\qquad \check{\mathbb{U}}_2=\{\bar{\Psi}_t\in\check{\mathbb{U}}:\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)<\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t)\}$$

Accordingly, the union and intersection membership functions select the corresponding maximum and minimum values on $\check{\mathbb{U}}_1$ and $\check{\mathbb{U}}_2$.

Therefore, $\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_1}\cup\bar{\mathscr{L}_2}\right)=\frac{1}{n}\sum_{\bar{\Psi}\in\check{\mathbb{U}}_1}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_1\cup\bar{\mathscr{L}}_2}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}_1\cup\bar{\mathscr{L}}_2}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|$ $+\frac{1}{n}\sum_{\bar{\Psi}\in\check{\mathbb{U}}_2}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_1\cup\bar{\mathscr{L}}_2}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}_1\cup\bar{\mathscr{L}}_2}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|$

which implies, $\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_1}\cup\bar{\mathscr{L}_2}\right)=\frac{1}{n}\sum_{\bar{\Psi}\in\check{\mathbb{U}}_1}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|$ $+\frac{1}{n}\sum_{\bar{\Psi}\in\check{\mathbb{U}}_2}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|$

and $\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_1}\cap\bar{\mathscr{L}_2}\right)=\frac{1}{n}\sum_{\bar{\Psi}\in\check{\mathbb{U}}_1}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|+\frac{1}{n}\sum_{\bar{\Psi}\in\check{\mathbb{U}}_2}\left|\frac{\pi^{-\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t)}-\pi^{-(1-\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t))}}{|\pi^{-1}-1|}\right|$

Hence we have proved the desired result i.e.
$\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_1}\cup\bar{\mathscr{L}_2}\right)+\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_1}\cap\bar{\mathscr{L}_2}\right)=\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_1}\right)+\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}_2}\right)$.

(ii) To prove that $0\le D^{\varphi}(\bar{\mathscr{L}})\le 1$, let us take $r(\mathfrak{z})=\left|\pi^{-\mathfrak{z}}-\pi^{-(1-\mathfrak{z})}\right|$.
As, $r(\mathfrak{z})$ is decreasing function for $\mathfrak{z}\le 0.5$ and is increasing function for $\mathfrak{z}\ge 0.5$ (as proved earlier in property N(3)), so

$$\begin{aligned}
r(0)\ge r(\mathfrak{z})\ge r(0.5)&\text{ and } r(0.5)\le r(\mathfrak{z})\le r(1)\\
r(0)=0.6815, r(0.5)&=0 \text{ and } r(1)=0.6815\\
\Rightarrow&0\le r(\mathfrak{z})\le 1/2,\quad\forall\mathfrak{z}\in[0,1]\\
\Rightarrow&0\le\sum_{i=1}^{n}\left|\pi^{-\mu_{\mathfrak{D}}(e_i)}-\pi^{-(1-\mu_{\mathfrak{D}}(e_i))}\right|\le n/2\\
\Rightarrow&0\le\frac{2}{n}\sum_{i=1}^{n}\left|\pi^{-\mu_{\mathfrak{D}}(e_i)}-\pi^{-(1-\mu_{\mathfrak{D}}(e_i))}\right|\le 1\\
\Rightarrow&\mathbb{D}^{\varphi}(\bar{\mathscr{L}})\in[0,1].
\end{aligned}$$

(iii) It is obvious from the equation that $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{c}\right)$, where $\bar{\mathscr{L}}^{c}$ is complement of $\bar{\mathscr{L}}$.

4. Comparative Study

This section presents a comparison of the proposed FKM as defined in eq.(23) with various established information and knowledge metrics. Additionally, the efficacy of the suggested FIM has been evaluated based on linguistic factors through some examples.

4.1 Ambiguity Associated with Fuzzy Sets

It is widely acknowledged that two fuzzy sets may differ in the ambiguity associated with them. However,some FIM/FKM provide the same ambiguity values for distinct FS. Therefore, a new fuzzy measure is essential to generalize existing fuzzy measures.The following example exhibits the performance of the suggested measure.

Example 1. Let $\check{\mathbb{U}} = \left\{\bar{\Psi}_1, \bar{\Psi}_2, \bar{\Psi}_3, \bar{\Psi}_4, \bar{\Psi}_5\right\}$ and $\bar{\mathscr{L}}_i$ (for i=1,2,3,4,5) be five FSs defined on $\check{\mathbb{U}}$ given as below:
$\bar{\mathscr{L}}_1 = \left\{\left(\bar{\Psi}_1, 0.14\right), \left(\bar{\Psi}_2, 0.337\right), \left(\bar{\Psi}_3, 0.602\right), \left(\bar{\Psi}_4, 0.617\right) \left(\bar{\Psi}_5, 0.5\right)\right\}$,
$\bar{\mathscr{L}}_2 = \left\{\left(\bar{\Psi}_1, 0.61\right), \left(\bar{\Psi}_2, 0.237\right), \left(\bar{\Psi}_3, 0.402\right), \left(\bar{\Psi}_4, 0.201\right) \left(\bar{\Psi}_5, 0.49\right)\right\}$,
$\bar{\mathscr{L}}_3 = \left\{\left(\bar{\Psi}_1, 0.6\right), \left(\bar{\Psi}_2, 0.25\right), \left(\bar{\Psi}_3, 0.40\right), \left(\bar{\Psi}_4, 0.1\right) \left(\bar{\Psi}_5, 0.49\right)\right\}$,
$\bar{\mathscr{L}}_4 = \left\{\left(\bar{\Psi}_1, 0.695\right), \left(\bar{\Psi}_2, 0.232\right), \left(\bar{\Psi}_3, 0.402\right), \left(\bar{\Psi}_4, 0.15\right) \left(\bar{\Psi}_5, 0.495\right)\right\}$,
$\bar{\mathscr{L}}_5 = \left\{\left(\bar{\Psi}_1, 0.5\right), \left(\bar{\Psi}_2, 0.25\right), \left(\bar{\Psi}_3, 0.49\right), \left(\bar{\Psi}_4, 0.209\right) \left(\bar{\Psi}_5, 0.195\right)\right\}$
Now, determine the degree of ambiguity linked to these imprecise collections by employing the knowledge metrics outlined in equations (17-22) and the suggested framework presented in equation (23). The computed level of uncertainty is displayed in Table 1.

Table 1
Values obtained for distinct fuzzy sets using different FKM

Fuzzy Sets ↓	⟵ Knowledge Measure's Values ⟶					
	$\mathscr{V}^S(\bar{\mathscr{L}})$	$\mathscr{V}^S_{\gamma=1.7}(\bar{\mathscr{L}})$	$\mathscr{V}^V(\bar{\mathscr{L}})$	$\mathscr{V}^S_{\gamma=1.4,\delta=2.8}(\bar{\mathscr{L}})$	$\mathscr{V}^{AS}(\bar{\mathscr{L}})$	$\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$
$\bar{\mathscr{L}}_1$	**0.1442**	**0.3383**	**0.1943**	**0.8972**	0.1892	0.2856
$\bar{\mathscr{L}}_2$	**0.1443**	**0.3379**	**0.1944**	**0.8972**	0.1981	0.2995
$\bar{\mathscr{L}}_3$	0.1940	0.3760	0.2559	**0.9030**	0.2423	0.2327
$\bar{\mathscr{L}}_4$	0.1935	0.3748	0.2553	**0.9030**	**0.2562**	0.3530
$\bar{\mathscr{L}}_5$	0.1922	0.3736	0.2536	0.9028	**0.2562**	0.3301

From Table 1, it is clear that Knowledge measures $\mathscr{V}^S, \mathscr{V}^S_\gamma, \mathscr{V}^V$, and $\mathscr{V}^S_{\gamma,\delta}$ compute an equivalent level of ambiguity for the fuzzy sets $\bar{\mathscr{L}}_1$ and $\bar{\mathscr{L}}_2$. The expression $\mathscr{V}^{AS}$ computes an equivalent level of ambiguity for the fuzzy sets $\bar{\mathscr{L}}_4$ and $\bar{\mathscr{L}}_5$ and the proposed measure yields varying levels of ambiguity across distinct fuzzy sets, resulting in enhanced accuracy and effectiveness in outcomes.

4.2 *Linguistic Comparison*

During the 1970s, Zadeh[30] introduced an idea for a linguistic component. Mathematical expressions are used to describe traits using a linguistic variable.The idea of the power of a fuzzy set will now be examined in this context in the manner described below.
Definition: (Zadeh[30]) Let $\bar{\mathscr{L}} \in FSs(\check{\mathbb{U}})$, then the strength of a fuzzy set is defined as: $\bar{\mathscr{L}}^n = \left\{\left(\bar{\Psi}_t, \mu^n_{\bar{\mathscr{L}}}\left(\bar{\Psi}_t\right)\right) : \bar{\Psi}_t \in \check{\mathbb{U}}\right\}$.
We consider fuzzy set $\bar{\mathscr{L}}$ as "True" on $\check{\mathbb{U}}$ by considering the description of linguistic variables. In a similar vein, the fuzzy sets $\bar{\mathscr{L}}^{1/2}, \bar{\mathscr{L}}^2, \bar{\mathscr{L}}^3, \bar{\mathscr{L}}^4, \bar{\mathscr{L}}^5$ can be classified as ''very true'', ''more or less true'', ''false'', ''possible false'',"not applicable" respectively. Information hidden within them evaporates less frequently from $\bar{\mathscr{L}}^{1/2}$ to $\bar{\mathscr{L}}^5$. As a result, the uncertainty level rises from $\bar{\mathscr{L}}^5$ to $\bar{\mathscr{L}}^{1/2}$.We compare different fuzzy information measures by examining these fuzzy sets.The FIM values for these FSs must keep certain order to ensure maximum efficiency.

$$\mathscr{E}\left(\bar{\mathscr{L}}^{1/2}\right) > \mathscr{E}(\bar{\mathscr{L}}) > \mathscr{E}\left(\bar{\mathscr{L}}^2\right) > \mathscr{E}\left(\bar{\mathscr{L}}^3\right) > \mathscr{E}\left(\bar{\mathscr{L}}^4\right) \tag{24}$$

Nonetheless, FKM acts as the multifaceted of FIM, then reversed sequence of FIM should be followed :

$$\mathscr{V}_m\left(\bar{\mathscr{L}}^{\mathbf{1/2}}\right) < \mathscr{V}_m(\bar{\mathscr{L}}) < \mathscr{V}_m\left(\bar{\mathscr{L}}^{\mathbf{2}}\right) < \mathscr{V}_m\left(\bar{\mathscr{L}}^{\mathbf{3}}\right) < \mathscr{V}_m\left(\bar{\mathscr{L}}^{\mathbf{4}}\right) < \mathscr{V}_m\left(\bar{\mathscr{L}}^{\mathbf{5}}\right) \tag{25}$$

Now,We present an example to investigate the effectiveness of $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$.
Example 2. Let $\bar{\mathscr{L}}=\left\{\left(\bar{\Psi}_1,0.2\right),\left(\bar{\Psi}_2,0.3\right),\left(\bar{\Psi}_3,0.4\right),\left(\bar{\Psi}_4,0.6\right)\left(\bar{\Psi}_2,0.8\right)\right\}$ be a fuzzy set defined on $\check{\mathbb{U}}=\left\{\bar{\Psi}_1,\bar{\Psi}_2,\bar{\Psi}_3,\bar{\Psi}_4,\bar{\Psi}_5\right\}$, then we have
$\bar{\mathscr{L}}^{1/2}=\{0.447,0.5477,0.632,0.8366,0.8944\}$,
$\bar{\mathscr{L}}^{2}=\{0.04,0.09,0.16,0.36,0.64\}$,
$\bar{\mathscr{L}}^{3}=\{0.008,0.027,0.064,0.216,0.512\}$,
$\bar{\mathscr{L}}^{4}=\{0.0016,0.0081,0.0256,0.1296,0.4096\}$ and
$\bar{\mathscr{L}}^{5}=\{0.00032,0.00243,0.1024,0.07776,0.32768\}$
Now, value of proposed FKM defined in eq.(23) is calculated for these fuzzy sets:

$$\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=0.38429,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{1/2}}\right)=0.37373,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{2}}\right)=0.58233,$$
$$\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{3}}\right)=0.67086,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{4}}\right)=0.7634,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{5}}\right)=0.78676$$

so that we obtain the desired order

$$\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{1/2}}\right)<\mathbb{D}^{\varphi}(\bar{\mathscr{L}})<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{2}}\right)<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{3}}\right)<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{4}}\right)<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{5}}\right)$$

So, propoesd measure satisfy the sequence of linguistic variable given in eq.(25).
Example 3. Let $\bar{\mathscr{L}}=\left\{\left(\bar{\Psi}_1,0.2\right),\left(\bar{\Psi}_2,0.3\right),\left(\bar{\Psi}_3,0.4\right),\left(\bar{\Psi}_4,0.7\right)\left(\bar{\Psi}_5,0.8\right)\right\}$ be a fuzzy set defined on $\check{\mathbb{U}}=\left\{\bar{\Psi}_1,\bar{\Psi}_2,\bar{\Psi}_3,\bar{\Psi}_4,\bar{\Psi}_5\right\}$, then we have
$\bar{\mathscr{L}}^{1/2}=\{0.447,0.5477,0.632,0.836,0.8944\}$,
$\bar{\mathscr{L}}^{2}=\{0.04,0.09,0.16,0.49,0.64\}$,
$\bar{\mathscr{L}}^{3}=\{0.008,0.027,0.064,0.343,0.512\}$,
$\bar{\mathscr{L}}^{4}=\{0.0016,0.0081,0.0256,0.2401,0.4096\}$, a n d
$\bar{\mathscr{L}}^{5}=\{0.00032,0.00243,0.1024,0.16807,0.32768\}$
The value of the proposed FKM, as defined in equation (23), is computed for the following fuzzy sets:

$$\mathbb{D}^{\varphi}(\bar{\mathscr{L}})=0.42277,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{1/2}}\right)=0.37349,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{2}}\right)=0.53284,$$
$$\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{3}}\right)=0.62115,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{4}}\right)=0.71874,\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{5}}\right)=0.74928$$

which meet the specified criteria.

$$\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{1/2}}\right)<\mathbb{D}^{\varphi}(\bar{\mathscr{L}})<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{2}}\right)<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{3}}\right)<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{4}}\right)<\mathbb{D}^{\varphi}\left(\bar{\mathscr{L}}^{\mathbf{5}}\right)$$

So, propoesd measure satisfy the sequence of linguistic variable given in eq.(25).

4.3 Comparison with Existing Fuzzy Information Measures

The results of the suggested FKM specified in eq. (23) are compared with the $FIMs$ provided in eqs.(10-16) in order to assess the efficacy and dependability of the suggested knowledge measure.
Example 4. Let fuzzy set $\bar{\mathscr{L}}$ be defined on $\check{\mathbb{U}}=\left\{\bar{\Psi}_1,\bar{\Psi}_2,\bar{\Psi}_3,\bar{\Psi}_4,\bar{\Psi}_5\right\}$ given as:

$$\begin{aligned}
\bar{\mathscr{L}}&=\left\{\left(\bar{\Psi}_1,0.2\right),\left(\bar{\Psi}_2,0.3\right),\left(\bar{\Psi}_3,0.5\right),\left(\bar{\Psi}_4,0.7\right)\left(\bar{\Psi}_5,0.9\right)\right\},\text{ then we have}\\
\bar{\mathscr{L}}^{1/2}&=\left\{\left(\bar{\Psi}_1,0.447\right),\left(\bar{\Psi}_2,0.5477\right),\left(\bar{\Psi}_3,0.7071\right),\left(\bar{\Psi}_4,0.836\right)\left(\bar{\Psi}_5,0.948\right)\right\},\\
\bar{\mathscr{L}}^{2}&=\left\{\left(\bar{\Psi}_1,0.04\right),\left(\bar{\Psi}_2,0.09\right),\left(\bar{\Psi}_3,0.25\right),\left(\bar{\Psi}_4,0.49\right)\left(\bar{\Psi}_5,0.81\right)\right\},\\
\bar{\mathscr{L}}^{3}&=\left\{\left(\bar{\Psi}_1,0.008\right),\left(\bar{\Psi}_2,0.027\right),\left(\bar{\Psi}_3,0.125\right),\left(\bar{\Psi}_4,0.343\right)\left(\bar{\Psi}_5,0.729\right)\right\},\\
\bar{\mathscr{L}}^{4}&=\left\{\left(\bar{\Psi}_1,0.0016\right),\left(\bar{\Psi}_2,0.0081\right),\left(\bar{\Psi}_3,0.0625\right),\left(\bar{\Psi}_4,0.2401\right)\left(\bar{\Psi}_5,0.6561\right)\right\},
\end{aligned}$$

and $\bar{\mathscr{L}}^5 = \left\{ \left(\bar{\Psi}_1, 0.00032\right), \left(\bar{\Psi}_2, 0.00243\right), \left(\bar{\Psi}_3, 0.03125\right), \left(\bar{\Psi}_4, 0.16807\right) \left(\bar{\Psi}_5, 0.59049\right) \right\}$

Therefore,the values calculated using different measures easily compared as shown in Table 2.

Table 2
The estimated outcomes of various FIM and the proposed FKM

$FSs\downarrow$	$\longleftarrow$ Values of FIMs and the proposed FKM $\longrightarrow$						
	$\mathscr{E}^{\text{Kapur}}_{\gamma=3}$	$\mathscr{E}^{\text{Bhandari}}_{\gamma=2}$	$\mathscr{E}^{\text{Pal}}$	$\mathscr{E}^{\text{Hooda}}_{\gamma=2,\delta=3}$	$\mathscr{E}^{\text{Joshi}}_{\gamma=0.9,\delta=1.3}$	$\mathscr{E}^{VS}_{\gamma=2}$	$\mathbb{D}^{\varphi}$
$\bar{\mathscr{L}}^{1/2}$	0.7098	0.6701	1.4640	1.4019	0.4964	0.4667	0.4257
$\bar{\mathscr{L}}$	**0.7360**	**0.6828**	**1.4819**	**1.4712**	**0.5169**	**0.4839**	0.4257
$\bar{\mathscr{L}}^2$	0.5692	0.5163	1.3741	1.1643	0.4125	0.3964	0.5634
$\bar{\mathscr{L}}^3$	0.4531	0.4093	1.2978	0.9279	0.3302	0.3238	0.6787
$\bar{\mathscr{L}}^4$	0.3810	0.3457	1.2503	0.7772	0.2770	0.2738	0.7279
$\bar{\mathscr{L}}^5$	0.3317	0.3049	1.2176	0.6714	0.2402	0.2382	0.7482

Table 2 makes it abundantly evident that the results of every information measure at the fuzzy set $\bar{\mathscr{L}}$ are greater than those at $\bar{\mathscr{L}}^{1/2}$, hence disrupting the linguistic variable pattern and only the proposed knowledge measure satisfies the hierarchy of linguistic variables specified in equation (25). Consequently, the suggested FKM yields more precise results.

5. Advanced Fuzzy Accuracy Measure

In this section, we propose a novel fuzzy accuracy measure for two fuzzy sets derived from the FKM defined in Equation (23), which is given as follows:

$$\mathbb{D}^{\varphi}Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2) = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\pi^{-\sqrt{\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t)\times\mu_{\bar{\mathscr{L}}_2}((\bar{\Psi}_t))}} - \pi^{-\sqrt{(1-\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t))\times(1-\mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t))}}}{\pi^{-1}-1}\right| \tag{26}$$

If we put $\bar{\mathscr{L}}_1 = \bar{\mathscr{L}}_2$ in Eq. (26), we get

$$\mathbb{D}^{\varphi}Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2) = \mathbb{D}^{\varphi}(\bar{\mathscr{L}}_1)$$

Theorem 3 : Let $\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2 \in FS(\check{\breve{U}}) \neq 0$. Define a mapping $\mathbb{D}^{\varphi}Accu : FS(\check{\breve{U}}) \times FS(\check{\breve{U}}) \rightarrow [0,1]$ given in eq(25).Then $\mathbb{D}^{\varphi}Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2)$ is an appropriate measure of accuracy if it meets the following requirements ;

1. $\mathbb{D}^{\varphi}Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2)$ has maximum value , i.e., equals to 1 *iff*

   $$\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t) = 0 = \mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t) \quad \text{or} \quad \mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t) = 1 = \mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t),$$

   i.e., both $\bar{\mathscr{L}}_1$ and $\bar{\mathscr{L}}_2$ are crisp sets. particularly, if $\bar{\mathscr{L}}_1 = \bar{\mathscr{L}}_1$, then

   $$\mathbb{D}^{\varphi}Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2) = \mathbb{D}^{\varphi}(\bar{\mathscr{L}})$$

   where $\mathbb{D}^{\varphi}(\bar{\mathscr{L}})$ is fuzzy knowledge measure.
   Figure 2 presents the graphical representation of the proposed fuzzy accuracy measure, illustrating its variation with respect to the membership and non-membership degrees.

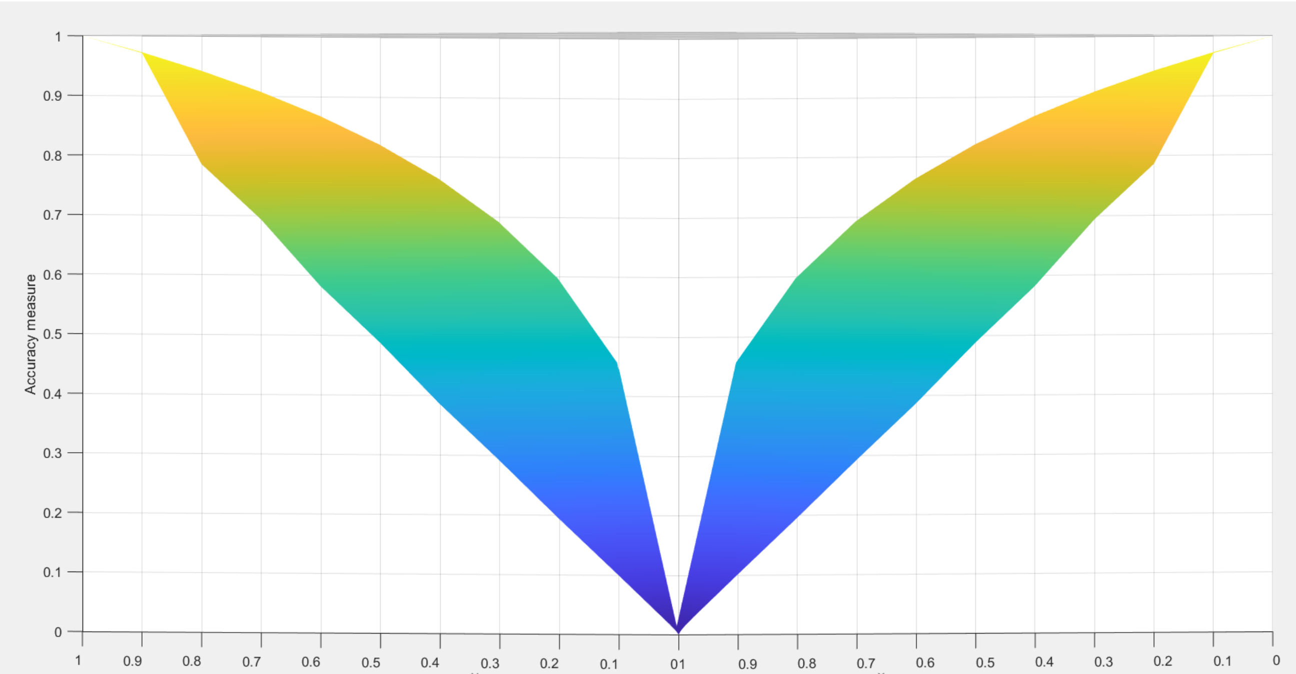

Fig. 2. Accuracy measure

2. If $\bar{\mathscr{L}_1}, \bar{\mathscr{L}_2}, \bar{\mathscr{L}_3} \in FS(\check{\bar{U}})$ such that $\bar{\mathscr{L}_1} \subseteq \bar{\mathscr{L}_2} \subseteq \bar{\mathscr{L}_3}$, then

    (i.) $\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \bar{\mathscr{L}_2}) \geq \mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \bar{\mathscr{L}_3})$ for $0 < \mu_{\mathcal{T}_1} \leq 0.5$

    (ii.) $\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \bar{\mathscr{L}_2}) \leq \mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \bar{\mathscr{L}_3})$ for $0.5 < \mu_{\bar{\mathscr{L}_1}} \leq 1$

3. If $\bar{\mathscr{L}_1}, \bar{\mathscr{L}_2} \in FS(\check{\bar{U}})$, then ...

(i.)

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}}1, \bar{\mathscr{L}_2}) = \mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}}1^c, \bar{\mathscr{L}_2^c})$$

(ii.)

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \mathcal{T}_2) = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\pi^{-\sqrt{\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t)\mu_{\bar{\mathscr{L}_2}}(\bar{\Psi}_t)}} - \pi^{-\sqrt{(1-\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t))(1-\mu_{\bar{\mathscr{L}_2}}(\bar{\Psi}_t))}}}{\pi^{-1} - 1} \right|$$

(iii.)

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \bar{\mathscr{L}_2^c}) = \mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1^c}, \bar{\mathscr{L}_2})$$

Proof:

1. Let us assume that $\bar{\mathscr{L}_1}$ and $\bar{\mathscr{L}_2}$ are crisp sets, i.e., $\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t) = 0 = \mu_{\bar{\mathscr{L}_2}}(\bar{\Psi}_t)$ or $\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t) = 1 = \mu_{\bar{\mathscr{L}_2}}(\bar{\Psi}_t)$, then

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \mathcal{T}_2) = \frac{1}{n} \sum i = 1^n \left| \frac{\pi^{-}\sqrt{\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t) \times \mu_{\bar{\mathscr{L}_2}}((\bar{\Psi}_t)} - \pi^{-}\sqrt{(1-\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t)) \times (1-\mu_{\bar{\mathscr{L}_2}}(\bar{\Psi}_t))}}{\pi^{-1} - 1} \right|$$

If we put $\mu_{\bar{\mathscr{L}_1}}(\bar{\Psi}_t) = 0 = \mu_{\bar{\mathscr{L}_2}}(\bar{\Psi}_t)$ in the above accuracy measure, then

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}_1}, \bar{\mathscr{L}_2}) = \frac{1}{n} \sum i = 1^n \, (1) = 1$$

Similarly, if we put $\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t) = 1 = \mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)$ in the above expression, then we get

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2) = \frac{1}{n} \sum i = 1^n \, (1) = 1$$

So,in both cases

$$\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}}_1, \bar{\mathscr{L}}_2) = 1$$

Thus, $\mathbb{D}^{\varphi} Accu(\bar{\mathscr{L}}1, \bar{\mathscr{L}}_2)$ is maximum if $\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t) = 0 = \mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)$ or $\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t) = 1 = \mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t)$. Also,if we put $\bar{\mathscr{L}}_1 = \bar{\mathcal{T}}_2$, then eq.(26) transforms into eq.(23).

2. Examine a function

$$y(y_1, y_2) = \left| \pi^{-\sqrt{y_1 y_2}} - \pi^{-\sqrt{(1-y_1)(1-y_2)}} \right|$$

It has been proved in the previous part that $\frac{\partial y}{\partial y_1}$ and $\frac{\partial y}{\partial y_2}$ are negative for $y_1, y_2 \in [0, 0.5]$ and positive for $y_1, y_2 \in [0.5, 1]$.

So, $y(y_1, y_2)$ exhibits a decreasing pattern within the interval $[0, 0.5]$ and an increasing trend in the interval $[0.5, 1]$.

As $\bar{\mathscr{L}}_1 \subseteq \bar{\mathscr{L}}_2 \subseteq \bar{\mathscr{L}}_3$, so

$$\mu_{\bar{\mathscr{L}}_1}(\bar{\Psi}_t) \leq \mu_{\bar{\mathscr{L}}_2}(\bar{\Psi}_t) \leq \mu_{\bar{\mathscr{L}}_3}(\bar{\Psi}_t).$$

Hence the result.

3. Clearly, from the definition of FAM given in eq.(26) all the properties are trivially true.

5.1 *Application of the Suggested FAM in identifying pattern*

The proposed accuracy measurement helps in analysing patterns by measuring similarities between fuzzy representations, allowing efficient comparison, classification, and recognition in uncertain conditions. In the following example,We analyse the proposed FAM with current accuracy measures for assessing pattern similarity.

Example 5 Consider six pattern in fuzzy environment i.e. $\bar{\mathscr{L}}_j$ where,$1 \leq j \leq 5$ given below :

$\bar{\mathscr{L}}$= $\left\{ \left(\bar{\Psi}_1, 0.5\right), \left(\bar{\Psi}_2, 0.3\right), \left(\bar{\Psi}_3, 0.25\right), \left(\bar{\Psi}_4, 0.7\right) \right\}$,
$\bar{\mathscr{L}}_1 = \left\{ \left(\bar{\Psi}_1, 0.2\right), \left(\bar{\Psi}_2, 0.15\right), \left(\bar{\Psi}_3, 0.3\right), \left(\bar{\Psi}_4, 0.35\right) \right\}$,
$\bar{\mathscr{L}}_2 = \left\{ \left(\bar{\Psi}_1, 0.2\right), \left(\bar{\Psi}_2, 0.4\right), \left(\bar{\Psi}_3, 0.25\right), \left(\bar{\Psi}_4, 0.15\right) \right\}$,
$\bar{\mathscr{L}}_3 = \left\{ \left(\bar{\Psi}_1, 0.9\right), \left(\bar{\Psi}_2, 0.25\right), \left(\bar{\Psi}_3, 0.3\right), \left(\bar{\Psi}_4, 0.5\right) \right\}$,
$\bar{\mathscr{L}}_4 = \left\{ \left(\bar{\Psi}_1, 0.2\right), \left(\bar{\Psi}_2, 0.4\right), \left(\bar{\Psi}_3, 0.3\right), \left(\bar{\Psi}_4, 0.1\right) \right\}$,
$\bar{\mathscr{L}}_5 = \left\{ \left(\bar{\Psi}_1, 0.1\right), \left(\bar{\Psi}_2, 0.4\right), \left(\bar{\Psi}_3, 0.3\right), \left(\bar{\Psi}_4, 0.15\right) \right\}$.

Table 3
Established values through various FKM for distinct fuzzy sets

Fuzzy Sets	VK[8]	MG[31]	Singh[12]	SG[29]	Proposed FAM
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_1)$	.15826	.27974	.1753	.70785	.33471
$(\bar{\mathcal{T}}, \bar{\mathscr{L}}_2)$	.10506	.25246	.1374	.68677	.31698
$(\bar{\mathscr{L}}, \bar{\mathcal{T}}_3)$	**.16485**	**.30001**	**.1801**	**.71009**	**.37892**
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_4)$	.08884	.2494	.1260	.67909	.32622
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_5)$	.09609	.25593	.1304	.68079	.34117
Result	$\bar{\mathscr{L}}_3$	$\bar{\mathscr{L}}_3$	$\bar{\mathscr{L}}_3$	$\bar{\mathscr{L}}_3$	$\bar{\mathscr{L}}_3$

We took, α=1.5 and β=2 in SG[29], also α=1.9 in Singh[12]

Let us examine the similarity of different patterns using proposed and existing $FAMs$. We say pattern $\bar{\mathscr{L}}$ has maximum similarity with pattern $\bar{\mathscr{L}}j$ where,$1 \leq j \leq 5$ when we have maximum FAM value between $\bar{\mathscr{L}}$ and $\bar{\mathscr{L}}j$.Then we say pattern $\bar{\mathscr{L}}$ belongs to pattern $\bar{\mathscr{L}}_j$.
Table 3 presents the accuracy measure values calculated between $\bar{\mathscr{L}}$ and $\bar{\mathscr{L}}j$ using the proposed FAM together with several existing accuracy measures. As shown in Table 3, all the compared measures identify $\bar{\mathscr{L}}4$ as the pattern most similar to $\bar{\mathscr{L}}$. The proposed accuracy measure reaches the same conclusion, confirming its consistency with the existing approaches in this pattern recognition example. *Example 6* Consider six pattern in fuzzy environment i.e. $\bar{\mathscr{L}}j$ where,$1 \leq j \leq 5$ given below :

$\bar{\mathscr{L}}$= $\{(\bar{\Psi}_1, 0.5), (\bar{\Psi}_2, 0.1), (\bar{\Psi}_3, 0.25), (\bar{\Psi}_4, 0.3)\}$,
$\bar{\mathscr{L}}_1 = \{(\bar{\Psi}_1, 0.1), (\bar{\Psi}_2, 0.4), (\bar{\Psi}_3, 0.3), (\bar{\Psi}_4, 0.15)\}$,
$\bar{\mathscr{L}}_2 = \{(\bar{\Psi}_1, 0.75), (\bar{\Psi}_2, 0.45), (\bar{\Psi}_3, 0.23), (\bar{\Psi}_4, 0.15)\}$,
$\bar{\mathscr{L}}_3 = \{(\bar{\Psi}_1, 0.19), (\bar{\Psi}_2, 0.45), (\bar{\Psi}_3, 0.15), (\bar{\Psi}_4, 0.2)\}$,
$\bar{\mathscr{L}}_4 = \{(\bar{\Psi}_1, 0.09), (\bar{\Psi}_2, 0.45), (\bar{\Psi}_3, 0.15), (\bar{\Psi}_4, 0.2)\}$,
$\bar{\mathscr{L}}_5 = \{(\bar{\Psi}_1, 0.95), (\bar{\Psi}_2, 0.18), (\bar{\Psi}_3, 0.45), (\bar{\Psi}_4, 0.195)\}$.

Table 4
Established values through various FKM for distinct fuzzy sets

Fuzzy Sets	VK[8]	MG[31]	Singh[12]	SG[29]	Proposed FAM
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_1)$	.2752	.40507	**.2620**	.73445	.49136
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_2)$	.27365	.39046	**.2620**	**.7378**	.44756
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_3)$	**.27985**	.40107	.2662	**.7378**	.47261
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_4)$	**.27985**	**.4113**	.2650	.73374	.51014
$(\bar{\mathscr{L}}, \bar{\mathscr{L}}_5)$	.3005	**.4113**	.2821	.74372	.49005
Result	Inappropriate	Inappropriate	Inappropriate	Inappropriate	$\bar{\mathscr{L}}_5$

we take α=1.9 in Singh[12]
As shown in Table 4, the existing accuracy measures produce identical similarity values for different patterns, making it impossible to uniquely classify the unidentified pattern into a known pattern. In contrast, the proposed FAM generates distinct similarity values and correctly classifies the unidentified pattern as $\bar{\mathscr{L}}_5$, demonstrating its effectiveness and improved discriminative capability in pattern recognition.

5.2 *Entropy Weighting Technique (EWT)*

The entropy weighting method (EWT) is one of the most widely used objective techniques for determining criterion weights, as it evaluates the variation among criteria without relying on subjective judgments or additional external information. The method assigns weights according to the amount of information contained in each criterion, which is reflected by the dispersion of its values. Criteria with greater variability provide more information and are therefore assigned higher weights, increasing their influence on the overall evaluation. Information-theoretic entropy [32], [33] is employed to quantify the degree of variation among the criteria. In EWT, the entropy value of the j-th criterion is used to determine its relative importance. A criterion with low variability among the alternatives provides limited discriminative information and therefore receives a lower weight [34]. The mathematical formulation of the EWT is presented in Section 6 through Eqs. (27)–(30).

5.3 TODIM

The TODIM (an acronym in Portuguese for Interactive and Multicriteria Decision Making) method, originally introduced by [35], is a well-established multi-criteria decision-making approach for solving complex decision problems. The method has been successfully applied in a wide range of application domains, including financial decision-making, investment analysis, urban planning, infrastructure development, and water resource management.

TODIM is based on Prospect Theory and explicitly considers the decision-makers' perception of gains and losses instead of assuming completely rational behaviour. This characteristic enables the method to model realistic decision-making preferences under uncertainty. Owing to its solid theoretical foundation and rigorous mathematical framework, TODIM has been widely recognized as an effective and reliable tool for addressing multi-criteria decision-making problems [36], [37], [38]. The mathematical formulation of the TODIM method is presented in Section 6 through Eqs. (31)–(34).

5.4 The Proposed Model

The proposed hybrid triangular fuzzy decision-making framework integrates the entropy weighting method with the TODIM approach to evaluate and compare the quality of e-commerce websites under uncertain conditions. The overall workflow of the proposed framework is illustrated in Figure 3. The methodology consists of three sequential stages, which are described below.

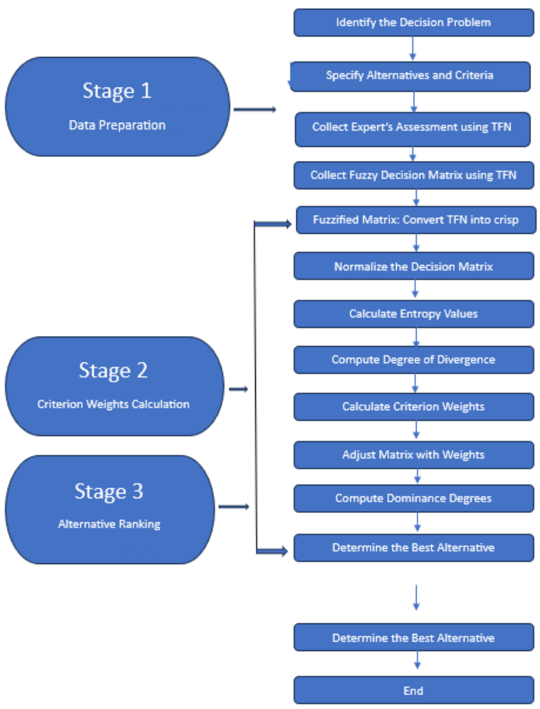

Fig. 3. Flow diagram for the suggested model

Stage 1. Establish the Decision Matrix(DM) utilising TFN
*Step 1.1*Specify the Criteria and Alternatives $\breve{\mathfrak{Ct}}_1, \breve{\mathfrak{Ct}}_2, \breve{\mathfrak{Ct}}_3, \ldots, \breve{\mathfrak{Ct}}_{11}$ for Criteria and $\breve{\mathfrak{At}}^1, \breve{\mathfrak{At}}^2, \breve{\mathfrak{At}}^3, \breve{\mathfrak{At}}^4, \breve{\mathfrak{At}}^5$ for Alternatives
Step 1.2 Gather Expert Views
Step 1.3 Establish the DM utilizing the score function presented in Equation (9).
Step 1.4 The matrix of decision contributors should be aggregated.

Stage 2. Execute the Entropy Weight Method (EWM): Below are the equations of EWT:
Step 2.1 DM Normalization
To achieve uniformity among various criteria, normalize the triangular fuzzy set decision matrix in the following manner:

$$\mathtt{n_{ij}} = \frac{d_{ij}}{\sum_{i=1}^{\hat{p}} d_{ij}} \text{ Where } 1 \leq i \leq \hat{p}, 1 \leq j \leq \hat{q} \tag{27}$$

Where, $\sum_{i=1}^{\hat{p}} d_{ij}$ is the aggregated matrix's total value for each criterion, computed for each column.
Step 2.2 Determine the Entropy for Each Criterion j utilizing a Nrml.DM, as outlined:

$$\breve{\mathcal{E}}_j = \frac{1}{\hat{p}} \sum_{i=1}^{\hat{p}} \left| \frac{\pi^{-\mathtt{n_{ij}}} - \pi^{-(1-\mathtt{n_{ij}})}}{\pi^{-1} - 1} \right| \tag{28}$$

Where $1 \leq j \leq \hat{q}$ and $\hat{p}$ is the list of options in ecommerce websites.
Step 2.3 Determine the degree of divergence across each criterion as shown below:

$$\breve{\mathcal{D}}_j = \left| 1 - \breve{\mathcal{E}}_j \right| \text{ Where } 1 \leq j \leq \hat{q} \tag{29}$$

Step 2.4 Determine weights for the criteria by applying the subsequent equation

$$\mathrm{W}_j = \frac{\breve{\mathcal{D}}_j}{\sum_{i=1}^{\hat{p}} \breve{\mathcal{D}}_j} \tag{30}$$

Stage 3. Apply TODIM Method Below are the mathematical equations of TODIM to place the alternatives according to the weight of each criterion[39] :
Step 3.1 Establishment of the DM.
Step 3.2 Normalize the DM.

$$\mathtt{n_{ij}} = \begin{cases} \frac{d_{ij}}{\sum_{i=1}^{\hat{p}} d_{ij}} & \text{for Beneficial criteria} \\ \frac{1/d_{ij}}{\sum_{i=1}^{\hat{p}} 1/d_{ij}} & \text{for Non-Beneficial criteria} \end{cases} \tag{31}$$

Step 3.3 Establish relative weight criteria using global weights of the criterion obtained from the following equation:

$$\bar{\mathrm{W}}_J = \frac{\mathrm{W}_j}{\breve{\mathrm{W}}_{max}} \tag{32}$$

Where, $\breve{\mathrm{W}}_{max}$ is the maximum value of weights
Step 3.4 Determine the dominance degree of the alternatives, for every combination of alternatives ($\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j$) calculate the $\delta\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right)$ as:

$$\boldsymbol{\sigma}\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right) = \sum_{j=1}^{\hat{p}} \lambda\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right) \quad \forall (i, j)$$

$$\lambda\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right) = \begin{cases} \sqrt{\frac{\mathrm{W}_j(\hat{r_i} - \hat{r_j})}{\sum_{j=1}^{\hat{q}} \bar{\mathrm{W}}_J}} & \text{if } (\hat{r_i} - \hat{r_j}) > 0 \\ 0 & \text{if } (\hat{r_i} - \hat{r_j}) = 0 \\ \frac{-1}{\theta} \sqrt{\frac{\sum_{j=1}^{\hat{q}} \bar{\mathrm{W}}_J(\hat{r_i} - \hat{r_j})}{\mathrm{W}_j}} & \text{if } (\hat{r_i} - \hat{r_j}) < 0 \end{cases} \tag{33}$$

Where, θ is the attenuation factor for disruptions varies between 1 and 10 .
Step 3.5 Assess the overall dominance level of each option as follows:

$$\bar{\zeta}_i = \frac{\sum_{j=1}^{\hat{q}} \sigma\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right) - \min \sum_{j=1}^{\hat{q}} \sigma\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right)}{\max \sum_{j=1}^{\hat{q}} \sigma\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right) - \min \sum_{j=1}^{\hat{q}} \sigma\left(\breve{\mathbb{A}}_i, \breve{\mathbb{A}}_j\right)} \tag{34}$$

Step 3.6 Rank alternatives according to their overall dominance values to detect the best alternative.

6. Case Study

The following section provides a comprehensive case study aimed at analysing the performance of five prominent e-commerce websites to assess the validity of the suggested approach. Figure 4 illustrates the organised framework for assessing these online retail platforms.

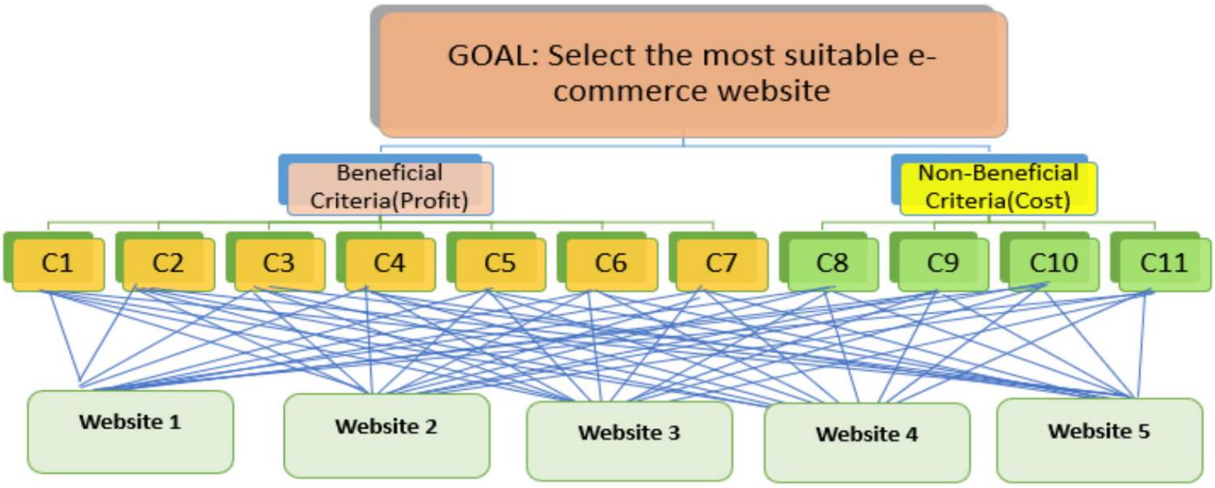

Fig. 4. The hierarchical structure of e-commerce website evaluation

6.1 *Establishing the Decision Matrix Using TNNs*

Step 6.1.1. Define the Criteria and the Alternatives:
Criteria are represented as $\breve{\mathfrak{C}}\mathfrak{t}_1, \breve{\mathfrak{C}}\mathfrak{t}_2, \breve{\mathfrak{C}}\mathfrak{t}_3, \ldots, \breve{\mathfrak{C}}\mathfrak{t}_{11}$ and alternatives as $\breve{\mathfrak{A}}\mathfrak{t}_1, \breve{\mathfrak{A}}\mathfrak{t}_2, \breve{\mathfrak{A}}\mathfrak{t}_3, \breve{\mathfrak{A}}\mathfrak{t}_4, \breve{\mathfrak{A}}\mathfrak{t}_5$. Five leading e-commerce websites from the USA, China, Japan, Egypt, and Russia were selected, as they offer consumers a wide range of products, including household appliances and technological devices. Based on studies by[[40],[41],[42],[43],[44],[45]], Eleven assessment standards were identified as the most significant factors for the assessment of platforms for online commerce. Out of these eleven criteria, seven are classified as being beneficial (profit-oriented), while four are regarded as non-beneficial (poor performance). The five e-commerce platforms were assessed according to the criteria presented in Tables 5 and 6.

Table 5
The selected criteria for alternative evaluation

$\check{\mathfrak{C}}\mathfrak{t}_i$	Criterion	Type	Description	Benefits	References
$\check{\mathfrak{C}}\mathfrak{t}_1$	Website Usability	Beneficial	Measures how easy and intuitive the website is for users, focusing on smooth navigation, clear layout, and an effective search function.	Simple exploration, organized presentation, and effective querying, improving user experience and boosting curiosity.	[44], [46]
$\check{\mathfrak{C}}\mathfrak{t}_2$	Search Functionality Quality	Beneficial	Evaluates how efficiently the website's search delivers accurate results, supports filters and suggestions, and handles errors or partial keywords.	Speeds product discovery, improves user experience, and boosts conversions.	[44], [41], [47]
$\check{\mathfrak{C}}\mathfrak{t}_3$	Security and Privacy Protection	Beneficial	Checks the site's capacity to safeguard user information via SSL/HTTPS, secure payment systems, and strong privacy protocols.	Establishes confidence, deters deception, and promotes ongoing transactions.	[48], [49], [45], [46]
$\check{\mathfrak{C}}\mathfrak{t}_4$	Data Integrity	Beneficial	Evaluates the exactness, accuracy, and immediateness of the product details presented on the website.	Ensures informed decisions, reduces errors, and builds trust.	[45]
$\check{\mathfrak{C}}\mathfrak{t}_5$	Price, Payment Options and Convenience	Beneficial	Evaluates the variety, cost, security, and ease of payment methods available to customers.	Simplifies purchasing, enhances conversion rates, and fosters customer confidence.	[40], [50], [51]
$\check{\mathfrak{C}}\mathfrak{t}_6$	Delivery Efficiency	Beneficial	Measures the speed, reliability, and coverage of product delivery, including tracking and timely fulfillment.	Ensures timely fulfillment, boosts satisfaction, and reduces complaints.	[49], [52]

Table 6
The selected criteria for alternative evaluation (continued)

$\check{\mathfrak{C}}\mathfrak{t}_i$	Criterion	Type	Description	Benefits	References
$\check{\mathfrak{C}}\mathfrak{t}_7$	Customer Reviews Ratings	Beneficial	Evaluates user feedback by analyzing average ratings and the proportion of positive reviews to gauge customer satisfaction and confidence.	Builds trust, guides buyers, and enhances reputation.	[45], [47]
$\check{\mathfrak{C}}\mathfrak{t}_8$	Website Error Rate	Non-beneficial	Examines website failures, such as crashes, broken links, or unsuccessful transactions, which hinder user interaction.	Minimizing errors improves reliability, user satisfaction, and trust.	[47]
$\check{\mathfrak{C}}\mathfrak{t}_9$	Issues in Order Handling	Non-beneficial	Assesses variations in shipments, including incorrect items or mismatched orders.	Reducing errors improves trust, reduces returns, and enhances efficiency.	[51], [53], [54]
$\check{\mathfrak{C}}\mathfrak{t}_{10}$	Policies for Returned Items and Refunds	Non-beneficial	Encompasses product exchange, return, and refund procedures, which should be straightforward and customer-friendly.	Reduces purchase risk and fosters greater trust, thereby supporting sales.	[41], [55]
$\check{\mathfrak{C}}\mathfrak{t}_{11}$	Extensions in Delivery	Non-beneficial	Quantifies the frequency of delayed deliveries affecting the anticipated customer schedule.	Minimizing delays improves satisfaction, retention, and reduces complaints.	[53], [54]

Step 6.1.2. Obtain expert views
The demographic characteristics of the experts involved in the evaluation are presented in Table 7.

Table 7
List of experts and their expertise

Expert	Job Title	Age and gender	Experience	Expertise
1	Digital Commerce Director	52 ,Male	20 years	broad range of strategic, technical, and business aspects.
2	Social Media Manager	45 ,Female	18 years	creating, managing, and optimizing an organization’s presence across social media platforms
3	Merchandising Specialist	35, Female	11 years	managing and optimizing the presentation,assortment, and sales of products
4	E-Commerce Growth Manager	36, Male	13 years	driving business expansion through data-driven strategies, marketing, and product optimization
5	Digital Marketing Executive	38, Female	12 years	running and managing online advertising campaigns⍰to promote a⍰company’s goods or services
6	Software Developer	30, Female	8 years	System Engineering

Collect expert assessments of all possibilities based on each standard, shown as TFN using linguistic expressions as outlined in Table 8 to produce the individual decision matrices presented in Tables 9–14.

Table 8
Linguistic Scale modified into the triangular Scale

Scale	Linguistic Scale	Triangular Scale
1	Low influential	$1 = \langle\langle 1, 1, 1\rangle; 0.2\rangle$
3	Moderately influential	$3 = \langle\langle 2, 3, 4\rangle; 0.06\rangle$
5	High influential	$5 = \langle\langle 4, 5, 6\rangle; 0.25\rangle$
7	Very high influential	$7 = \langle\langle 6, 7, 8\rangle; 0.25\rangle$
9	Absolutely high (Extreme)	$9 = \langle\langle 9, 9, 9\rangle; 0.9\rangle$
2,4,6,8	Sporadic values within two adjacent scales	$2 = \langle\langle 1, 2, 3\rangle; 0.10\rangle$
	(Intermediate values)	$4 = \langle\langle 3, 4, 5\rangle; 0.30\rangle$
		$6 = \langle\langle 5, 6, 7\rangle; 0.30\rangle$
		$8 = \langle\langle 7, 8, 9\rangle; 0.40\rangle$

Table 8 illustrates that each linguistic term signifies a qualitative assessment of the relative impact of a criterion, whereas the triangular fuzzy scale (⍰a,b,c⍰; T) reflects the aspect of uncertainty. The triangular fuzzy scale consists of TFNs represented as <(a, b, c);T> where, T: Truth-membership.
For example, a high influential value can be expressed as 5=⍰⍰4,5,6⍰;0.25⍰, indicating that the triangular fuzzy⍰number is ⍰4,5,6⍰ (with 5⍰being the most probable, but it may vary between 4⍰and 6). At T=0.25, there is a low level of confidence regarding ”high influence.”

Table 9
Decision matrix of Expert's 1 opinion

	$\breve{\mathfrak{At}}^1$	$\breve{\mathfrak{At}}^2$	$\breve{\mathfrak{At}}^3$	$\breve{\mathfrak{At}}^4$	$\breve{\mathfrak{At}}^5$
$\breve{\mathfrak{Ct}}_1$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{Ct}}_2$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{Ct}}_3$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{Ct}}_4$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{Ct}}_5$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_6$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_7$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{Ct}}_8$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 1,1,1\rangle;0.2\rangle$
$\breve{\mathfrak{Ct}}_9$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$
$\breve{\mathfrak{Ct}}_{10}$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$
$\breve{\mathfrak{Ct}}_{11}$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$

Table 10
Decision matrix of Expert's 2 opinion

	$\breve{\mathfrak{At}}^1$	$\breve{\mathfrak{At}}^2$	$\breve{\mathfrak{At}}^3$	$\breve{\mathfrak{At}}^4$	$\breve{\mathfrak{At}}^5$
$\breve{\mathfrak{Ct}}_1$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_2$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{Ct}}_3$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{Ct}}_4$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{Ct}}_5$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_6$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{Ct}}_7$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_8$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_9$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$
$\breve{\mathfrak{Ct}}_{10}$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,1,1\rangle;0.20\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$
$\breve{\mathfrak{Ct}}_{11}$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.03\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$

Table 11
Decision matrix of Expert's 3 opinion

	$\breve{\mathfrak{At}}^1$	$\breve{\mathfrak{At}}^2$	$\breve{\mathfrak{At}}^3$	$\breve{\mathfrak{At}}^4$	$\breve{\mathfrak{At}}^5$
$\breve{\mathfrak{Ct}}_1$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_2$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_3$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_4$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_5$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_6$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_7$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$
$\breve{\mathfrak{Ct}}_8$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_9$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$
$\breve{\mathfrak{Ct}}_{10}$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{Ct}}_{11}$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$

Table 12
Decision matrix of Expert's 4 opinion

	$\breve{\mathfrak{A}}\mathfrak{t}^1$	$\breve{\mathfrak{A}}\mathfrak{t}^2$	$\breve{\mathfrak{A}}\mathfrak{t}^3$	$\breve{\mathfrak{A}}\mathfrak{t}^4$	$\breve{\mathfrak{A}}\mathfrak{t}^5$
$\breve{\mathfrak{C}}\mathfrak{t}_1$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_2$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_3$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_4$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_5$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_6$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_7$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_8$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,1,1\rangle;0.2\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_9$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$
$\mathfrak{Ctr}_{10}$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_{11}$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$

Table 13
Decision matrix of Expert's 5 opinion

	$\breve{\mathfrak{A}}\mathfrak{t}^1$	$\breve{\mathfrak{A}}\mathfrak{t}^2$	$\breve{\mathfrak{A}}\mathfrak{t}^3$	$\breve{\mathfrak{A}}\mathfrak{t}^4$	$\breve{\mathfrak{A}}\mathfrak{t}^5$
$\breve{\mathfrak{C}}\mathfrak{t}_1$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_2$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_3$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.90\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_4$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_5$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_6$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_7$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_8$	$\langle\langle 3,4,5\rangle;30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_9$	$\langle\langle 3,4,5\rangle;30\rangle$	$\langle\langle 3,4,5\rangle;30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;30\rangle$	$\langle\langle 3,4,5\rangle;30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_{10}$	$\langle\langle 3,4,5\rangle;30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_{11}$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 3,4,5\rangle;30\rangle$	$\langle\langle 3,4,5\rangle;30\rangle$

Table 14
Decision matrix of Expert's 6 opinion

	$\breve{\mathfrak{A}}\mathfrak{t}^1$	$\breve{\mathfrak{A}}\mathfrak{t}^2$	$\breve{\mathfrak{A}}\mathfrak{t}^3$	$\breve{\mathfrak{A}}\mathfrak{t}^4$	$\breve{\mathfrak{A}}\mathfrak{t}^5$
$\breve{\mathfrak{C}}\mathfrak{t}_1$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_2$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_3$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$	$\langle\langle 9,9,9\rangle;0.9\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_4$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_5$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_6$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 3,4,5\rangle;30\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_7$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$	$\langle\langle 5,6,7\rangle;0.30\rangle$	$\langle\langle 6,7,8\rangle;0.25\rangle$	$\langle\langle 7,8,9\rangle;0.40\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_8$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 1,1,1\rangle;0.2\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_9$	$\langle\langle 1,1,1\rangle;0.2\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_{10}$	$\langle\langle 4,5,6\rangle;0.25\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$
$\breve{\mathfrak{C}}\mathfrak{t}_{11}$	$\langle\langle 3,4,5\rangle;0.30\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$	$\langle\langle 1,2,3\rangle;0.10\rangle$	$\langle\langle 2,3,4\rangle;0.06\rangle$

Step 6.1.3. Use Eq. (2) and Eq. (6) to form the decision maker's matrix to produce the matrix shown in Table 15.

Table 15
Aggregated Triangular Fuzzy Decision Matrix

	$\breve{\mathfrak{At}}^1$	$\breve{\mathfrak{At}}^2$	$\breve{\mathfrak{At}}^3$	$\breve{\mathfrak{At}}^4$	$\breve{\mathfrak{At}}^5$
$\breve{\mathfrak{Ct}}_1$	$\langle\langle 4, 6.83, 9\rangle; 0.2\rangle$	$\langle\langle 1, 6.5, 9\rangle; 0.10\rangle$	$\langle\langle 2, 6.83, 9\rangle; 0.06\rangle$	$\langle\langle 3, 7.83, 9\rangle; 0.25\rangle$	$\langle\langle 3, 7, 9\rangle; 0.25\rangle$
$\breve{\mathfrak{Ct}}_2$	$\langle\langle 3, 7.166, 9\rangle; 0.30\rangle$	$\langle\langle 1, 6.833, 9\rangle; 0.10\rangle$	$\langle\langle 2, 6.166, 9\rangle; 0.06\rangle$	$\langle\langle 3, 7.83, 9\rangle; 0.25\rangle$	$\langle\langle 3, 7, 9\rangle; 0.25\rangle$
$\breve{\mathfrak{Ct}}_3$	$\langle\langle 1, 6.166, 9\rangle; 0.10\rangle$	$\langle\langle 3, 6.833, 9\rangle; 0.25\rangle$	$\langle\langle 2, 6.666, 9\rangle; 0.06\rangle$	$\langle\langle 3, 6.83, 9\rangle; 0.25\rangle$	$\langle\langle 3, 7.5, 9\rangle; 0.25\rangle$
$\breve{\mathfrak{Ct}}_4$	$\langle\langle 5, 7, 8\rangle; 0.25\rangle$	$\langle\langle 2, 6.166, 9\rangle; 0.06\rangle$	$\langle\langle 4, 6.5, 9\rangle; 0.25\rangle$	$\langle\langle 4, 6.666, 9\rangle; 0.25\rangle$	$\langle\langle 3, 6.5, 9\rangle; 0.30\rangle$
$\breve{\mathfrak{Ct}}_5$	$\langle\langle 3, 7.166, 9\rangle; 0.25\rangle$	$\langle\langle 2, 6.33, 9\rangle; 0.06\rangle$	$\langle\langle 4, 6.166, 9\rangle; 0.25\rangle$	$\langle\langle 2, 6.833, 9\rangle; 0.06\rangle$	$\langle\langle 3, 6.833, 9\rangle; 0.25\rangle$
$\breve{\mathfrak{Ct}}_6$	$\langle\langle 2, 6.333, 9\rangle; 0.06\rangle$	$\langle\langle 2, 6.166, 9\rangle; 0.25\rangle$	$\langle\langle 2, 6.833, 9\rangle; 0.25\rangle$	$\langle\langle 2, 6.166, 9\rangle; 0.06\rangle$	$\langle\langle 3, 6.166, 9\rangle; 0.25\rangle$
$\breve{\mathfrak{Ct}}_7$	$\langle\langle 3, 6.833, 9\rangle; 0.25\rangle$	$\langle\langle 3, 7.166, 9\rangle; 0.25\rangle$	$\langle\langle 4, 6.666, 9\rangle; 0.25\rangle$	$\langle\langle 1, 5.833, 8\rangle; 0.25\rangle$	$\langle\langle 4, 7.666, 9\rangle; 0.25\rangle$
$\breve{\mathfrak{Ct}}_8$	$\langle\langle 1, 3.833, 6\rangle; 0.06\rangle$	$\langle\langle 2, 4.166, 6\rangle; 0.06\rangle$	$\langle\langle 1, 3.5, 6\rangle; 0.10\rangle$	$\langle\langle 1, 3.166, 7\rangle; 0.06\rangle$	$\langle\langle 1, 2.5, 7\rangle; 0.10\rangle$
$\breve{\mathfrak{Ct}}_9$	$\langle\langle 1, 2.333, 5\rangle; 0.06\rangle$	$\langle\langle 2, 3.833, \rangle); 06\rangle$	$\langle\langle 1, 4, 6\rangle; 0.06\rangle$	$\langle\langle 1, 3.166, 6\rangle; 0.06\rangle$	$\langle\langle 1, 2.66, 5\rangle; 0.06\rangle$
$\breve{\mathfrak{Ct}}_{10}$	$\langle\langle 1, 3.66, 6\rangle; 0.06\rangle$	$\langle\langle 1, 2.833, 6\rangle; 0.06\rangle$	$\langle\langle 1, 3.333, 5\rangle; 0.06\rangle$	$\langle\langle 1, 3.166, 5\rangle; 0.06\rangle$	$\langle\langle 1, 3.33, 6\rangle; 0.06\rangle$
$\breve{\mathfrak{Ct}}_{11}$	$\langle\langle 1, 3.5, 6\rangle; 0.06\rangle$	$\langle\langle 2, 4, 6\rangle; 0.06\rangle$	$\langle\langle 1, 2.666, 6\rangle; 0.06\rangle$	$\langle\langle 1, 3.166, 5\rangle; 0.06\rangle$	$\langle\langle 1, 3.66, 6\rangle; 0.06\rangle$

Step 6.1.4 Determine the score function utilizing Eq. (9) as presented in Table 16.

Table 16
Aggregated Decision Matrix

	$\breve{\mathfrak{Ct}}_1$	$\breve{\mathfrak{Ct}}_2$	$\breve{\mathfrak{Ct}}_3$	$\breve{\mathfrak{Ct}}_4$	$\breve{\mathfrak{Ct}}_5$	$\breve{\mathfrak{Ct}}_6$	$\breve{\mathfrak{Ct}}_7$	$\breve{\mathfrak{Ct}}_8$	$\breve{\mathfrak{Ct}}_9$	$\breve{\mathfrak{Ct}}_{10}$	$\breve{\mathfrak{Ct}}_{11}$
$\breve{\mathfrak{At}}^1$	6.665	6.5830	5.583	6.75	6.583	5.9165	6.416	3.665	2.6665	3.58	3.5
$\breve{\mathfrak{At}}^2$	5.75	5.9165	6.4165	5.833	5.915	5.833	6.583	4.083	3.9165	3.1665	4
$\breve{\mathfrak{At}}^3$	6.165	5.833	6.08	6.5	6.333	6.166	6.58	3.5	3.75	3.165	3.083
$\breve{\mathfrak{At}}^4$	6.9150	6.5	6.415	6.58	6.1665	5.833	5.15	3.583	3.33	3.083	3.083
$\breve{\mathfrak{At}}^5$	6.5	6.5	6.75	6.25	6.4165	6.083	7.08	3.25	2.83	3.415	3.58

Using score function TFNs are converted into a crisp number in Aggr. Decision Matrix, as provided in the preceding table.

6.2 Application of the Entropy Weight Technique

Step 6.2.1. Using Eq. (27), normalize Table 16 to obtain the EWT aggregated decision matrix, as shown in Table 17.

Table 17
Normalized Decision Matrix

	$\breve{\mathfrak{Ct}}_1$	$\breve{\mathfrak{Ct}}_2$	$\breve{\mathfrak{Ct}}_3$	$\breve{\mathfrak{Ct}}_4$	$\breve{\mathfrak{Ct}}_5$	$\breve{\mathfrak{Ct}}_6$	$\breve{\mathfrak{Ct}}_7$	$\breve{\mathfrak{Ct}}_8$	$\breve{\mathfrak{Ct}}_9$	$\breve{\mathfrak{Ct}}_{10}$	$\breve{\mathfrak{Ct}}_{11}$
$\breve{\mathfrak{At}}^1$	0.2083	0.2101	0.2041	0.2115	0.2095	0.1983	0.2017	0.2024	0.1674	0.2702	0.2029
$\breve{\mathfrak{At}}^2$	0.1797	0.1888	0.1989	0.1827	0.1883	0.1955	0.2069	0.2258	0.2465	0.2390	0.2319
$\breve{\mathfrak{At}}^3$	0.1926	0.1861	0.1885	0.2036	0.2015	0.2066	0.2068	0.1936	0.1982	0.2389	0.1787
$\breve{\mathfrak{At}}^4$	0.2161	0.2074	0.1989	0.2062	0.1963	0.1955	0.1619	0.1982	0.2096	0.2327	0.1787
$\breve{\mathfrak{At}}^5$	0.2031	0.2074	0.2094	0.1958	0.2042	0.2039	0.2225	0.1797	0.1781	0.2578	0.2075

Step 6.2.2. Find the entropy $\mathcal{E}_j$ over each criterion by using Equation (28) as shown in Table 17.

Table 18
Entropy for every criterion Table

	$\breve{\mathfrak{C}}t_1$	$\breve{\mathfrak{C}}t_2$	$\breve{\mathfrak{C}}t_3$	$\breve{\mathfrak{C}}t_4$	$\breve{\mathfrak{C}}t_5$	$\breve{\mathfrak{C}}t_6$	$\breve{\mathfrak{C}}t_7$	$\breve{\mathfrak{C}}t_8$	$\breve{\mathfrak{C}}t_9$	$\breve{\mathfrak{C}}t_{10}$	$\breve{\mathfrak{C}}t_{11}$
$\mathcal{E}_j$	0.5797	0.5797	0.5796	0.5797	0.5797	0.5797	0.5798	0.5797	0.5799	0.4846	0.5798

Step 6.2.3. Using Eq(29) , we calculate degree of divergence $\mathcal{D}_j$ as shown in Table 18.

Table 19
Divergence of Degree

	$\breve{\mathfrak{C}}t_1$	$\breve{\mathfrak{C}}t_2$	$\breve{\mathfrak{C}}t_3$	$\breve{\mathfrak{C}}t_4$	$\breve{\mathfrak{C}}t_5$	$\breve{\mathfrak{C}}t_6$	$\breve{\mathfrak{C}}t_7$	$\breve{\mathfrak{C}}t_8$	$\breve{\mathfrak{C}}t_9$	$\breve{\mathfrak{C}}t_{10}$	$\breve{\mathfrak{C}}t_{11}$
$\mathcal{D}_j$	0.4202	0.4202	0.4203	0.4202	0.4202	0.4203	0.4201	0.4022	0.4200	0.5153	0.4201

Step 6.2.4. Analyze the criteria's weights making use of Eq. (30). as presented in Table 20.

Table 20
Criteria Weights

	$\breve{\mathfrak{C}}t_1$	$\breve{\mathfrak{C}}t_2$	$\breve{\mathfrak{C}}t_3$	$\breve{\mathfrak{C}}t_4$	$\breve{\mathfrak{C}}t_5$	$\breve{\mathfrak{C}}t_6$	$\breve{\mathfrak{C}}t_7$	$\breve{\mathfrak{C}}t_8$	$\breve{\mathfrak{C}}t_9$	$\breve{\mathfrak{C}}t_{10}$	$\breve{\mathfrak{C}}t_{11}$
W_j	0.0894	0.0894	0.0894	0.0894	0.0894	0.0894	0.0894	0.0855	0.0893	0.1096	0.0894

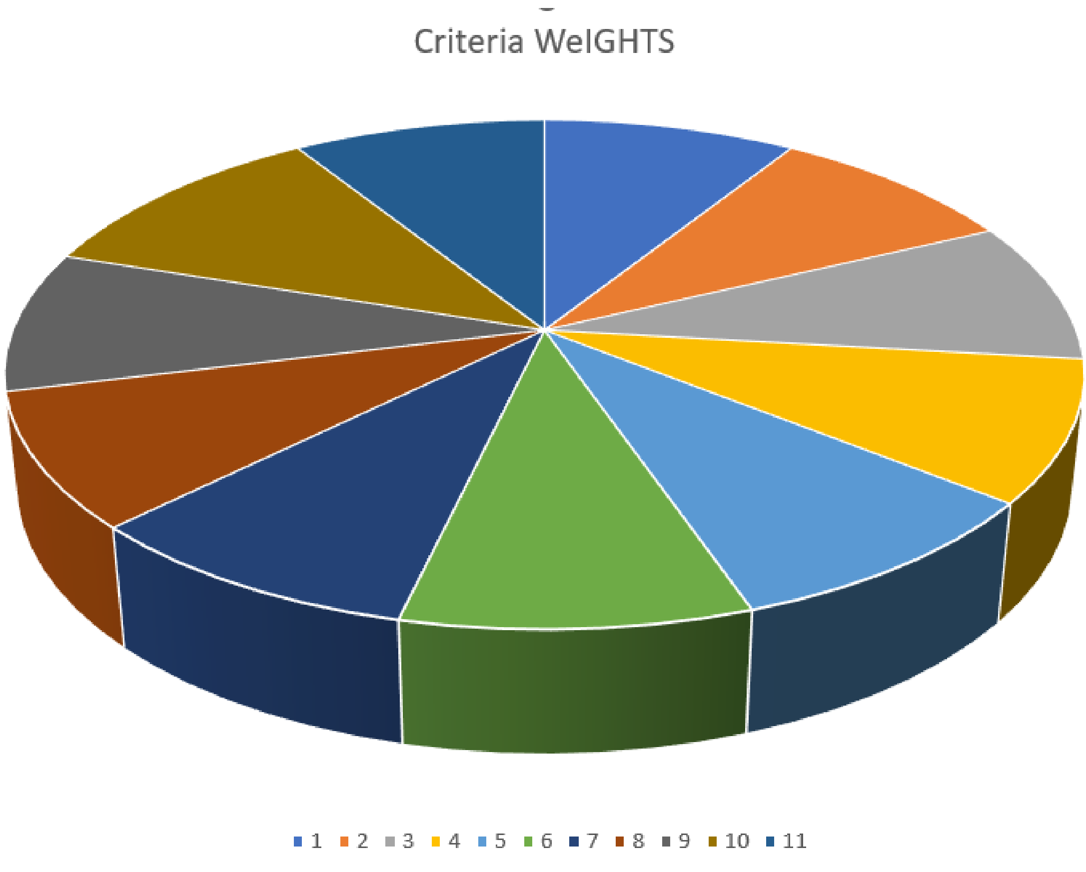

Fig. 5. Criteria weights values

Subsequently, employ the entropy weight approach to allocate a weight to each of the criterion. Applying Equation 27 to transform Table 16 into the entropy-normalized matrix (Table 17), facilitates the comparison of criteria by removing scale effects among them. To assess the deficiency of valuable information in each criterion, apply Eq. 28 to evaluate the overall entropy of each criteria, $\mathcal{E}_j$, as illustrated in Table 18. Utilising Equation 29, assess a difference of degree (1-$\mathcal{E}_j$) which indicates the existence of valuable information (variability) within each criterion. By utilising Eq. 30, it allocates relative significance according to deviation from the criteria.The substantial variability in the criterion, indicating considerable divergence, leads to an increased weighting for that criterion. Consequently, Table 20 and Figure 5 illustrate the significance of each criterion $\breve{\mathfrak{C}}\mathfrak{t}_j$; Criterion $\breve{\mathfrak{C}}\mathfrak{t}_{10}$ (Policies for Returned items and Refunds)stand out compared to the other criteria, while Criterion $\breve{\mathfrak{C}}\mathfrak{t}_8$ (Website error rate) holds the least weight.

6.3 Application of the TODIM Method

Step 6.3.1. Using Eq(31) to establish the Nrml. DM. $\breve{\mathfrak{C}}\mathfrak{t}_1$, $\breve{\mathfrak{C}}\mathfrak{t}_2$, $\breve{\mathfrak{C}}\mathfrak{t}_3$, $\breve{\mathfrak{C}}\mathfrak{t}_4$, $\breve{\mathfrak{C}}\mathfrak{t}_5$, $\breve{\mathfrak{C}}\mathfrak{t}_6$, and $\breve{\mathfrak{C}}\mathfrak{t}_7$ are beneficial- criteria, however $\breve{\mathfrak{C}}\mathfrak{t}_8$, $\breve{\mathfrak{C}}\mathfrak{t}_9$, $\breve{\mathfrak{C}}\mathfrak{t}_{10}$, and $\breve{\mathfrak{C}}\mathfrak{t}_{11}$ are non-beneficial criteria. The normalized decision matrix for the TODIM analysis is presented in Table 21.

Table 21
TODIM Normalized Decision Matrix

	Beneficial							Non-Beneficial			
	$\breve{\mathfrak{C}}\mathfrak{t}_1$	$\breve{\mathfrak{C}}\mathfrak{t}_2$	$\breve{\mathfrak{C}}\mathfrak{t}_3$	$\breve{\mathfrak{C}}\mathfrak{t}_4$	$\breve{\mathfrak{C}}\mathfrak{t}_5$	$\breve{\mathfrak{C}}\mathfrak{t}_6$	$\breve{\mathfrak{C}}\mathfrak{t}_7$	$\breve{\mathfrak{C}}\mathfrak{t}_8$	$\breve{\mathfrak{C}}\mathfrak{t}_9$	$\breve{\mathfrak{C}}\mathfrak{t}_{10}$	$\breve{\mathfrak{C}}\mathfrak{t}_{11}$
$\breve{\mathfrak{A}}\mathfrak{t}_1$	0.2083	0.2101	0.2041	0.2115	0.2095	0.1983	0.2017	0.1964	0.2346	0.1827	0.1951
$\breve{\mathfrak{A}}\mathfrak{t}_2$	0.1797	0.1888	0.1989	0.1827	0.1883	0.1955	0.2069	0.1761	0.1593	0.2066	0.1707
$\breve{\mathfrak{A}}\mathfrak{t}_3$	0.1926	0.1861	0.1885	0.2036	0.2015	0.2066	0.2068	0.2054	0.1981	0.2067	0.2215
$\breve{\mathfrak{A}}\mathfrak{t}_4$	0.2161	0.2062	0.1989	0.2024	0.1963	0.1955	0.1619	0.2006	0.1874	0.2122	0.2215
$\breve{\mathfrak{A}}\mathfrak{t}_5$	0.2031	0.2074	0.2094	0.1958	0.2042	0.2039	0.2225	0.2212	0.2205	0.1915	0.1908

Step 6.3.2. Calculate the relative weight using Eq(32) as presented in Table 22, utilizing the information from Table 16.

Table 22
Relative Weight

	$\breve{\mathfrak{C}}\mathfrak{t}_1$	$\breve{\mathfrak{C}}\mathfrak{t}_2$	$\breve{\mathfrak{C}}\mathfrak{t}_3$	$\breve{\mathfrak{C}}\mathfrak{t}_4$	$\breve{\mathfrak{C}}\mathfrak{t}_5$	$\breve{\mathfrak{C}}\mathfrak{t}_6$	$\breve{\mathfrak{C}}\mathfrak{t}_7$	$\breve{\mathfrak{C}}\mathfrak{t}_8$	$\breve{\mathfrak{C}}\mathfrak{t}_9$	$\breve{\mathfrak{C}}\mathfrak{t}_{10}$	$\breve{\mathfrak{C}}\mathfrak{t}_{11}$
W_{Cr}	0.8155	0.8155	0.8157	0.8155	0.8156	0.8156	0.8153	0.7805	0.8151	1	0.8153

Step 6.3.3. determine the Dominance degree for every alternative with respect to each criteria

Table 23
The matrix for $\breve{\mathfrak{A}}\mathfrak{t}^1$

	Beneficial							Non-Beneficial			
	$\breve{\mathfrak{C}}\mathfrak{t}_1$	$\breve{\mathfrak{C}}\mathfrak{t}_2$	$\breve{\mathfrak{C}}\mathfrak{t}_3$	$\breve{\mathfrak{C}}\mathfrak{t}_4$	$\breve{\mathfrak{C}}\mathfrak{t}_5$	$\breve{\mathfrak{C}}\mathfrak{t}_6$	$\breve{\mathfrak{C}}\mathfrak{t}_7$	$\breve{\mathfrak{C}}\mathfrak{t}_8$	$\breve{\mathfrak{C}}\mathfrak{t}_9$	$\breve{\mathfrak{C}}\mathfrak{t}_{10}$	$\breve{\mathfrak{C}}\mathfrak{t}_{11}$
$\breve{\mathfrak{A}}\mathfrak{t}^1$	0.0505	0.0436	0.0215	0.0507	0.0436	0.0157	-0.2423	0.0417	0.0820	-0.4668	0.0467
$\breve{\mathfrak{A}}\mathfrak{t}^2$	0.0373	0.0462	0.0373	0.0264	0.0268	-0.3061	-0.2401	-0.3239	0.0571	-0.4674	-0.5434
$\breve{\mathfrak{A}}\mathfrak{t}^3$	-0.5655	0.0153	0.0215	0.0218	0.0344	0.0157	-0.0596	-0.2221	0.0649	-0.5183	-0.5434
$\breve{\mathfrak{A}}\mathfrak{t}^4$	0.0214	0.0153	-0.2436	0.0374	0.0218	0.2501	0.4832	-0.5381	0.0354	-0.2837	0.0197
$\breve{\mathfrak{A}}\mathfrak{t}^5$	0.1364	0.1370	0.1351	0.1375	0.1369	0.1331	0.1342	0.1296	0.1448	0.1415	0.1321

Table 24
The matrix for $\breve{\mathfrak{A}t}^2$

	Beneficial							Non-Beneficial			
	$\breve{\mathfrak{C}t}_1$	$\breve{\mathfrak{C}t}_2$	$\breve{\mathfrak{C}t}_3$	$\breve{\mathfrak{C}t}_4$	$\breve{\mathfrak{C}t}_5$	$\breve{\mathfrak{C}t}_6$	$\breve{\mathfrak{C}t}_7$	$\breve{\mathfrak{C}t}_8$	$\breve{\mathfrak{C}t}_9$	$\breve{\mathfrak{C}t}_{10}$	$\breve{\mathfrak{C}t}_{11}$
$\breve{\mathfrak{A}t}^1$	-0.5655	-0.4879	-0.2406	-0.5677	-0.4876	-0.1764	0.0216	-0.4770	-0.9172	0.0511	-0.5224
$\breve{\mathfrak{A}t}^2$	-0.3808	0.0153	0.0305	0.4845	-0.3843	-0.3532	-0.0325	-0.5854	-0.6584	-0.0243	-0.7537
$\breve{\mathfrak{A}t}^3$	-0.6381	-0.4565	0.0016	-0.5126	-0.2989	0	0.0634	-0.5358	-0.5601	-0.2252	-0.7537
$\breve{\mathfrak{A}t}^4$	-0.5119	-0.4565	-0.3424	-0.3836	0.4223	-0.3061	-0.4180	-0.7262	-0.8271	-0.0406	-0.4734
$\breve{\mathfrak{A}t}^5$	0.1267	0.1299	0.1333	0.1278	0.1297	0.1322	0.1360	0.1227	0.1193	0.1505	0.1235

Table 25
The matrix for $\breve{\mathfrak{A}t}^3$

	Beneficial							Non-Beneficial			
	$\breve{\mathfrak{C}t}_1$	$\breve{\mathfrak{C}t}_2$	$\breve{\mathfrak{C}t}_3$	$\breve{\mathfrak{C}t}_4$	$\breve{\mathfrak{C}t}_5$	$\breve{\mathfrak{C}t}_6$	$\breve{\mathfrak{C}t}_7$	$\breve{\mathfrak{C}t}_8$	$\breve{\mathfrak{C}t}_9$	$\breve{\mathfrak{C}t}_{10}$	$\breve{\mathfrak{C}t}_{11}$
$\breve{\mathfrak{A}t}^1$	-0.4180	-0.5173	-0.4176	-0.2959	-0.3000	0.0273	0.0214	0.0277	-0.6389	0.0512	0.0485
$\breve{\mathfrak{A}t}^2$	0.0340	-0.1721	-0.3412	0.0433	0.0343	0.0315	-0.0325	0.0501	0.0588	0.0026	0.0673
$\breve{\mathfrak{A}t}^3$	-0.5119	-0.4879	-0.3407	-0.1674	0.0216	0.0315	0.0633	0.0201	0.0309	-0.2239	0
$\breve{\mathfrak{A}t}^4$	-0.3421	-0.4879	-0.4834	0.0264	-0.1749	0.0157	-0.4193	-0.4297	-0.5006	-0.0407	0.0524
$\breve{\mathfrak{A}t}^5$	0.1312	0.1290	0.1298	0.1349	0.1342	0.1359	0.1359	0.1326	0.1330	0.1505	0.1407

Table 26
The matrix for $\breve{\mathfrak{A}t}^4$

	Beneficial							Non-Beneficial			
	$\breve{\mathfrak{C}t}_1$	$\breve{\mathfrak{C}t}_2$	$\breve{\mathfrak{C}t}_3$	$\breve{\mathfrak{C}t}_4$	$\breve{\mathfrak{C}t}_5$	$\breve{\mathfrak{C}t}_6$	$\breve{\mathfrak{C}t}_7$	$\breve{\mathfrak{C}t}_8$	$\breve{\mathfrak{C}t}_9$	$\breve{\mathfrak{C}t}_{10}$	$\breve{\mathfrak{C}t}_{11}$
$\breve{\mathfrak{A}t}^1$	0.0264	-0.1721	-0.2413	-0.2440	-0.3852	-0.1764	-0.6672	0.0190	-0.7267	0.0568	0.0485
$\breve{\mathfrak{A}t}^2$	0.0570	0.0408	-0.0186	0.0458	0.0267	0	-0.5358	0.0458	0.0500	0.0246	0.0673
$\breve{\mathfrak{A}t}^3$	0.0457	0.0436	0.0304	0.0149	0.3532	0.7091	0.2358	0.3461	-0.2239	0.0245	0
$\breve{\mathfrak{A}t}^4$	0.0340	0	-0.3429	0.0304	0.3061	-0.8238	-0.4901	-0.6086	-0.4337	0.0475	0.0524
$\breve{\mathfrak{A}t}^5$	0.1390	0.1362	0.1333	0.1357	0.1324	0.1322	0.1203	0.1310	0.1294	0.1525	0.1407

Table 27
The matrix for $\breve{\mathfrak{A}t}^5$

	Beneficial							Non-Beneficial			
$\breve{\mathfrak{A}t}^i$	$\breve{\mathfrak{C}t}_1$	$\breve{\mathfrak{C}t}_2$	$\breve{\mathfrak{C}t}_3$	$\breve{\mathfrak{C}t}_4$	$\breve{\mathfrak{C}t}_5$	$\breve{\mathfrak{C}t}_6$	$\breve{\mathfrak{C}t}_7$	$\breve{\mathfrak{C}t}_8$	$\breve{\mathfrak{C}t}_9$	$\breve{\mathfrak{C}t}_{10}$	$\breve{\mathfrak{C}t}_{11}$
$\breve{\mathfrak{A}t}^1$	-0.2401	-0.1721	-0.0217	-0.4185	-0.2438	0.0223	0.0432	0.0460	-0.3970	0.0311	-0.2208
$\breve{\mathfrak{A}t}^2$	0.4578	0.4082	0.0306	0.0343	0.0377	0.0273	0.0373	0.0621	0.0739	-0.3707	0.0423
$\breve{\mathfrak{A}t}^3$	-0.0305	0.0436	0.0432	-0.2959	0.0156	-0.1763	0.0374	0.0367	0.0447	-0.3715	-0.5865
$\breve{\mathfrak{A}t}^4$	-0.3808	0	0.0306	-0.3400	0.0266	0.2737	0.0736	0.0419	0.0543	-0.4337	-0.5865
$\breve{\mathfrak{A}t}^5$	0.1347	0.1362	0.1368	0.1323	0.1351	0.1350	0.1410	0.1376	0.1403	0.1449	0.1306

Step 6.3.3. Calculate the dominance degree by making use of Eq(33). The criterion-level dominance matrices for Alternatives 1–5 are presented in Tables 23–27, while the aggregated dominance degrees are presented in Table 28.

Table 28
Dominance Degree of the Alternatives

$\breve{\mathfrak{A}}\mathfrak{t}^1$	-3.727251
$\breve{\mathfrak{A}}\mathfrak{t}^2$	-15.29297
$\breve{\mathfrak{A}}\mathfrak{t}^3$	-5.414105
$\breve{\mathfrak{A}}\mathfrak{t}^4$	-5.725055
$\breve{\mathfrak{A}}\mathfrak{t}^5$	-2.626157

*Step 6.3.4.*Calculate the aggregation of the dominance degrees making use of Equation (34) as presented in Table 29.

Table 29
Overall, Dominance Degree of the Alternatives

$\breve{\mathfrak{A}}\mathfrak{t}^1$	5.742392
$\breve{\mathfrak{A}}\mathfrak{t}^2$	-5.823326
$\breve{\mathfrak{A}}\mathfrak{t}^3$	4.055538
$\breve{\mathfrak{A}}\mathfrak{t}^4$	3.744584
$\breve{\mathfrak{A}}\mathfrak{t}^5$	6.843486

Step 6.3.5. Now, arrange the available alternatives according to their overall degree of dominance., as presented in Table 30.

Table 30
Ranking the Alternatives

$\breve{\mathfrak{A}}\mathfrak{t}^1$	2
$\breve{\mathfrak{A}}\mathfrak{t}^2$	5
$\breve{\mathfrak{A}}\mathfrak{t}^3$	3
$\breve{\mathfrak{A}}\mathfrak{t}^4$	4
$\breve{\mathfrak{A}}\mathfrak{t}^5$	1

The TODIM normalization process is first performed by transforming the aggregated decision matrix (Table 16) into the normalized decision matrix (Table 21) using Eq. (31). In this matrix, higher values are preferred for beneficial criteria, whereas lower values are preferred for non-beneficial criteria. Subsequently, Eq. (32) is applied to determine the relative weights with respect to the reference criterion, $\mathfrak{Ct}_{10}$, as presented in Table 22. The dominance degrees of the alternatives are then calculated using Eq. (33), and the obtained results are presented in Table 28. Based on these dominance values, the overall dominance degrees are computed using Eq. (34), as shown in Table 29. Finally, the ranking of the e-commerce websites is presented in Table 30. The results indicate that Website 5 is the best-performing e-commerce platform, followed by Website 1 and Website 3, whereas Website 2 is ranked last according to the proposed TFN–Entropy–TODIM framework.

6.4 Comparison with Other MCDM Methods

To assess the efficacy of the TFN-EWT-TODIM model, we compare the outcomes of the model that we suggested with those obtained from various other multi-criteria decision-making methods, including CORPAS [56], MOORA [57], ARAS [58], TOPSIS [59],[40], Singh et al. [11] and WASPAS [60].

Table 31
Ranking of websites in other comparative MCDM methods

MCDM Methods	$Website_1$	$Website_2$	$Website_3$	$Website_4$	$Website_5$
TOPSIS [59],[40]	3	5	4	2	1
MOORA [57]	2	5	3	4	1
ARAS [58]	2	5	3	4	1
DMM by Singh et al. [11]	2	3	4	5	1
CORPAS [56]	3	4	5	2	1
WASPAS [60] ($\theta = 0.7$)	2	5	3	4	1
Proposed Method (TODIM)[11] ($\theta = 1$)	2	5	3	4	1

Table 32
Best Alternative for the MCDM Methods

MCDM Methods	**Ranking order**	**Best option**
TOPSIS	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
MOORA	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
ARAS	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
DMM	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_2 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_4$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
CORPAS	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_2 \geq \breve{\mathfrak{A}}\mathfrak{t}_3$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
WASPAS	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
Proposed Method (TODIM)	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$

The differences between the proposed model and the other MCDM methods are presented in Table 31. The complete ranking orders and the best alternative identified by each method are summarized in Table 32 and illustrated in Figure **??**. The proposed model identifies Website 5 as the most preferred alternative and Website 2 as the least preferred one. Website 5 is consistently ranked first by all the compared MCDM methods, whereas Website 2 is ranked last by the TOPSIS, MOORA, ARAS, and WASPAS methods.

The results indicate that the proposed framework effectively addresses uncertainty through the use of Triangular Fuzzy Numbers (TFNs). Furthermore, the integration of the entropy weighting method and the TODIM approach enables the model to combine objective criterion weighting with prospect theory-based ranking. This integrated framework provides a robust decision-support tool for evaluating e-commerce websites under uncertain conditions. Although some differences in the intermediate rankings are observed when compared with other MCDM methods, the overall results confirm the effectiveness and practical applicability of the proposed model.

6.5 Sensitivity Analysis

The sensitivity study is performed to make sure that the TFN-EWT-TODIM model works well on different e-commerce sites with different values of attenuation factor (θ).

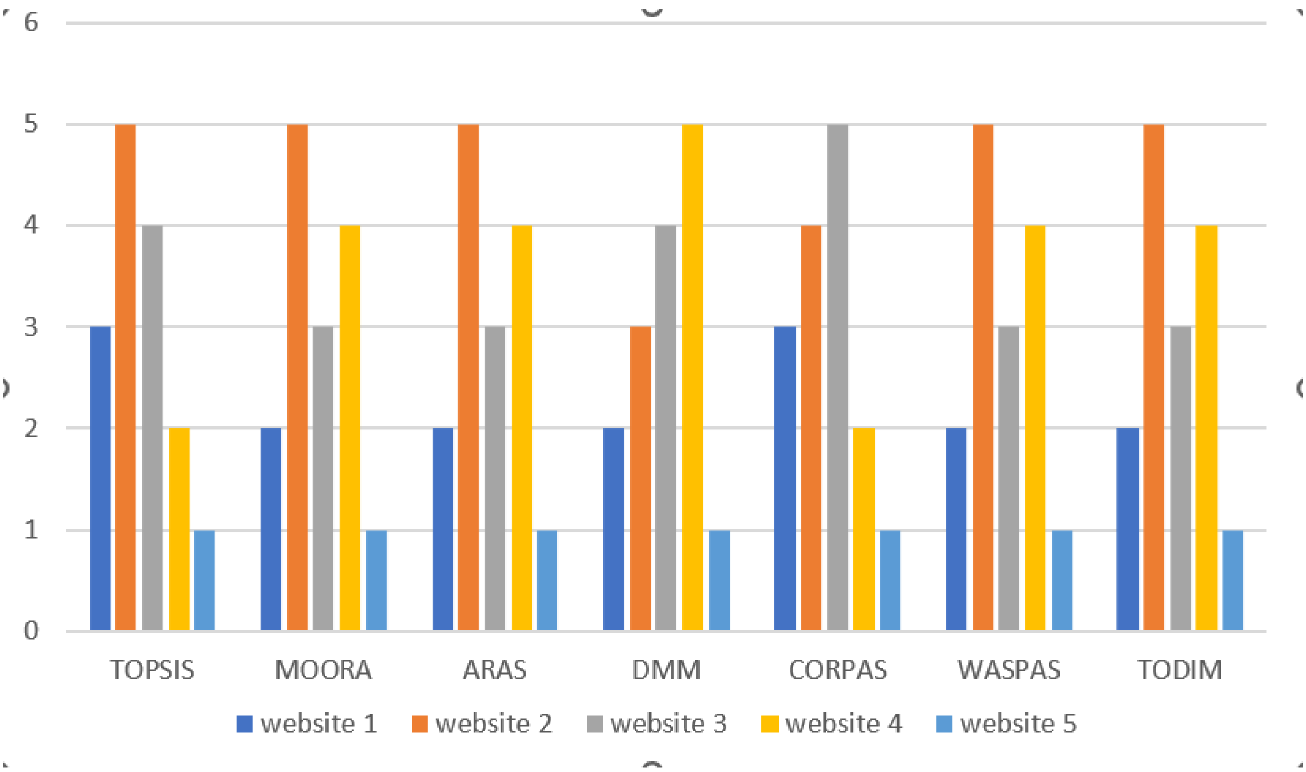

Fig. 6. Ranking order of the five websites

Table 33
Ranking of E-commerce Websites for Different θ Values

θ	$Website_1$	$Website_2$	$Website_3$	$Website_4$	$Website_5$
1	2	5	3	4	1
1.5	2	5	3	4	1
2.5	1	5	4	3	2
4.5	1	5	4	3	2
6	2	5	3	4	1
7.5	2	5	3	4	1
8.3	2	5	3	4	1
10	2	5	3	4	1

Table 33 presents the different values of the θ parameter used in the TODIM method to evaluate the robustness of the proposed framework through sensitivity analysis. As shown in Table 33, Website 5 achieves the highest ranking for most values of θ, indicating its stable performance under different decision-making scenarios. Variations in the θ parameter lead to changes in the rankings of several websites, confirming that this parameter has a direct influence on the final ranking results. Since the θ parameter controls the relative impact of losses with respect to gains in the TODIM method, increasing its value changes the decision-makers' risk perception and consequently affects the ranking outcomes. The corresponding overall dominance values, ranking orders, and the best alternative obtained for different values of θ are presented in Table 34. Overall, the sensitivity analysis demonstrates that the proposed framework produces stable and reliable rankings across a wide range of θ values, confirming its robustness and practical applicability for evaluating e-commerce websites under different decision-making preferences.

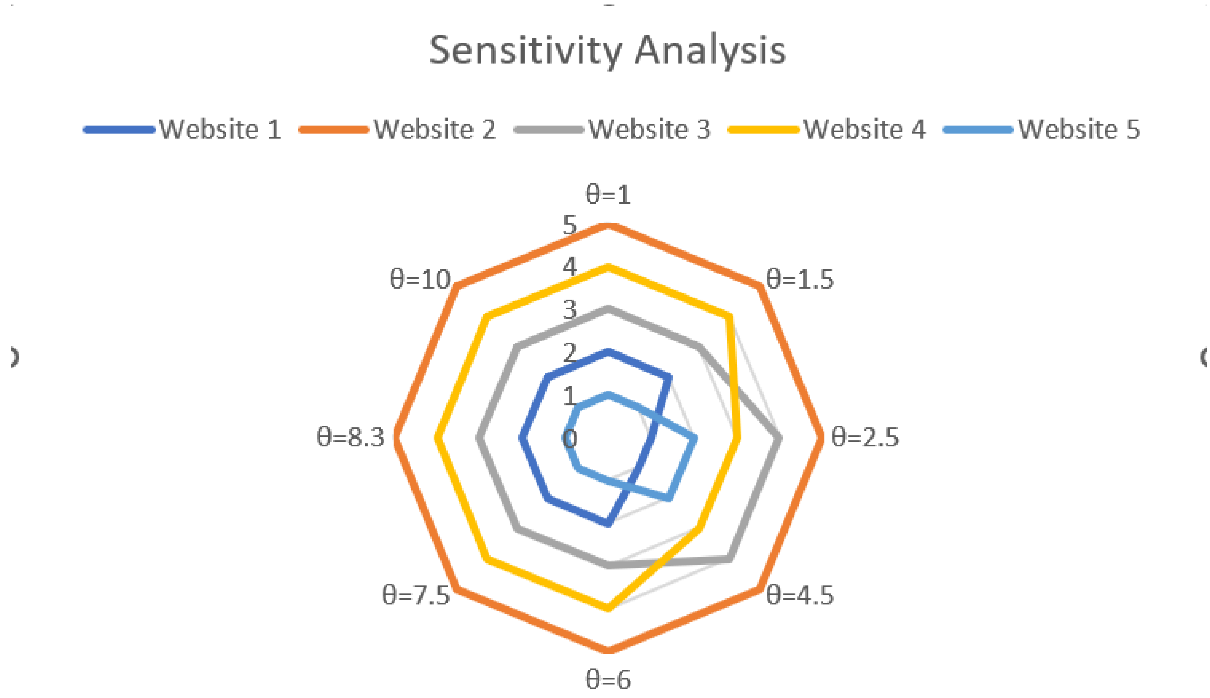

Fig. 7. Sensitivity analysis for different values of θ

Table 34
Ranking of alternatives for different attenuation factorθ values

θ	$Website_1$	$Website_2$	$Website_3$	$Website_4$	$Website_5$	Ranking order	Best alternative
$\theta = 1$	5.2669	-5.3650	3.3102	2.9430	6.0981	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
$\theta = 1.5$	-1.4677	-9.6502	-2.8459	-3.0820	-0.8811	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
$\theta = 2.5$	58.0255	51.5161	56.3467	56.9789	57.7835	$\breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_1$
$\theta = 4.5$	8.1802	3.3048	6.6364	7.5647	8.0103	$\breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_1$
$\theta = 6$	1.5088	-1.1362	1.0145	0.9743	1.7344	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
$\theta = 7.5$	1.7156	0.6046	1.2629	1.2367	1.9109	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
$\theta = 8.3$	1.7023	-0.3866	1.3620	1.3408	1.9781	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$
$\theta = 10$	1.9145	-0.0393	1.5198	1.5067	2.0854	$\breve{\mathfrak{A}}\mathfrak{t}_5 \geq \breve{\mathfrak{A}}\mathfrak{t}_4 \geq \breve{\mathfrak{A}}\mathfrak{t}_1 \geq \breve{\mathfrak{A}}\mathfrak{t}_3 \geq \breve{\mathfrak{A}}\mathfrak{t}_2$	$\breve{\mathfrak{A}}\mathfrak{t}_5$

7. Conclusion

This study proposed an integrated decision-making framework for evaluating the reliability of e-commerce platforms under uncertain and subjective conditions. By combining Triangular Fuzzy Numbers, the Entropy Weight Method, and the TODIM approach, the proposed framework effectively models uncertainty, objectively determines the importance of evaluation criteria, and produces a reliable ranking of e-commerce websites. The integration of these techniques reduces the influence of subjective judgments while providing a structured and transparent decision-support process.

The applicability of the proposed framework was demonstrated through a case study involving five international e-commerce platforms evaluated against eleven criteria. The obtained results confirmed the effectiveness of the proposed model in distinguishing the performance of competing websites. Comparative analysis with several well-established MCDM methods showed a high level of consistency in the obtained rankings, while the sensitivity analysis further verified the robustness and stability of the proposed approach under different weighting scenarios. These findings indicate that the framework provides reliable decision support even when variations in criterion importance occur.

The proposed TFN–Entropy–TODIM framework is not limited to e-commerce website evaluation and can be readily extended to a wide range of complex decision-making problems characterized by uncertainty and conflicting criteria. Potential application areas include tourism, healthcare, supplier selection, e-government, sustainability assessment, renewable energy planning, cloud computing, and other decision-support problems. Future research may focus on extending the framework to other fuzzy environments, such as Intuitionistic Fuzzy Sets, Picture Fuzzy Sets, Interval-Valued Fuzzy Sets, Hesitant Fuzzy Sets, and Bipolar Fuzzy Sets, to improve its capability of representing different types of uncertainty. In addition, the proposed framework can be integrated with other objective and subjective weighting techniques or alternative ranking methods to further enhance its flexibility and applicability. Future studies may also investigate its performance in large-scale real-world decision problems and explore its integration with artificial intelligence and machine learning techniques for intelligent decision-support systems.

Abbreviations

(i)	FS	fuzzy set
(ii)	FSs	Set of all fuzzy sets of $\check{\mathbb{U}}$
(iii)	FIM	Fuzzy Information measure
(iv)	FKM	Fuzzy Knowledge measure
(v)	FAM	Fuzzy Accuracy measure
(vi)	MCDM	Multi criteria decision making
(vii)	TFN	Triangular Fuzzy Number
(viii)	Aggr. DM	Aggregated Decision Matrix
(ix)	Nrml. DM	Normal Decision Matrix

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Conflicts of Interest
The authors declare no conflicts of interest.

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