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A Hamacher Aggregation-Based Pythagorean Fuzzy Z-Number MCDM Framework for Sustainable Urban Green Space Planning

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ABSTRACT

Evaluation of alternatives is a complex task in real-world decision-making, where ambiguity and uncertainty are inherent. To address these challenges, this study proposes an integrated Pythagorean Fuzzy Z-Number (PyFZN)-based framework for multi-attribute decision-making (MADM) in urban green space planning. PyFZNs extend intuitionistic fuzzy sets by incorporating both fuzzy membership information and reliability measures, enabling a more realistic representation of uncertainty and a more effective modeling of expert judgments. Hamacher norms with adjustable parameters are employed to improve the aggregation process, providing greater flexibility and consistency in information fusion. Based on these norms, a family of aggregation operators is developed to enhance decision-making performance under uncertain conditions. The proposed PyFZN-MADM framework is applied to evaluate urban green space alternatives using criteria such as population coverage, accessibility, environmental impact, cost, and social benefits. A metropolitan case study demonstrates the applicability of the proposed framework for ranking alternatives, optimizing resource allocation, and improving urban livability. The results indicate that the proposed approach is robust, sensitive, and reliable, outperforming existing methods in handling uncertainty and prioritizing alternatives. Comparative analysis further confirms its effectiveness in supporting informed, consistent, and reliable decision-making in complex urban planning environments.

1. Introduction

Decision-making (DM) plays a fundamental role in problem-solving, goal achievement, and effective resource allocation, while also supporting innovation, risk management, and organizational adaptability [1]. To address ambiguity in real-world data, Zadeh [2] introduced fuzzy sets (FSs), which represent uncertainty through membership degrees (MDs). Atanassov [3] later extended this concept by proposing intuitionistic fuzzy sets (IFSs), which incorporate both membership and non-membership degrees (NMDs), thereby providing a more expressive framework for modeling

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uncertainty. However, IFSs impose the constraint that the sum of the membership and non-membership degrees must not exceed one.

To overcome this limitation, Yager [4] introduced Pythagorean fuzzy sets (PyFSs), in which the squared sum of the membership and non-membership degrees is constrained to be less than or equal to one, offering greater flexibility in representing uncertain information. Subsequently, Zadeh [5] proposed Z-numbers, which combine an uncertain assessment with its associated reliability, providing a richer representation of human judgments. By explicitly incorporating both restriction and reliability, Z-numbers enable more accurate computational decision-making [6]. Moreover, mathematical operations and optimization techniques, including linear programming and function construction, have been developed to facilitate the manipulation of Z-numbers through operations such as addition, subtraction, multiplication, division, maximum, minimum, and square root, thereby reducing uncertainty in MADM problems [7].

Recent studies have also focused on developing advanced aggregation operators (AOs) for the PyFS framework. Yager and Abbasov [8] introduced averaging and geometric aggregation operators, while Zhang and Xu [9] extended the TOPSIS method to Pythagorean fuzzy decision-making problems. Subsequent research further enriched this field by proposing optimization-based approaches and various aggregation mechanisms, including Einstein-based Pythagorean fuzzy aggregation, Z-valuations, and Hamacher-based aggregation operators [10]. These advances have significantly strengthened the applicability of Pythagorean Fuzzy Z-Numbers (PyFZNs) in multi-attribute decision-making by simultaneously addressing ambiguity, uncertainty, and reliability. The basic structure of a PyFZN is illustrated in Figure 1, where the membership and non-membership information are represented by the membership and non-membership values, respectively, while the reliability and non-reliability information are represented by the corresponding reliability and non-reliability values.

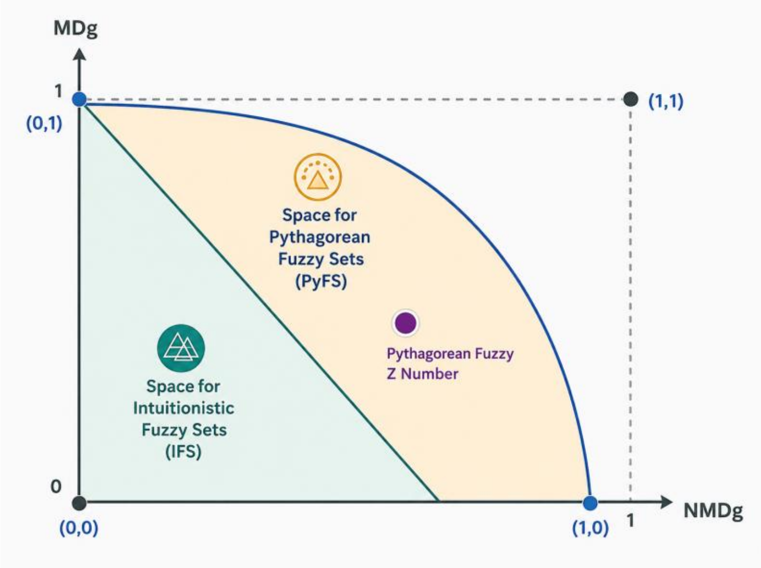

Fig. 1. Pythagorean fuzzy Z number system

Researchers have proposed a wide range of approaches to enhance aggregation operators (AOs) within fuzzy set frameworks. Conventional AOs are generally based on simple algebraic operations, whereas t-norms (t-Nr) and t-conorms (t-CNr) provide more flexible and mathematically consistent aggregation mechanisms. Among these, Einstein, Archimedean, Frank, and Hamacher norms have been widely adopted in the literature. Tešić and Marinković [11] introduced normalized aggregation operators based on Bonferroni mean operators. Ali *et al.,* [12] developed power aggregation operators based on Hamacher t-norms and t-conorms for complex intuitionistic fuzzy information and demonstrated their applicability to decision-making problems. Garg [13,14] employed both

Einstein and Hamacher t-norms in multi-attribute decision-making (MADM), while Huang [15] proposed several aggregation operators based on Hamacher norms. Furthermore, Yager [16] introduced Z-valuations derived from Zadeh's Z-numbers, and Deb *et al.,* [17] further extended aggregation operators using Hamacher t-norms. In addition, numerous studies have successfully applied these aggregation frameworks to various decision-making problems under uncertainty [18–22].

1.1 Review of the Literature to Identify Gaps

Although fuzzy sets (FSs), Pythagorean fuzzy sets (PyFSs), and Pythagorean Fuzzy Z-Numbers (PyFZNs) have been widely applied to multi-criteria decision-making problems, several important research gaps remain, particularly in urban green space planning under uncertainty. Existing studies have primarily focused on general decision-making applications, while the integration of PyFZNs with Hamacher aggregation operators for evaluating and prioritizing urban green space alternatives has received limited attention. Furthermore, many existing aggregation approaches are unable to simultaneously capture the uncertainty of expert assessments and the reliability of the provided information, reducing their effectiveness in complex urban planning scenarios.

Another limitation of the current literature is the absence of comprehensive decision-making frameworks capable of integrating environmental, social, and economic criteria within a unified PyFZN-based structure. Although numerous fuzzy MCDM models have been proposed, only a few studies have performed systematic comparisons between PyFZN-Hamacher-based approaches and established MCDM techniques. In addition, advanced fuzzy extensions and the treatment of uncertainty associated with population distribution, accessibility, and environmental indicators remain insufficiently investigated.

To address these limitations, this study proposes a novel PyFZN-based MCDM framework integrated with Hamacher aggregation operators. The proposed approach is designed to improve the reliability and robustness of decision-making under uncertainty while providing a more effective tool for prioritizing urban green space development.

1.2 Research Objectives

The primary objective of this study is to develop a robust multi-criteria decision-making framework for urban green space planning under uncertainty by integrating Pythagorean Fuzzy Z-Numbers with Hamacher aggregation operators. The proposed framework aims to model both uncertainty and reliability in expert evaluations while enhancing the aggregation of decision information through mathematically consistent Hamacher operators.

To achieve this objective, the study develops a family of Hamacher-based aggregation operators suitable for PyFZN environments and incorporates them into a structured decision-making framework. The applicability of the proposed model is demonstrated through a real-world metropolitan case study involving the evaluation and prioritization of parks, urban forests, and green belts using multiple sustainability criteria. Finally, the performance of the proposed approach is assessed through comparative analysis with existing MCDM methods to evaluate its robustness, stability, and effectiveness in handling uncertainty. Overall, the framework seeks to improve the quality of urban green space planning and support sustainable urban development.

1.3 Main Contributions

This study makes several theoretical and practical contributions to the field of fuzzy multi-criteria decision-making. First, it introduces a novel aggregation framework based on Pythagorean Fuzzy Z-Numbers for handling uncertainty and reliability in complex decision environments. Second, it develops a comprehensive mathematical formulation of Hamacher aggregation operators within the PyFZN framework, providing a flexible and effective mechanism for information fusion.

In addition, the study presents a structured decision-making algorithm accompanied by a clear implementation procedure that facilitates practical application. The proposed methodology is validated through a real-world urban green space planning case study involving multiple sustainability indicators and alternative types, including parks, green belts, and urban forests. Finally, the effectiveness of the proposed framework is demonstrated through comparative analyses with existing MCDM approaches, highlighting its improved robustness, flexibility, and decision accuracy under uncertain conditions.

1.4 Motivation of the Proposed Study

Classical fuzzy sets (FSs), intuitionistic fuzzy sets (IFSs), and Pythagorean fuzzy sets (PyFSs) have been extensively applied to decision-making problems involving uncertain information. Although these models effectively represent vagueness and ambiguity, they do not explicitly account for the reliability or credibility of the available information. While PyFSs provide greater flexibility than IFSs by relaxing the relationship between membership and non-membership degrees, they still cannot distinguish between uncertain information that is highly reliable and information whose credibility is questionable.

Zadeh addressed this limitation through the introduction of Z-numbers, which incorporate both an uncertain assessment and its associated reliability. Building upon this concept, the proposed study integrates Pythagorean fuzzy sets with Z-numbers to construct Pythagorean Fuzzy Z-Numbers, enabling the simultaneous representation of uncertainty and reliability. This integration significantly enhances the expressive capability of fuzzy models by considering both the fuzziness of expert judgments and the credibility of the underlying information.

The proposed framework is particularly suitable for urban green space planning, where decisions depend on heterogeneous environmental, social, and economic information that is frequently incomplete, uncertain, and inconsistent. By combining PyFZNs with Hamacher aggregation operators, the proposed model provides a more reliable and flexible mechanism for information fusion, leading to improved evaluation accuracy, greater robustness, and more realistic decision outcomes. Consequently, the framework offers an effective tool for supporting sustainable urban planning and strategic decision-making. Figure 2 presents the overall structure of the paper, illustrating the relationship between its main sections from the introduction to the conclusion.

2. Preliminaries

This section will discuss some basic definitions and rules that will help construct our paper.

Definition 1 [4]. A PyFS S defined in Y is an ordered pair presented by $S = \{\langle \mathrm{y}, g(\mathrm{y}), h(\mathrm{y})\rangle | \mathrm{y} \in \mathrm{Y}\}$ where $g(\mathrm{y}), h(\mathrm{y}): \mathrm{Y} \rightarrow [0,1]$ represent the MDg and NMDg of the element y such that $g(\mathrm{y}), h(\mathrm{y}) \in [0,1]$ and $(g(\mathrm{y}))^2 + (h(\mathrm{y}))^2 \leq 1$ for all y.

Definition 2 [5]. A PyFZN is an ordered pair $\omega = \{y, < \big(g(y), h(y)\big), (r(y), s(y) > | y \in Y\}$ where $A = (g, h)$ and $B = (r, s)$ are Pythagorean fuzzy numbers satisfying
$0 \leq g(y)^2 + h(y)^2 \leq 1$ and $0 \leq r(y)^2 + s(y)^2 \leq 1$ for all y.

A represents the uncertain evaluation of a variable and B represents the reliability (confidence) of the evaluation.

Definition 3 [5]. Let $\omega = \{y, < \big(g(y), h(y)\big), (r(y), s(y) > | y \in Y\}$, then score function ($SF$) for any PyFZN is given as $SF(\omega) = \frac{1 + g\mathrm{h} - \mathrm{rs}}{2} \in [0,1]$.

Fig. 2. Flowchart summarizing the structure of the paper

3. Improved Operational Laws and Aggregation Operators by PyFZNS

We will present some new rules and laws which will be helpful in the study of AOs.

3.1 Hamacher Operational Laws for PyFZNS

Definition 4. Suppose $\underline{\omega} = \langle (g,h),(r,s) \rangle$ and $\underline{\omega}_i = \langle (g_i,h_i),(r_i,s_i) \rangle, (\mathrm{i} = 1,2,\dots,\mathrm{n})$ be a set of PyFZNs and Let $\mathfrak{Y} \in (0,\infty)$ and $\text{þ} > 0$. Then the new operational laws using the Hamacher norm for these numbers can be defined as:

$$\text{(i)}\quad \underline{\omega}_1 \oplus \underline{\omega}_2 \dots \oplus \underline{\omega}_i = \left\langle \begin{array}{l} \left(\sqrt{\frac{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(g_i)^2-\oplus_{i=1}^{n}(1-(g_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(g_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(g_i)^2}}, \sqrt{\frac{\oplus_{i=1}^{n}(1-(g_i)^2-\mathfrak{Y}\oplus_{i=1}^{n}(1-(g_i)^2-(\mathrm{h}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(g_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(g_i)^2}} \right) \\ \left(\sqrt{\frac{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{r}_i)^2-\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{r}_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2}}, \sqrt{\frac{\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2-\mathfrak{Y}\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2-(\mathrm{s}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{r}_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2}} \right) \end{array} \right\rangle$$

$$\text{(ii)}\quad \underline{\omega}_1 \otimes \underline{\omega}_2 \dots \otimes \underline{\omega}_i = \left\langle \begin{array}{l} \left(\sqrt{\frac{\mathfrak{Y}\oplus_{i=1}^{n}[1-(\mathrm{h}_i)^2-\mathfrak{Y}\oplus_{i=1}^{n}(1-(g_i)^2-(\mathrm{h}_i)^2)}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{h}_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{h}_i)^2}}, \sqrt{\frac{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{h}_i)^2-\oplus_{i=1}^{n}(1-(\mathrm{h}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{h}_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{h}_i)^2}} \right) \\ \left(\sqrt{\frac{\mathfrak{Y}\oplus_{i=1}^{n}[1-(\mathrm{s}_i)^2-\mathfrak{Y}\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2-(\mathrm{s}_i)^2)}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{s}_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{s}_i)^2}}, \sqrt{\frac{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{s}_i)^2-\oplus_{i=1}^{n}(1-(\mathrm{s}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{s}_i)^2-(\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{s}_i)^2}} \right) \end{array} \right\rangle$$

$$\text{(iii)}\quad \text{þ}\omega = \left\langle \begin{array}{l} \left(\sqrt{\frac{[1+(\mathfrak{Y}-1)(g)^2]^{\text{þ}}(1-(g)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(g)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(g)^2)^{\text{þ}}}}, \sqrt{\frac{\mathfrak{Y}(1-(g)^2)^{\text{þ}}-\mathfrak{Y}(1-(g)^2-(\mathrm{h})^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(g)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(g)^2)^{\text{þ}}}} \right) \\ \left(\sqrt{\frac{[1+(\mathfrak{Y}-1)(r)^2]^{\text{þ}}(1-(r)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(r)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(r)^2)^{\text{þ}}}}, \sqrt{\frac{\mathfrak{Y}(1-(r)^2)^{\text{þ}}-\mathfrak{Y}(1-(r)^2-(s)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(r)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(r)^2)^{\text{þ}}}} \right) \end{array} \right\rangle$$

$$\text{(iv)}\quad \underline{\omega}^{\text{þ}} = \left\langle \begin{array}{l} \left(\sqrt{\frac{(\mathfrak{Y}(1-(h)^2]^{\text{þ}}-\mathfrak{Y}(1-(g)^2-(h)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(h)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(h)^2)^{\text{þ}}}}, \sqrt{\frac{(1+(\mathfrak{Y}-1)(h)^2)^{\text{þ}}-(1-(h)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(h)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(h)^2)^{\text{þ}}}} \right) \\ \left(\sqrt{\frac{(\mathfrak{Y}(1-(s)^2]^{\text{þ}}-\mathfrak{Y}(1-(r)^2-(s)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(s)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(s)^2)^{\text{þ}}}}, \sqrt{\frac{(1+(\mathfrak{Y}-1)(s)^2)^{\text{þ}}-(1-(s)^2)^{\text{þ}}}{[1+(\mathfrak{Y}-1)(s)^2]^{\text{þ}}-(\mathfrak{Y}-1)(1-(s)^2)^{\text{þ}}}} \right) \end{array} \right\rangle.$$

where $\text{þ} > 0$ and $\mathfrak{Y} \in (0, \infty)$ are the real numbers.

3.2 Hamacher Averaging Type Aggregation Operators

For the set of $\underline{\omega}_i = \langle(\mathscr{g}_i, h_i), (r_i, s_i)\rangle, (\mathrm{i} = 1, 2, \dots, \mathrm{n})$ with $\mathscr{g}_i, r_i \neq 1$ for all i and relying on the above-mentioned rules, we will present new PyFH interactive weighted (PyFHIWA) and order weighted (PyFHIOWA) averaging operators.

Definition 5. Suppose Y be the collection of PyFZN and PyFHIWA : $\mathrm{Y}^{\mathrm{n}} \rightarrow \mathrm{Y}$, such that:

$$\mathrm{PFHIWA}(\omega_1, \omega_2, \dots, \omega_n) = \mathrm{d}_1\omega_1 \oplus \mathrm{d}_2\omega_2 \oplus \dots \oplus \mathrm{d}_n\omega_n$$

where $\mathrm{d} = (\mathrm{d}_1, \mathrm{d}_2, \dots, \mathrm{d}_n)^{\mathrm{T}}$ is the weight vector of ω_{i} such that $\mathrm{d}_i > 0$ and $\sum \mathrm{d}_i = 1$, $(\mathrm{i} = 1 \text{ to } \mathrm{n})$ Then, PyFHIWA is defined as the PyFH interactive weighted averaging operator.

Theorem 1. The aggregated value of PyFZN $\underline{\omega}_i = \langle(\mathscr{g}_i, h_i), (r_i, s_i)\rangle$, $\mathscr{g}_i, r_i \neq 1 (\mathrm{i} = 1, 2, \dots, \mathrm{n})$ using PyFHIWA is also a PyFZN and is given by:

$$PyFHIWA(\omega_1, \omega_2, \dots \omega_n) =$$

$$\left\langle \begin{array}{c} \left(\frac{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathscr{g}_i)^2]^{\text{đ}_i} - \oplus_{i=1}^{n}(1-(\mathscr{g}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathscr{g}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathscr{g}_i)^2)^{\text{đ}_i}}, \frac{(\mathfrak{Y}\oplus_{i=1}^{n}(1-(\mathscr{g}_i)^2]^{\text{đ}_i} - \mathfrak{Y}\oplus_{i=1}^{n}(1-(\mathscr{g}_i)^2-(\mathrm{h}_i)^2)^{\text{đ}_i}}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathscr{g}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathscr{g}_i)^2)^{\text{đ}_i}} \right) \\ \left(\frac{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{r}_i)^2]^{\text{đ}_i} - \oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{r}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2)^{\text{đ}_i}}, \frac{(\mathfrak{Y}\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2]^{\text{đ}_i} - \mathfrak{Y}\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2-(\mathrm{r}_i)^2)^{\text{đ}_i}}{\oplus_{i=1}^{n}[1+(\mathfrak{Y}-1)(\mathrm{r}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{n}(1-(\mathrm{r}_i)^2)^{\text{đ}_i}} \right) \end{array} \right\rangle$$

4. Proposed Framework for Multi Attribute Decision Making

This section presents a decision-making (DM) framework for solving multi-attribute decision-making (MADM) problems in a Pythagorean Fuzzy Z-Number (PyFZN) environment. In practical applications, decision-making involves selecting the most appropriate alternative from a set of feasible options based on multiple, often conflicting, criteria. Such decisions typically rely on the knowledge, experience, and subjective judgments of experts. However, real-world decision-making problems are inherently complex and are frequently characterized by ambiguity, uncertainty, and imprecision. Consequently, there is a need for a mathematical framework capable of effectively representing both the uncertainty and the reliability of expert assessments. The proposed PyFZN-based framework addresses these challenges by providing a flexible and robust mechanism for modeling uncertain information and supporting reliable multi-attribute decision-making.

This section presents the MCDM approach for PyFZNs using the Hamacher aggregation operators.

Let $\hat{\mathrm{E}} = \{\hat{\mathrm{E}}_1, \hat{\mathrm{E}}_2, \hat{\mathrm{E}}_3, \dots, \hat{\mathrm{E}}_p\}$ be a set of alternatives and $\Xi = \{\Xi_1, \Xi_2, \Xi_3, \dots, \Xi_q\}$ be the set of criterion. Let $\breve{\mathrm{G}}$ be the WV such that $\sum_{j=1}^{q} \breve{\mathrm{G}}_j = 1$, $(j = 1, 2, \dots, n)$ where $\breve{\mathrm{G}}_j \in [0, 1]$ and each $\breve{\mathrm{G}}_j$ represent the weight of Ξ_j. The decision-maker (DM) reviews the alternatives on an attribute, and the assessment measurements must be in the PyFZNs. Let $\hat{\mathrm{W}} = (\hat{B}_{ij})_{p\times q}$ be the decision matrix that DM has supplied where $\hat{B}_{ij}$ shows the PyFZNs. The representation of $\hat{B}_{ij}$ is for alternatives $\hat{\mathrm{E}}$ related to criterion Ξ_j.

Accordingly, a novel algorithm is proposed to solve this problem:

Step 1: In the form of PyFZNs, the DM has expressed its personal viewpoint. Create a PyFZNs decision matrix P by moving $\hat{\mathrm{W}} = (\hat{B}_{ij})_{p\times q}$ in the direction of the alternative $\hat{\mathrm{E}}$ with $\hat{B}_{ij} = \langle(\mathscr{g}_{ij}, h_{ij}), (r_{ij}, s_{ij})\rangle$.

Step 2: The decision matrix should be normalized. If various criteria or features, such as cost ($\acute{\alpha}_c$) and benefit ($\acute{\alpha}_b$), are present. We handle each criterion or attribute uniformly by normalizing the decision matrix. If not, distinct criteria or traits ought to be combined in various ways.

$$\Xi_{ij} = \begin{cases} \hat{B}_{ij} & j \in \acute{\alpha}_b \\ \hat{B}_{ij}^{c} & j \in \acute{\alpha}_c \end{cases} \tag{1}$$

$\hat{B}_{ij}^{c}$ is the compliment of $\hat{B}_{ij}$.

Step 3: The value of the alternative $\hat{E}_i$ for different parameters Ξ_j is determined using PyFHWA, PyFHWG operators, etc. Thus, from step 2, the decision matrix is being generated. This yields the collective value Ξ_i for each alternative $\hat{E}_i$ $(i = 1,2, \dots, p)$

Step 4: Utilizing Definition 3, determine the scoring functions for each Ξ_i for PyFZNs.

Step 5: To select the best option, rank each $\hat{E}_i$ according to the score values.

The flow of the proposed MADM procedure based on PyFZNs and Hamacher interactive averaging operators is illustrated in Figure 3.

Fig. 3. Demonstration of flowchart for MADM technique

5. Practical Example

This study considers a real-world urban sustainability decision-making problem in the context of urban green space planning in South Korea, where rapid urbanization has intensified the need for effective green infrastructure and sustainable urban development. The proposed decision model is designed to assist policymakers in evaluating and prioritizing urban green space projects under conditions of uncertainty and imprecision. The considered decision alternatives include urban parks, green belts, and urban forests, all of which play a vital role in improving environmental quality, public health, and urban livability.

The complexity of this problem is addressed through a multi-attribute decision-making (MADM) framework based on Pythagorean Fuzzy Z-Numbers (PyFZNs) and Hamacher aggregation operators. The proposed framework effectively captures both the uncertainty and the reliability of expert judgments while accommodating the variability inherent in environmental and socio-economic data. Consequently, it provides a more comprehensive and reliable basis for evaluating competing urban green space alternatives.

The overall methodology, illustrated in Figure 4, consists of a structured step-by-step computational procedure that integrates uncertainty modeling, reliability assessment, and multi-criteria evaluation into a unified decision-making framework. This systematic process enables the robust ranking of urban green space alternatives and supports consistent decision outcomes. The results obtained from the proposed model assist decision-makers in identifying the most sustainable and beneficial projects, thereby contributing to more effective urban planning strategies and long-term sustainable development in South Korea.

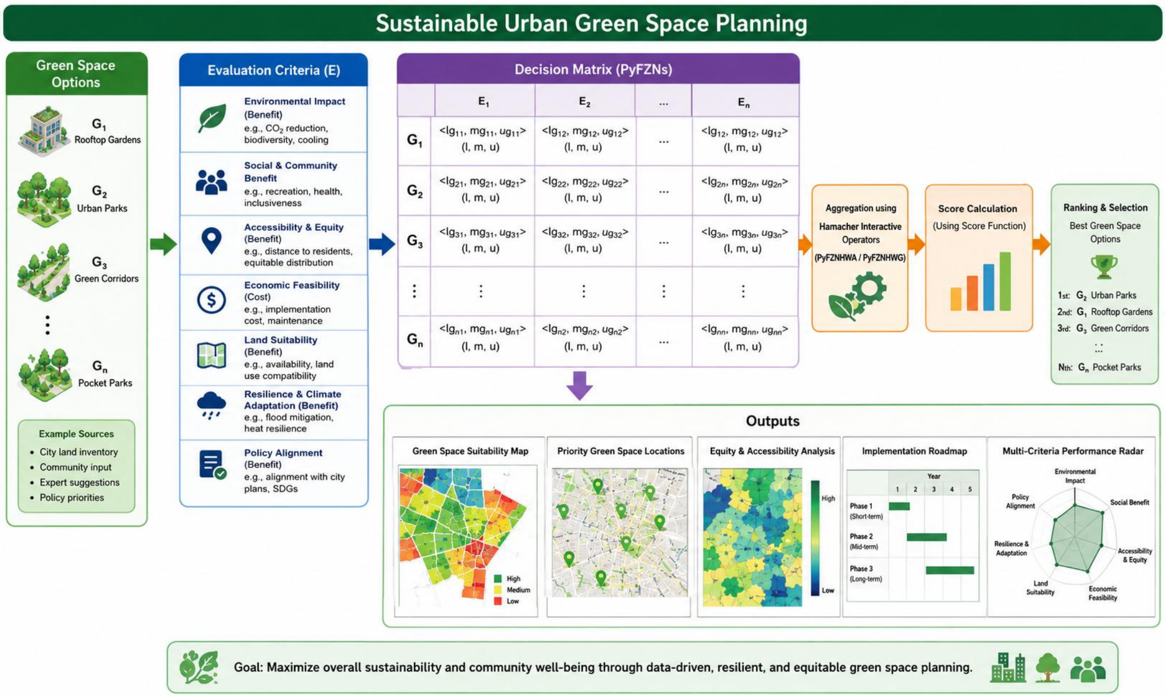

Fig. 4 Proposed ranking of urban green space alternatives using the MADM technique

5.1 Decision-Making Criteria for Urban Green Space Planning

In the proposed framework, Pythagorean Fuzzy Z-Numbers (PyFZNs) are employed to determine the importance weights of the evaluation criteria based on expert judgments. Experts evaluate the relative importance of each criterion using linguistic terms, which are subsequently transformed into PyFZN representations to simultaneously capture uncertainty and the reliability of their assessments. The individual evaluations are then aggregated and normalized to ensure that the resulting criterion weights sum to one. By incorporating both the ambiguity of expert opinions and the credibility of the provided information, the proposed weighting procedure produces a more balanced and realistic representation of criterion importance. The developed PyFZN-based decision-making framework is applied to prioritize urban green space alternatives in South Korea with the objective of promoting urban sustainability, improving environmental quality, and enhancing urban livability.

The proposed PyFZN–Hamacher MCDM framework evaluates urban green space alternatives using eight criteria that collectively capture the environmental, economic, social, and planning dimensions of sustainable urban development. *Environmental Quality* (Ξ_1) measures the contribution of green spaces to improving air quality, reducing pollution, and enhancing ecological conditions. *Accessibility* (Ξ_2) evaluates the ease with which residents can access parks, green belts, and urban forests. *Cost of Development and Maintenance* (Ξ_3) represents the financial resources required for the construction, operation, and long-term maintenance of green infrastructure. *Population Coverage* (Ξ_4) assesses the proportion of the urban population that can benefit from the proposed green space projects.

The remaining criteria address broader sustainability considerations. *Biodiversity Enhancement* (Ξ_5) evaluates the ability of green spaces to support and preserve urban flora and fauna. *Social and Health Benefits* (Ξ_6) measure the positive effects of green infrastructure on physical health, mental well-being, and social interaction among residents. *Sustainability Compliance* (Ξ_7) assesses the consistency of each alternative with environmental regulations, sustainability objectives, and green city policies. Finally, *Urban Expansion Compatibility* (Ξ_8) evaluates the capability of green space projects to coexist with future urban growth while maintaining their environmental and social functions. Collectively, these criteria provide a comprehensive basis for evaluating and prioritizing urban green space alternatives under uncertain decision-making conditions.

To demonstrate the applicability of the proposed framework, a set of decision alternatives is considered, denoted by $\hat{E} = \{\hat{E}_1, \hat{E}_2, \hat{E}_3, \hat{E}_4, \hat{E}_5, \hat{E}_6, \hat{E}_7, \hat{E}_8\}$, where each alternative represents a different urban green space development project. These alternatives are evaluated with respect to the predefined set of evaluation criteria, $\Xi = \{\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5, \Xi_6, \Xi_7, \Xi_8\}$, which collectively capture the environmental, social, economic, and sustainability dimensions that are essential for urban green space planning. The evaluation process is conducted within the proposed PyFZN–Hamacher MCDM framework, enabling a comprehensive assessment of the alternatives under conditions of uncertainty and reliability.

5.2 Application of the PyFZNIWA Operator

Step 1. The decision-maker provides the evaluation matrix in the form of Pythagorean Fuzzy Z-Numbers (PyFZNs). The corresponding decision matrix is presented in **Table 1**, where each element represents the assessment of an alternative with respect to a specific evaluation criterion under uncertainty and reliability.

Table 1
PyFZNs decision matrix taking by decision maker by PyFZNIWA Operator

	Ξ_1	Ξ_2	Ξ_3	Ξ_4
$\hat{E}_1$	⟨(0.72, 0.61), (0.66, 0.57)⟩	⟨(0.81, 0.52), (0.74, 0.62)⟩	⟨(0.63, 0.57), (0.69, 0.63)⟩	⟨(0.58, 0.69), (0.62, 0.74)⟩
$\hat{E}_2$	⟨(0.64, 0.67), (0.69, 0.61)⟩	⟨(0.73, 0.59), (0.67, 0.67)⟩	⟨(0.59, 0.61), (0.73, 0.65)⟩	⟨(0.66, 0.64), (0.66, 0.69)⟩
$\hat{E}_3$	⟨(0.81, 0.55), (0.71, 0.65)⟩	⟨(0.69, 0.63), (0.65, 0.70)⟩	⟨(0.77, 0.62), (0.67, 0.69)⟩	⟨(0.64, 0.61), (0.69, 0.71)⟩
$\hat{E}_4$	⟨(0.79, 0.60), (0.68, 0.69)⟩	⟨(0.61, 0.58), (0.72, 0.68)⟩	⟨(0.74, 0.62), (0.70, 0.68)⟩	⟨(0.72, 0.67), (0.68, 0.68)⟩
$\hat{E}_5$	⟨(0.71, 0.63), (0.72, 0.67)⟩	⟨(0.68, 0.66), (0.69, 0.65)⟩	⟨(0.62, 0.64), (0.65, 0.67)⟩	⟨(0.69, 0.62), (0.70, 0.67)⟩
$\hat{E}_6$	⟨(0.68, 0.69), (0.70, 0.63)⟩	⟨(0.63, 0.64), (0.68, 0.66)⟩	⟨(0.66, 0.65), (0.72, 0.66)⟩	⟨(0.65, 0.63), (0.65, 0.70)⟩
$\hat{E}_7$	⟨(0.75, 0.60), (0.73, 0.66)⟩	⟨(0.70, 0.62), (0.71, 0.67)⟩	⟨(0.68, 0.63), (0.74, 0.62)⟩	⟨(0.71, 0.64), (0.69, 0.68)⟩
$\hat{E}_8$	⟨(0.62, 0.70), (0.65, 0.72)⟩	⟨(0.66, 0.67), (0.68, 0.70)⟩	⟨(0.64, 0.66), (0.70, 0.71)⟩	⟨(0.67, 0.65), (0.66, 0.69)⟩
	Ξ_5	Ξ_6	Ξ_7	Ξ_8
$\hat{E}_1$	⟨(0.69, 0.58), (0.70, 0.60)⟩	⟨(0.74, 0.51), (0.73, 0.55)⟩	⟨(0.67, 0.60), (0.65, 0.64)⟩	⟨(0.76, 0.49), (0.71, 0.58)⟩
$\hat{E}_2$	⟨(0.72, 0.55), (0.68, 0.62)⟩	⟨(0.68, 0.60), (0.72, 0.57)⟩	⟨(0.61, 0.66), (0.64, 0.66)⟩	⟨(0.70, 0.57), (0.70, 0.59)⟩
$\hat{E}_3$	⟨(0.63, 0.67), (0.72, 0.58)⟩	⟨(0.71, 0.58), (0.66, 0.63)⟩	⟨(0.75, 0.52), (0.74, 0.54)⟩	⟨(0.66, 0.62), (0.68, 0.62)⟩
$\hat{E}_4$	⟨(0.59, 0.66), (0.66, 0.64)⟩	⟨(0.67, 0.60), (0.71, 0.59)⟩	⟨(0.73, 0.55), (0.69, 0.61)⟩	⟨(0.65, 0.63), (0.73, 0.55)⟩
$\hat{E}_5$	⟨(0.76, 0.54), (0.73, 0.56)⟩	⟨(0.64, 0.65), (0.67, 0.66)⟩	⟨(0.70, 0.59), (0.71, 0.60)⟩	⟨(0.72, 0.57), (0.66, 0.63)⟩
$\hat{E}_6$	⟨(0.69, 0.61), (0.69, 0.61)⟩	⟨(0.72, 0.56), (0.74, 0.53)⟩	⟨(0.60, 0.68), (0.67, 0.65)⟩	⟨(0.71, 0.58), (0.71, 0.58)⟩
$\hat{E}_7$	⟨(0.65, 0.66), (0.68, 0.62)⟩	⟨(0.77, 0.51), (0.70, 0.60)⟩	⟨(0.62, 0.67), (0.72, 0.57)⟩	⟨(0.69, 0.60), (0.66, 0.64)⟩
$\hat{E}_8$	⟨(0.70, 0.58), (0.71, 0.59)⟩	⟨(0.61, 0.69), (0.69, 0.65)⟩	⟨(0.73, 0.55), (0.73, 0.55)⟩	⟨(0.68, 0.61), (0.67, 0.63)⟩

Step 2. The evaluations in Table 1 are normalized according to the proposed normalization procedure. The normalized decision matrix, which serves as the basis for the subsequent aggregation process, is presented in Table 2.

Table 2
Normalized PyFZN decision matrix by PyFZNIWA Operator

	Ξ_1	Ξ_2	Ξ_3	Ξ_4
$\hat{E}_1$	⟨(0.13, 0.12), (0.12, 0.11)⟩	⟨(0.15, 0.11), (0.13, 0.12)⟩	⟨(0.12, 0.11), (0.12, 0.12)⟩	⟨(0.11, 0.13), (0.12, 0.13)⟩
$\hat{E}_2$	⟨(0.11, 0.13), (0.12, 0.12)⟩	⟨(0.13, 0.12), (0.12, 0.13)⟩	⟨(0.11, 0.12), (0.13, 0.12)⟩	⟨(0.12, 0.12), (0.12, 0.12)⟩
$\hat{E}_3$	⟨(0.14, 0.11), (0.13, 0.13)⟩	⟨(0.13, 0.13), (0.12, 0.13)⟩	⟨(0.14, 0.12), (0.12, 0.13)⟩	⟨(0.12, 0.12), (0.13, 0.13)⟩
$\hat{E}_4$	⟨(0.14, 0.12), (0.12, 0.13)⟩	⟨(0.11, 0.12), (0.13, 0.13)⟩	⟨(0.14, 0.12), (0.12, 0.13)⟩	⟨(0.14, 0.13), (0.13, 0.12)⟩
$\hat{E}_5$	⟨(0.12, 0.12), (0.13, 0.13)⟩	⟨(0.12, 0.13), (0.12, 0.12)⟩	⟨(0.12, 0.13), (0.12, 0.13)⟩	⟨(0.13, 0.12), (0.13, 0.12)⟩
$\hat{E}_6$	⟨(0.12, 0.14), (0.13, 0.12)⟩	⟨(0.11, 0.13), (0.12, 0.12)⟩	⟨(0.12, 0.13), (0.13, 0.12)⟩	⟨(0.12, 0.12), (0.12, 0.13)⟩
$\hat{E}_7$	⟨(0.13, 0.12), (0.13, 0.13)⟩	⟨(0.13, 0.13), (0.13, 0.13)⟩	⟨(0.13, 0.13), (0.13, 0.12)⟩	⟨(0.13, 0.12), (0.13, 0.12)⟩
$\hat{E}_8$	⟨(0.11, 0.14), (0.12, 0.14)⟩	⟨(0.12, 0.14), (0.12, 0.13)⟩	⟨(0.12, 0.13), (0.12, 0.13)⟩	⟨(0.13, 0.13), (0.12, 0.12)⟩
	Ξ_5	Ξ_6	Ξ_7	Ξ_8
$\hat{E}_1$	⟨(0.13, 0.12), (0.13, 0.12)⟩	⟨(0.13, 0.11), (0.13, 0.12)⟩	⟨(0.12, 0.12), (0.12, 0.13)⟩	⟨(0.14, 0.10), (0.13, 0.12)⟩
$\hat{E}_2$	⟨(0.13, 0.11), (0.12, 0.13)⟩	⟨(0.12, 0.13), (0.13, 0.12)⟩	⟨(0.11, 0.14), (0.12, 0.14)⟩	⟨(0.13, 0.12), (0.13, 0.12)⟩
$\hat{E}_3$	⟨(0.12, 0.14), (0.13, 0.12)⟩	⟨(0.13, 0.12), (0.12, 0.13)⟩	⟨(0.14, 0.11), (0.13, 0.11)⟩	⟨(0.12, 0.13), (0.12, 0.13)⟩
$\hat{E}_4$	⟨(0.11, 0.14), (0.12, 0.13)⟩	⟨(0.12, 0.13), (0.13, 0.12)⟩	⟨(0.13, 0.11), (0.12, 0.13)⟩	⟨(0.12, 0.13), (0.13, 0.11)⟩
$\hat{E}_5$	⟨(0.14, 0.11), (0.13, 0.12)⟩	⟨(0.12, 0.14), (0.12, 0.14)⟩	⟨(0.13, 0.12), (0.13, 0.12)⟩	⟨(0.13, 0.12), (0.12, 0.13)⟩
$\hat{E}_6$	⟨(0.13, 0.13), (0.12, 0.13)⟩	⟨(0.13, 0.12), (0.13, 0.11)⟩	⟨(0.11, 0.14), (0.12, 0.13)⟩	⟨(0.13, 0.12), (0.13, 0.12)⟩
$\hat{E}_7$	⟨(0.12, 0.14), (0.12, 0.13)⟩	⟨(0.14, 0.11), (0.12, 0.13)⟩	⟨(0.11, 0.14), (0.13, 0.12)⟩	⟨(0.12, 0.13), (0.12, 0.13)⟩
$\hat{E}_8$	⟨(0.13, 0.12), (0.13, 0.12)⟩	⟨(0.11, 0.15), (0.12, 0.14)⟩	⟨(0.13, 0.11), (0.13, 0.11)⟩	⟨(0.12, 0.13), (0.12, 0.13)⟩

Step 3. The normalized evaluations are aggregated using the proposed PyFZNIWA operator. Specifically, the aggregated value for each criterion is computed as $\Xi_i = \mathrm{PyFZNIWA}(\Xi_{i1}, \Xi_{i2}, \dots, \Xi_{ip})$. The resulting aggregated criterion values are presented in Table 3.

Table 3
Evaluation by averaging operator

	Ξ_1	Ξ_2	Ξ_3	Ξ_4
$\hat{E}_1$	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.03), (0.04, 0.04)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.05), (0.04, 0.05)⟩
$\hat{E}_2$	⟨(0.04, 0.05), (0.04, 0.04)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.04), (0.05, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_3$	⟨(0.05, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.04, 0.05)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_4$	⟨(0.05, 0.04), (0.04, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_5$	⟨(0.04, 0.04), (0.05, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.05), (0.04, 0.05)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_6$	⟨(0.04, 0.05), (0.04, 0.04)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.05), (0.05, 0.05)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_7$	⟨(0.05, 0.04), (0.05, 0.04)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_8$	⟨(0.04, 0.05), (0.04, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.05), (0.05, 0.05)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
	Ξ_5	Ξ_6	Ξ_7	Ξ_8
$\hat{E}_1$	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.04), (0.05, 0.04)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.04), (0.05, 0.05)⟩
$\hat{E}_2$	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_3$	⟨(0.04, 0.05), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.04, 0.05)⟩	⟨(0.04, 0.03), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_4$	⟨(0.04, 0.05), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.04)⟩
$\hat{E}_5$	⟨(0.05, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.05), (0.04, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_6$	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.04)⟩	⟨(0.04, 0.05), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_7$	⟨(0.04, 0.05), (0.04, 0.04)⟩	⟨(0.05, 0.04), (0.05, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩
$\hat{E}_8$	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.04, 0.06), (0.05, 0.05)⟩	⟨(0.04, 0.04), (0.04, 0.04)⟩	⟨(0.05, 0.05), (0.05, 0.05)⟩

Step 4. The proposed aggregation operator is applied to the normalized decision matrix to obtain the aggregated evaluation matrix. The resulting aggregated score matrix is presented below (Table 4).

Table 4.
Aggregated Evaluation Matrix of the Alternatives

	Ξ_1	Ξ_2	Ξ_3	Ξ_4	Ξ_5	Ξ_6	Ξ_7	Ξ_8
$\hat{E}_1$	0.5001	0.5004	0.5001	0.4996	0.5000	0.5003	0.4999	0.5003
$\hat{E}_2$	0.4999	0.5000	0.5000	0.5000	0.5001	0.5000	0.4997	0.5001
$\hat{E}_3$	0.5003	0.4999	0.5001	0.5000	0.4999	0.4999	0.5003	0.4998
$\hat{E}_4$	0.5001	0.5000	0.5001	0.5001	0.4997	0.4999	0.5001	0.5000
$\hat{E}_5$	0.5000	0.4999	0.4998	0.5002	0.5003	0.4996	0.5001	0.4999
$\hat{E}_6$	0.4999	0.4999	0.5000	0.4999	0.5000	0.5003	0.4997	0.5001
$\hat{E}_7$	0.5001	0.5000	0.5001	0.5001	0.4998	0.5002	0.4999	0.4998
$\hat{E}_8$	0.4996	0.4998	0.4998	0.5000	0.5001	0.4995	0.5002	0.4998

Step 5. The score function is computed for each alternative as follows:
$S(\hat{E}_1) = 0.5513$
$S(\hat{E}_2) = 0.3496$
$S(\hat{E}_3) = 0.5427$
$S(\hat{E}_4) = 0.3601$
$S(\hat{E}_5) = 0.6687$
$S(\hat{E}_6) = 0.3643$
$S(\hat{E}_7) = 0.3457$
$S(\hat{E}_8) = 0.5319$
The alternatives are then ranked in descending order according to their score-function values, yielding the following preference order:
$\hat{E}_5 \succ \hat{E}_1 \succ \hat{E}_3 \succ \hat{E}_8 \succ \hat{E}_6 \succ \hat{E}_4 \succ \hat{E}_2 \succ \hat{E}_7$
Since $\hat{E}_5$ achieves the highest score value ($S(\hat{E}_5) = 0.6687$), it is identified as the optimal alternative. This result indicates that $\hat{E}_5$ provides the most suitable solution among the considered urban green space development projects according to the proposed PyFZN–Hamacher MCDM framework.

6. Sensitivity Analysis and Validation

An extensive sensitivity analysis is conducted to evaluate the robustness and stability of the proposed PyFZN-based MCDM framework. The analysis investigates the influence of different values of the parameter Ξon the ranking of the alternatives while keeping the remaining components of the decision model unchanged. Examining the impact of parameter variations enables the assessment of the consistency and reliability of the proposed decision-making framework under different decision environments.

To evaluate the effect of the parameter Ξ, several values (10, 15, 18, 22, 25, and 30) are considered. The corresponding score values and ranking results obtained using the proposed PyFZNHIWA operator are presented in Table 5. The results indicate that the score values of the alternatives gradually increase as the value of Ξincreases. However, changes in the relative ranking of some alternatives are also observed, particularly for larger parameter values.

At the initial parameter value, $\Xi = 10$, the ranking order is $\hat{E}_5 \succ \hat{E}_1 \succ \hat{E}_3 \succ \hat{E}_8 \succ \hat{E}_6 \succ \hat{E}_4 \succ \hat{E}_2 \succ \hat{E}_7$, where $\hat{E}_5$ is identified as the best alternative. For $\Xi = 15$and $\Xi = 18$, $\hat{E}_5$ remains the preferred alternative, while $\hat{E}_8$ moves ahead of $\hat{E}_1$ and $\hat{E}_3$, indicating only minor changes in the ranking structure. When the parameter increases to $\Xi = 22$, $\hat{E}_2$ improves considerably and becomes the second-ranked alternative. For the highest parameter values, $\Xi = 25$and $\Xi = 30$, $\hat{E}_2$ becomes the best-ranked alternative, whereas $\hat{E}_5$ moves to the second position.

These findings demonstrate that the proposed model is sensitive to variations in the parameter Ξ, particularly for larger parameter values. Nevertheless, the ranking changes occur in a systematic and interpretable manner, indicating that the proposed framework responds consistently to parameter variations rather than producing unstable or erratic results. Therefore, the sensitivity analysis confirms that the proposed PyFZNHIWA-based decision-making model provides a flexible and reliable evaluation framework capable of adapting to different decision-makers' preferences and parameter settings. Table 5 presents the score values and corresponding ranking results obtained for the different values of the parameter Ξ.

Table 5
Score functions and corresponding ranking

Ξ	$S(\hat{E}_1)$	$S(\hat{E}_2)$	$S(\hat{E}_3)$	$S(\hat{E}_4)$	$S(\hat{E}_5)$	$S(\hat{E}_6)$	$S(\hat{E}_7)$	$S(\hat{E}_8)$
10	0.5513	0.3496	0.5427	0.3601	0.6687	0.3643	0.3457	0.5319
15	0.5374	0.3918	0.5486	0.3867	0.6549	0.3715	0.3526	0.5581
18	0.5231	0.4552	0.5358	0.4214	0.6412	0.3839	0.3694	0.5667
22	0.5078	0.5834	0.4976	0.4625	0.6268	0.3612	0.4029	0.5415
25	0.4926	0.6127	0.4862	0.4489	0.6035	0.3375	0.4268	0.5592
30	0.4758	0.6369	0.5053	0.4316	0.5894	0.3187	0.4472	0.5711

Ξ	Ordering
10	$\hat{E}_5 \succ \hat{E}_1 \succ \hat{E}_3 \succ \hat{E}_8 \succ \hat{E}_6 \succ \hat{E}_4 \succ \hat{E}_2 \succ \hat{E}_7$
15	$\hat{E}_5 \succ \hat{E}_8 \succ \hat{E}_3 \succ \hat{E}_1 \succ \hat{E}_2 \succ \hat{E}_4 \succ \hat{E}_6 \succ \hat{E}_7$
18	$\hat{E}_5 \succ \hat{E}_8 \succ \hat{E}_3 \succ \hat{E}_1 \succ \hat{E}_2 \succ \hat{E}_4 \succ \hat{E}_6 \succ \hat{E}_7$
22	$\hat{E}_5 \succ \hat{E}_2 \succ \hat{E}_8 \succ \hat{E}_1 \succ \hat{E}_3 \succ \hat{E}_4 \succ \hat{E}_7 \succ \hat{E}_6$
25	$\hat{E}_2 \succ \hat{E}_5 \succ \hat{E}_8 \succ \hat{E}_1 \succ \hat{E}_3 \succ \hat{E}_4 \succ \hat{E}_7 \succ \hat{E}_6$
30	$\hat{E}_2 \succ \hat{E}_5 \succ \hat{E}_8 \succ \hat{E}_3 \succ \hat{E}_1 \succ \hat{E}_7 \succ \hat{E}_4 \succ \hat{E}_6$

Figure 5 presents the sensitivity analysis results for different values of the parameter Ξ, illustrating the corresponding changes in the score values and ranking of the alternatives. The figure highlights the robustness of the proposed framework under varying parameter settings.

Finally, urban green spaces play a crucial role in promoting environmental sustainability, enhancing public health, and improving the overall quality of urban life [23]. The planning and prioritization of new green space developments require the simultaneous consideration of multiple criteria, including population density, environmental benefits, accessibility, economic feasibility, and long-term sustainability objectives [24]. However, the decision-making process is often complicated by incomplete demographic information, dynamic urban growth patterns, and the inherent uncertainty associated with expert judgments [25].

To address these challenges, this study employs a Pythagorean Fuzzy Z-Number (PyFZN)-based MCDM framework integrated with Hamacher aggregation operators. The proposed approach effectively combines multiple evaluation criteria and expert opinions while simultaneously accounting for both uncertainty and the reliability of the available information. Consequently, it provides a robust and flexible decision-support tool for prioritizing urban green space projects and facilitating more effective resource allocation under uncertain decision-making environments.

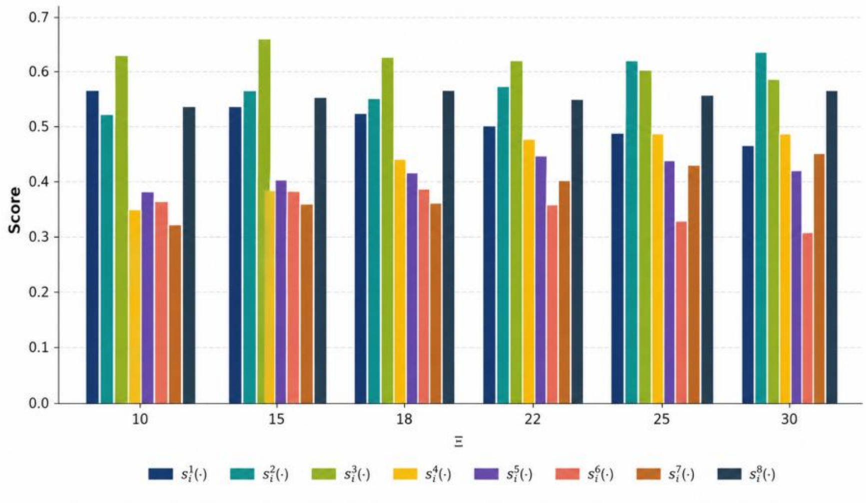

Fig. 5. Chart Illustrating the Ranking Results

7. Conclusion

This study proposed a novel multi-attribute decision-making (MADM) framework by integrating Pythagorean Fuzzy Z-Numbers (PyFZNs) with Hamacher aggregation operators to address uncertainty, vagueness, and reliability in expert-based decision-making. The proposed framework simultaneously models membership, non-membership, and reliability information, providing a more comprehensive representation of uncertain assessments than conventional fuzzy approaches. A family of Hamacher-based aggregation operators was developed to effectively combine expert judgments while preserving the underlying uncertainty and credibility of the available information.

The applicability of the proposed framework was demonstrated through a real-world urban green space planning problem in South Korea. Eight sustainability-related evaluation criteria were considered to assess alternative green space development projects, including parks, green belts, and urban forests. The obtained results show that the proposed approach effectively ranks the alternatives under uncertain decision environments while producing consistent and interpretable outcomes. The sensitivity analysis further confirmed the robustness of the proposed framework by demonstrating its ability to respond systematically to changes in the aggregation parameter without producing unstable decision behavior.

The proposed PyFZN–Hamacher framework contributes to the existing literature by providing a flexible aggregation mechanism capable of simultaneously handling uncertainty and reliability in multi-attribute decision-making problems. From a practical perspective, the framework offers policymakers a reliable decision-support tool for prioritizing sustainable urban green space projects while considering environmental, social, economic, and urban development objectives. Although the case study focused on South Korea, the proposed methodology is sufficiently general to be applied to a broad range of complex decision-making problems characterized by uncertain and imprecise information.

Despite these promising results, several limitations should be acknowledged. The proposed framework was validated using a single case study, and its generalizability should be further examined through applications in different geographical regions and planning contexts. In addition, the model relies on expert judgments, which inevitably introduce a degree of subjectivity into the evaluation process. Furthermore, the criterion weights are assumed to remain constant throughout the analysis, whereas real-world planning environments often evolve over time due to policy, environmental, and socioeconomic changes.

Future research may extend the proposed framework by incorporating dynamic criterion weights, real-time urban data obtained from GIS platforms, environmental sensors, and IoT technologies, as well as advanced artificial intelligence and machine learning techniques for adaptive decision support. The integration of the proposed Hamacher aggregation mechanism with other advanced fuzzy and probabilistic models, together with dynamic group decision-making strategies, also represents a promising direction for improving decision robustness and supporting sustainable planning in smart-city environments.

Conflicts of Interest

The authors declare no conflicts of interest.

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