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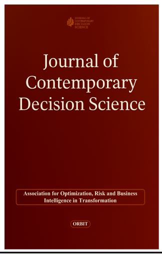

Sustainability Challenges for Non-Governmental Funded Higher Educational Institutions: A q-rung Orthopair Fuzzy Sets Coupled MCDM Approach

Tripti Basuri[1,2], Kamal Hossain Gazi[1], Prodip Bhaduri[1], Srabani Guria Das[3], Sankar Prasad Mondal[1,*]

[1] Department of Applied Mathematics, Maulana Abul Kalam Azad University of Technology, West Bengal, Nadia, West Bengal, India
[2] Uttar Dum Dum Vidyapith (Girls) Primary section, Birati, West Bengal, India
[3] Department of Basic Science, NSHM Knowledge Campus, Durgapur, West Bengal, India

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ABSTRACT

Financial aid plays a crucial role in the sustainability and development of higher educational institutions, particularly those operating without governmental funding. This study proposes a decision-making framework for identifying and prioritizing the key challenges faced by non-governmentally funded higher educational institutions. To address this problem, a hybrid Multi-Criteria Decision-Making (MCDM) approach is employed, where the MEREC method is used to determine the weights of evaluation criteria and the MARCOS method is applied to rank the identified challenges. To effectively handle uncertainty and ambiguity in expert judgments, q-Rung Orthopair Fuzzy Numbers (q-ROFNs) are integrated into the framework. Furthermore, sensitivity analysis is conducted to assess the robustness and reliability of the obtained results under different conditions. The results reveal the most influential challenges affecting the financial sustainability and operational performance of these institutions, providing valuable insights for strategic decision-making. In addition, the proposed framework offers a systematic and flexible tool that can be adapted to similar decision-making problems in the education sector. Overall, the q-ROF-MEREC-MARCOS framework supports policymakers, institutional administrators, and stakeholders in developing effective strategies for enhancing the resilience and sustainable growth of non-governmentally funded higher educational institutions.

*Corresponding author.
E-mail address: sankar.mondal02@gmail.com

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1. Introduction

Education plays a key role in the development of a society as well as of a country. In a progressive country like India, it is very important to focus on establishing a strong educational system that is compatible with the times. To provide quality education to students, it is essential to advance the facilities and infrastructure of educational institutions. There are mainly 3 types of educational institutions based on financial support, such as government institutions, government-aided institutions and non-government funded institutions. In a government institution, all expenditures, such as salaries and infrastructure, of an institution are fully funded by the government, whereas government-aided institutions are partially funded by the government. Non-governmentally funded institutions do not receive any financial assistance from the government, so they are running through using the fees collected from students and private resources. Thus, the expenses of studying at a self-financed private institution are so high that it is difficult to bear for students coming from economically weaker families. After completing the basic education, students desire to take admission in a higher educational institution for their future study and here they initially preferred government or government-aided institutions.

1.1 *Higher educational institutions structures in India*

Education helps to understand the unknown using the light of knowledge. Education not only improves one's thinking but also helps in building a better society. Education helps in developing some good qualities like ethics, values, a respectful nature and honesty, which make a good human being. Higher Education Institutions (HEIs) [1] in India are established by an Act of Parliament or a State Legislature, which is administered under the Ministry of Education. State Universities are governed by the respective Acts. On the other hand, an Act of Parliament governs Central Universities. Deemed-to-be-Universities were declared by the University Grants Commission in 1956 under Section 3 of the UGC Act. To maintain and coordinate the quality of education, the University Grants Commission distributes grants to universities and colleges. Established technical institutions are regulated by the All India Council for Technical Education (AICTE). Whereas medical education is regulated by the National Medical Commission. According to the hierarchical structure, most universities have a Chancellor, Vice-Chancellor, Registrar and Finance Officer. The executive council, academic council and board of studies play an important role in academic bodies to make the right decisions in critical situations. Affiliated colleges are under the universities and follow the curriculum and examination system prescribed by the university. Autonomous colleges enjoy autonomy in the academic field but are under the overall supervision of the university and the regulatory bodies.

1.2 *Role of higher educational institutions for societal benefits and sustainability*

Higher education institutions play a critical role in advancing social good and sustainability [2] by fostering knowledge creation, research, and innovation in a variety of fields, and by addressing community-centered issues, including poverty, health, environment, and equitable development. It develops students' ethical leadership skills, socio-economic development [3], and the skills and values necessary for sustainable practices in business, technology, and public policy. Originally, the framework for higher education in India was established through an Act of Parliament, which empowered regulatory bodies to set institutional standards. For that purpose, the University Grants Commission Act, 1956, was enacted. It was responsible for improving and maintaining the quality of university education throughout the country, allocating grants to eligible institutions, and advising the government on higher education policy. Again, the All India Council for Technical Education was given statutory status by Parliament in 1987 to plan, coordinate and ensure quality in technical and professional education, including engineering management and applied sciences. Both the UGC and AICTE, which were formed, work under the Ministry of Education, Government of India. They approve institutes, encourage research through their grants and projects, which in turn set academic standards, and approve new courses in various institutions. The UGC also encourages collaborative programs to expand global connections between students and engaged faculty in the internationalization of education. Higher education institutions, such as universities and colleges, directly contribute to development by implementing quality education and integrating community engagement, industry partnerships and social inclusion into the curriculum. Academic councils, boards of studies, research committees and extension cells play a key role in connecting universities to larger societal issues. India is currently moving towards a unified and robust regulatory framework through bills such as the proposed Higher Education Commission of India (HECI). Its aim is to streamline oversight and focus on enhancing quality in all higher education sectors through their regulation. Ultimately, ethical governance, while maintaining standards, serves as an engine for innovation for social resilience and equitable opportunities for all learners.

1.3 *Motivation of this study*

This section summarizes the motivation for this study. Higher education is the gateway to deeper study of any subject, research-oriented work and skill development. The expenses of getting higher education [4] are increasing nowadays. There are a limited number of government-funded educational institutions, so most of the students have to rely on private institutions to pursue higher studies. Employability after completing higher study doesn't rely only on the degree or certificates, but rather than it depends upon the skills and practical knowledge to deal with any situation in the working zone. So, it is also necessary for private institutes to maintain the quality of teaching, advanced lab facilities, modern equipment for research work, and collaboration with industries to give students internship facilities, etc. The primary objective of this study is to identify the challenges of privately funded higher education institutions. Generally, privately funded higher education institutions face various sustainability challenges, such as rising operational costs, dependence on student fees, limited and unstable funding sources of the managed higher education institutions, etc. They also struggle to balance infrastructural development, educational quality and long-term financial stability to remain competitive and socially responsible. The motivation for this study is described as follows:

(a) Multiple criteria are considered to assess the sustainability challenges of privately funded higher education institutions.

(b) q-rung orthopair fuzzy numbers (q-ROFNs) are taken to capture the uncertainty of the data set.

(c) New score function and accuracy function were developed to covert the fuzzy values into crisp value and applied to simplify the adopted model.

(d) Using the MEREC method, decision matrices are constructed and weight of each criterion is evaluated. A weight-based ranking mechanism called 'MARCOS' has been used to rank alternatives.

(e) Information is obtained from three decision makers (DMs) in a neutral process, in a linguistic way.

(f) A sensitivity analysis helps to test the stability of the proposed model.

1.4 Research outline

Based on the above research and motivations, we outline the research in this section. Its primary objective is to properly process the challenges of privately funded higher education institutions by considering them. For this, six important criteria are adopted, with five alternatives taken for evaluation. Two MCDM methods are considered as optimization tools, while q-rung orthopair fuzzy numbers are evaluated as uncertainty tools. Data are collected in an unbiased manner and numerical calculations are performed to generate results. Finally, sensitivity and comparative analyses were conducted to address the resilience of the entire model.

1.5 Structure of this paper

In this section, the paper's structure is presented concisely. An introduction to the sustainability challenges of non-governmentally funded higher education institutions is presented in Section 1. A brief literature survey on various fields is given in Section 2. Then, Section 3 demonstrates the criteria as sustainable challenges for non-government funded higher educational institutions. The preliminaries of mathematical tools are presented in Section 4. Furthermore, Section 5 outlines the MCDM methodologies considered. Model formulation and data collection of this study are covered in Section 6. After that, the numerical illustrations of the proposed research are discussed in Section 7. The sensitivity analysis is demonstrated in Section 8. Additionally, Section 9 focuses on the research implications of this research. Lastly, the conclusions and the scope for future research are discussed in Section 10.

2. Literature survey of this study

We investigate the literature survey of this research article in this section. We will have an initial discussion on the background of non-government funded higher educational institutions, focusing on their sustainability challenges. Then we will discuss some studies on q-ROFSs [5]. At the end, a discussion on the literature on MCDM methodologies, MEREC [6] and MARCOS [7] will be presented.

2.1 Background on sustainability challenges of HEIs

In this section, a brief literature survey on sustainability challenges for non-governmental funded higher educational institutions is performed. The sustainability of the institutions depends on multi-dimensional bridges balancing majorly in-between financial constraints and educational efficiency. Multiple current studies have investigated sustainability challenges in HEI, focusing on governance, financial and intuitional standpoints. In 2025, Trinh, T. M. et al. [8] described in their paper about the unseen components influencing the financial stability in the expanding HEIs. A study in 2024 by Connie, B.D. et al. [9] explained the opportunities and some particular problems of the private educational sectors of Philippine. Hassan, M.M. et al. [10] explored the dimensions and research gaps in their research work on a literature review on sustainability procedures in the year of 2025. Acosta, A. N. et al. [11] investigated the importance of performing leadership roles for preserving sustainable performance in the area of research in private universities in their study in the year of 2024. Yassim, K. et al. [12] described the difficulties and administrative complications of the roles of the stakeholders for sustainable development of a university in their research article in the year 2025. Elmassri, M. et al. [13] have discussed about the perceptions of the curricula in business schools in their study in the year 2023. Opportunities and difficulties for sustainability challenges in a educational organization are described by Dahlström, N. [14] in their study of 2025. The research by Carvalho, O. M. et al. [15] in the year of 2025 dived into the literature review of campus operations of HEIs for sustainability. Ramírez-Correa, P. E. et al. [16] investigated about the digital sustainable education under social inclusion in their research in the year 2025. Barnett-Itzhaki, Z. et al. [17] described a holistic perspective for sustainability of HEIs through exploring economic, social and transportability in their research in 2025. With the help of existing literature, we get to know about the sustainability struggles of HEIs due to various critical issues related to restricted funding structures, constraints because of weak governance and non-focused multilayer objectives of the administration panels.

2.2 *Literature of Mathematical tool*

In real-world critical situations, crisp numbers are unable to deal with the uncertainty associated with the scenario. For dealing with the ambiguity in essential conflicting data sets, there is a strong requirement for fuzzy numbers. In the year 1965, Lotfi A. Zadeh [18] first introduced us to fuzzy sets and fuzzy numbers, having the capability of dealing with uncertainties. Several mathematicians worked on fuzzy sets and fuzzy numbers and extended the concepts in numerous ways. The popular and most useful extensions of fuzzy sets and fuzzy numbers are: triangular fuzzy numbers [19], trapezoidal fuzzy numbers [20], pentagonal fuzzy numbers [21], intuitionistic fuzzy sets [22], neutrosophic sets [23], fuzzy soft sets [24], etc. In this paper, for model formulation and computation, we have applied the concept of q-rung orthopair fuzzy numbers (q-ROFNs). In q-ROFNs, there are two membership functions with boundary conditions, as the qth power sum of membership functions is always less than 1. To achieve more realistic outcomes, the concept of membership function is introduced to q-rung orthopair fuzzy numbers [25], forming the q-rung orthopair fuzzy sets [5] . In the year 2022, Zulqarnain, R. M. et al. [26] applied Einstein geometric aggregation operators in a q-rung orthopair fuzzy soft set environment to perform diagnosis of medical patients. Yusoff, B. et al. [27] in their paper proposed a new extension of fuzzy number known as circular q-rung orthopair fuzzy set (Cq-ROFS) and introduced some algebraic properties of Cq-ROFS. Pinar, A. et al. [28] introduced a multi-criteria group decision-making approach under q-rung orthopair fuzzy environment for supplier selection. Wang, D. et al. [29] presented two novel distance measures of q-ROFS and applied those proposed distance measures in different practical situations to check their efficiency. Xiao, L. et al. [30] introduced a decision-making model for manufacturer selection under q-rung orthopair fuzzy environment and also proposed a new score function. Bilal, M. A. et al. [31] used rough q-rung orthopair fuzzy sets to tackle the decision-making problem. AlSalman, H. et al. [32] in their study focused on the graphical analysis of information given in q-rung orthopair fuzzy environment and also evaluated the ideas of domination theory (DT) and double domination theory (DDT) in q-ROFGs. Khan, A. et al. [33] in their study extended a popular MCDM method, namely the CoCoSo method, in a q-rung orthopair fuzzy environment. Kumar, M. et al. [34] introduced a TOPSIS algorithm for Q-Rung Orthopair Z-Numbers (Q-ROZNs) and implemented this in decision-making problem.

2.3 *Background of MCDM methodologies*

Multi-Criteria Decision-Making (MCDM) [35] is a very efficient and useful tool for optimization, used for solving complicated real-life decision-making problems containing conflicting criteria. MCDM obtains optimum solutions for problems consisting of several criteria and multiple alternatives. For calculating weight of criteria, we have various methodologies like Analytic Hierarchy Process (AHP) [36], Step-wise Weight Assessment Ratio Analysis (SWARA) [37], Entropy [38], CRiteria Importance Through Intercriteria Correlation (CRITIC) [39], MEthod based on the Removal Effects of Criteria (MEREC) [40] etc and several techniques for ranking the best alternatives such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [41], Vlse Kriterijumska Optimizacija Kompromisno Resenje

(VIKOR) [42], Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) [43], Multi-Objective Optimization on the Basis of Ratio Analysis (MOORA) [44], Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) [45] etc. Decision makers like policy makers, company owners, investors and management specialists benefit from using MCDM methodologies by obtaining the most preferable alternative while prioritising all the related criteria of a dynamic and serious situation. Some examples of real life problems where MCDM are used are as follows: site selection of a new canteen in an uncertain environment [46], IoT adaptation in a sustainable specialist hospital [47], selection of a green supplier in the textile industry [48], evaluation of multiple solar technologies through various parameters [49], supply chain management in health care sector [50] and so on.

To calculate the criteria weights, we have used the MEthod based on the Removal Effects of Criteria (MEREC) method. Ghorabaee, M. K. et al. [6] first introduced the MEREC method. Some recent application-based studies using the MEREC method are described in Table 1, as follows:

Table 1
Some recent studies on fuzzy-MEREC method with applications

Author	Year	Uncertainty taken	Application area
[51] Mishra, A.R. et al.	2025	Single-valued neutrosophic set	Sustainable energy storage technology
[52] Elsayed, A.	2024	Neutrosophic fuzzy set	Smart building selection for new capital
[53] Petrovic, N. et al.	2025	Triangular fuzzy number	Annual operational efficiency in passenger and freight air transport
[54] Wang, CN. et al.	2025	Fuzzy number	Renewable technologies
[55] Mishra, A.R. et al.	2024	Fermatean fuzzy number	Household waste recycling plant location
[56] Hasnan, Q.H. et al.	2024	Triangular fuzzy number	Assessing halal suppliers
[57] Saranya, M. et al.	2025	Cubic fermatean fuzzy set	Healthcare waste management system
[58] Irvanizam, I. et al.	2025	Spherical fuzzy number	Banking financial aid recipients
[59] Seikh, M.R. et al.	2025	Interval-valued spherical fuzzy number	Adaption of electric vehicle
[60] Ali, J.	2025	Linguistic q-rung orthopair fuzzy set	Plastic waste management
[61] Rani, P. et al.	2021	Fermatean fuzzy set	Technology for food waste management
[62] Mishra, A. R. et al.	2022	Single-valued neutrosophic numbers	Assessment strategy of low carbon tourism

The ranking of the alternatives is evaluated by the Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) method. Stević, Ž. et al. [7] first introduced the MARCOS method. Some recent studies using the MARCOS method with associated details are described in Table 2, as follows:

Table 2
Some recent studies on fuzzy-MARCOS method with applications

Author	Year	Uncertainty taken	Application area
[63] Younis, M. et al.	2025	Pythagorean hesitant fuzzy number	Renewable energy investments
[64] Radovanović, M. et al.	2025	Spherical fuzzy number	Evaluation of university professors
[65] Baydaş, M. et al.	2025	Fuzzy number	IoT-based healthcare systems
[66] Mondal, M. K. et al.	2023	Pythagorean fuzzy number	Sustainable forest resources
[67] Ali, J.	2022	q-rung orthopair fuzzy set	Solid waste management
[68] El-Araby, A.	2023	Fuzzy number	Different engineering applications
[69] Muhammad, A. et al.	2023	q-rung picture fuzzy number	Framework for high-stakes industries
[70] Abualkishik, A. Z. et al.	2022	Triangular fuzzy number	Smart agricultural production efficiency
[71] Stanković, M. et al.	2020	Triangular fuzzy number	Risk analysis of road traffic
[72] Bakır, M. et al.	2021	Fuzzy number	E-service in airlines
[73] Puška, A. et al.	2021	Triangular fuzzy number	Sustainable supplier selection
[74] Bakır, M. et al.	2021	Triangular fuzzy number	Selection of regional aircraft

3. Sustainable Challenges for Non-Government Funded Higher Educational Institutions

In a developing country like India, education plays a major role in the progression of society and economy [3]. For providing higher education, there are primarily two categories of authorities: government-funded institutes and non-government funded institutes. Without any financial assistance from the government, it is challenging for the non-funded institutions to sustain. The major sustainability factors for non-governmentally funded higher educational institutions are: sustainable financial support [75], infrastructure & technological support [76], academic Quality [77], institutional governance & management [78], research & innovation [79], and industrial engagement & employability [80]. Private institutions directly depend on the fees collected by the students for their financial sustainability. Highly qualified, experienced professors can improve the academic environment and research outcomes, but hiring them for long can create a financial burden on the institute. Further, digital smart classrooms, modern and technically equipped laboratories, high-quality libraries and everything need a lot of capital too, but the fees of students cannot be increased abruptly. So, balancing between collected fees and provided academic quality, keeping the relevance to industrial collaboration is a real challenge. The criteria for the non-governmental funded higher educational institutions are graphically presented in Figure 1. The potential factors of sustainability challenges are discussed below:

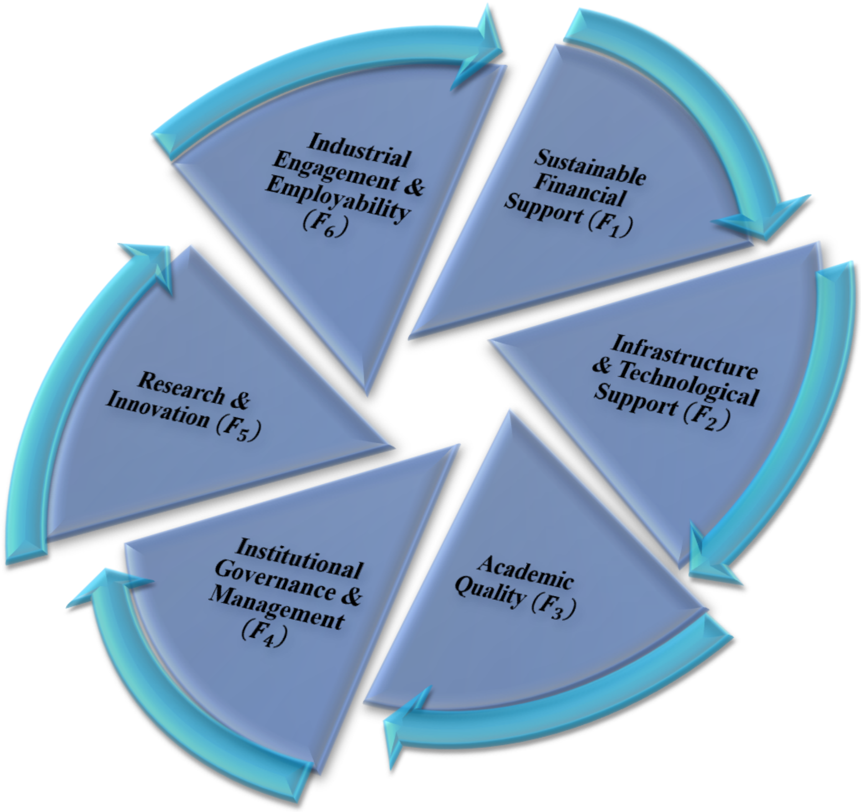

Fig. 1. Criteria related to non-governmental funded higher educational institutions

3.1 *Sustainable Financial Support* (F_1)

Non-governmental institutions are run in the absence of financial support from the government. This makes it challenging for them to sustain [81], especially in the modern era of a competitive market. Thus, the sustainability of a private educational organization is highly dependent on the admission of students and consultancy services. Fees collected from the students are the main source for paying salaries of faculty and infrastructural development of the institution. The incapability to diversify the revenue sources [75] highly impacts the faculty retention and overall improvement of the organization in long-term scenarios. To survive against the economic downfalls and volatile student admissions, organizations have to ensure funding [82] from corporate partnerships, alumni donations and grants from project-based services.

3.2 *Infrastructure & Technological Support* $(F2)$

In the modern era of technological advancement, all the educational institutions [83] have to adapt to the current technologies, furthermore advancing the infrastructure of the organizations. To stand out from the competition, organizations have to implement high-speed internet connectivity, digital classrooms, advanced laboratories, libraries, canteens and other required essential facilities [76]. But to improve the mentioned features, organizations have to spend a huge amount of capital. But to sustain the developments, they are bound to increase the fees, which will be a potential burden [84] to the students and that would immediately have an effect on the admission. This step may reduce the number of admissions instead of attracting more students, making the sustainability of the higher educational institution challenging.

3.3 Academic Quality (F_3)

The academic quality of an educational institution is the primary key factor in enrolling students. An organization without providing proper academic guidance and preparation towards modern job standards cannot attract students for a long period of time. Academic quality [85] is dependent on the quality of factualities, along with the classroom facilities. Highly qualified individuals from reputed institutions [77] with years of experience usually ask for a high paycheck, but it creates a certain burden on the institutions if the student sources are limited. Digital smart classrooms and modern simulation laboratories are also very expensive to sustain. To maintain sustainability, the organizations [86] have to ensure regular external audits, peer review systems and NAAC compliance to amplify academic relevance and determination.

3.4 Institutional Governance & Management (F_4)

Efficient governance and management systems [78] are a crucial sustainability goal for long-term sustainability in the case of non-governmental institutions. Many such organizations face problems like shadiness in arbitrary decision-making, authoritarianism and non-accountability due to the owner-led or family-run unprofessional management systems [87]. Data-driven, transparent management systems are essential for long-term strategic sustainable governance, consisting of professional administrative actions paired with decentralized decision-making. Rational decisions need to be taken in search of longevity in sustainability [88] instead of short-term profits. Ethical leadership, along with transparent policy frameworks and management, enhances organizational trust, efficiency, influence and structural strength.

3.5 Research & Innovation (F_5)

Research and innovation [89] are the key factors to determine the academic capabilities of an institution. But due to a lack of funds and governmental support, it becomes difficult for them to shine in the key area of research. To publish quality research papers, there will be a need to hire highly qualified faculty, which will cause a financial burden on non-government-funded higher education institutes with limited resources [79]. Innovations necessarily need a well-structured infrastructure ecosystem of advanced labs along with experienced and organized faculty teams. Private non-government institutions [90] look for external research grants and research collaboration ideas, leaning on industry requirements for getting additional funding for sustainable advancement.

3.6 Industrial Engagement & Employability (F_6)

The modern era is the era of technology, and to survive in the competitive job market, a student needs to have the ability to work with and work for recently developed technologies [91]. But most of the students suffer because there is a clear gap between academic learning and its applications in industries. The academic curriculum is not updated as per modern technological advancements, which are evolving regularly. All these things directly make the situation of the non-governmental higher educational institute challenging because students will be less interested in taking admission if there is no job [80] available after completing the degree with a high amount of fees. To sustain in this critical situation, institutions need to collaborate on live projects conducted by industry experts and enhance industrial engagements. Employability [92] of students actively boosts the reputation of the institutions as well as the student admissions.

4. Preliminaries of Mathematical Tools

This section discussed the preliminaries of mathematical tools in detail. In the crisp set theory, there are no uncertainties, only certainties. Either any element fully belongs to the set or does not belong to the set, but there are no intermediate states. To solve this problem, Lotfi A. Zadeh [18] introduced the concept of a fuzzy set in 1965.

4.1 Fuzzy set and fuzzy number

This section describes the basic concepts of fuzzy sets and their extensions.

Definition 1. ***Fuzzy set*** *[93]*

Consider $\mathscr{X}$ to be a universal set of discourse. A fuzzy set $(\tilde{\mathcal{A}})$ defined on $\mathscr{X}$ and describes as

$$\tilde{\mathcal{A}} = \{(\eta; \mu_{\tilde{\mathcal{A}}}(\eta)) : \eta \in \mathscr{X}\} \tag{1}$$

where $\mu_{\tilde{\mathcal{A}}}(\eta) : \mathscr{X} \to [0,1]$ be the membership function of $\tilde{\mathcal{A}}$ with $\eta \in \mathscr{X}$.

Remark 1. *The membership function $(\mu_{\tilde{\mathcal{A}}}(\eta))$ of a fuzzy set $(\mathcal{A})$ represents the belongness of the element (η) in the fuzzy set $(\mathcal{A})$. It is bounded by the $[0,1]$ for all elements $(\eta \in \mathscr{X})$ in the set of the universe. If two elements ξ and ζ are elements in the fuzzy set $(\mathcal{A})$ with membership values 0.75 and 0.50, respectively, then the element ξ belongs more as compare to the element ζ in the fuzzy set $(\mathcal{A})$.*

Example 1. *Consider $\mathscr{Y} = \{1, 2, \ldots, 10\}$ be a universal set and a fuzzy set $\tilde{\mathcal{B}}$ defined on it. Then $\tilde{\mathcal{B}}$ represents as*

$$\begin{aligned}\tilde{\mathcal{B}} =& \{(x, \mu_{\tilde{\mathcal{B}}}(x)) : x \in \mathscr{Y}\} \\ =& \{(x, |\sin(x)|) : x \in \mathscr{Y}\}\end{aligned} \tag{2}$$

It is easy to see that the membership value of the element x is $|\sin(x)|$, which is always in $[0,1]$.

Remark 2. ***Degree of hesitancy***

Let $\tilde{\mathcal{A}}$ be a fuzzy set on $\mathscr{X}$, define in Definition 1, with its membership function $(\mu_{\tilde{\mathcal{A}}}(\eta))$. Then the degree of hesitancy $(\pi_{\tilde{\mathcal{A}}}(\eta))$ of the element η on the fuzzy set $\tilde{\mathcal{A}}$ is

$$\pi_{\tilde{\mathcal{A}}}(\eta) = 1 - \mu_{\tilde{\mathcal{A}}}(\eta) \tag{3}$$

Since, $\mu_{\tilde{\mathcal{A}}}(\eta)$ always in $[0,1]$ for all $\eta \in \mathscr{X}$, then $\pi_{\tilde{\mathcal{A}}}(\eta) \in [0,1]$ for all η.

The degree of hesitancy of the fuzzy set $\tilde{\mathcal{B}}$ in Example 1 is $\pi_{\tilde{\mathcal{B}}}(x) = 1 - |\sin(x)|$, where $x \in \mathscr{Y}$.

Definition 2. ***Fuzzy number*** *[93]*

Assume the set of real numbers $(\mathbb{R})$ is the universal set of discourse. Consider $\tilde{\mathcal{B}}$ be a fuzzy set defined on $\mathbb{R}$, is called a fuzzy number, if it satisfies the following conditions:

1. *$\tilde{\mathcal{B}}$ be a normal fuzzy set, i.e., $\exists\, \psi \in \mathbb{R}$ such that $\mu_{\tilde{\mathcal{B}}}(\psi) = 1$.*
2. *$\tilde{\mathcal{B}}$ be a convex fuzzy set, i.e., $\mu_{\tilde{\mathcal{B}}}(\lambda\psi + (1-\lambda)\chi) \geq \min\{\mu_{\tilde{\mathcal{B}}}(\psi), \mu_{\tilde{\mathcal{B}}}(\chi)\}$, for all $\psi, \chi \in \mathbb{R}$ and $\lambda \in [0,1]$.*
3. *The membership function is piecewise continuous, i.e., if $\mathbb{R} = \mathscr{I}_1 \cup \mathscr{I}_2 \cup \mathscr{I}_3 \cup \mathscr{I}_4 \cup \cdots \cup \mathscr{I}_n \cup \ldots$ are disjoints intervals, then $\mu_{\tilde{\mathcal{B}}}(\psi)$ is continuous on every $\mathscr{I}_n$ for $n \in \mathbb{N}$.*

4. *Support of $\tilde{\mathcal{B}}$ is bounded, i.e., $support\left(\tilde{\mathcal{B}}\right) = \{\psi : \mu_{\tilde{\mathcal{B}}}(\psi) > 0 \;\&\; \psi \in \mathbb{R}\}$ is bounded.*

Example 2. *Consider the set of real numbers $(\mathbb{R})$ be the universal set and a fuzzy number $\tilde{\mathcal{C}}$ defined on it. The fuzzy number $\tilde{\mathcal{C}}$ represents as*

$$\tilde{\mathcal{C}} = \{(x, |x - [x]|) : x \in \mathbb{R}\}. \tag{4}$$

Here, the membership function $(\mu_{\tilde{\mathcal{C}}})$ is $|x - [x]|$, where $[x]$ be the greatest integer function of x.

4.2 Intuitionistic fuzzy set and its extension

This section describes the intuitionistic fuzzy set (IFS) and its extension in detail. There are several extensions on fuzzy sets; however, in this study, we consider another membership function of the elements, namely the non-membership function.

Definition 3. ***Intuitionistic fuzzy set*** *[94]*

Assume $\mathscr{Y}$ to be a universal set of discourse. An intuitionistic fuzzy set $(\tilde{\mathcal{C}})$ defined on $\mathscr{Y}$ and describes as

$$\tilde{\mathcal{C}} = \{(\eta; \mu_{\tilde{\mathcal{C}}}(\eta), \nu_{\tilde{\mathcal{C}}}(\eta)) : \eta \in \mathscr{Y}\} \tag{5}$$

where $\mu_{\tilde{\mathcal{C}}}(\eta) : \mathscr{Y} \to [0,1]$ be the membership function and $\nu_{\tilde{\mathcal{C}}}(\eta) : \mathscr{Y} \to [0,1]$ be the non-membership function of $\tilde{\mathcal{C}}$ and satisfies $0 \leq \mu_{\tilde{\mathcal{C}}}(\eta) + \nu_{\tilde{\mathcal{C}}}(\eta) \leq 1$ for all $\eta \in \mathscr{Y}$.

Remark 3. ***Degree of hesitancy***

Let $\tilde{\mathcal{C}}$ be an intuitionistic fuzzy set on $\mathscr{Y}$, define in Definition 3, with its membership function $(\mu_{\tilde{\mathcal{C}}}(\eta))$ and non-membership function $(\nu_{\tilde{\mathcal{C}}}(\eta))$. Then the degree of hesitancy $(\pi_{\tilde{\mathcal{C}}}(\eta))$ of the element η on the intuitionistic fuzzy set $(\tilde{\mathcal{C}})$ is

$$\pi_{\tilde{\mathcal{C}}}(\eta) = 1 - \mu_{\tilde{\mathcal{C}}}(\eta) - \nu_{\tilde{\mathcal{C}}}(\eta) \tag{6}$$

Since, $\mu_{\tilde{\mathcal{C}}}(\eta)$ and $\nu_{\tilde{\mathcal{C}}}(\eta)$ always in $[0,1]$ for all $\eta \in \mathscr{Y}$, then $\pi_{\tilde{\mathcal{C}}}(\eta) \in [0,1]$ for all η.

Example 3. *Let $\mathscr{X} = \{1,2,3,4,5\}$ be a universal set of discourse and an IFS $(\tilde{\mathcal{C}})$ defined on it. The $\tilde{\mathcal{D}}$ represent as*

$$\tilde{\mathcal{D}} = \{(1; 0.5, 0.2), (2; 0.6, 0.3), (3; 0.75, 0.15), (4; 0.8, 0.1), (5; 0.6, 0.2)\} \tag{7}$$

In IFS $\tilde{\mathcal{D}}$, the membership and non-membership values of 1 is 0.5 and 0.2, respectively and so on.

Definition 4. ***Pythagorean fuzzy set*** *[47]*

Assume $\mathscr{Y}$ to be a universal set of discourse. A Pythagorean fuzzy set $(\tilde{\mathcal{D}})$ defined on $\mathscr{Y}$ and describes as

$$\tilde{\mathcal{D}} = \{(\eta; \mu_{\tilde{\mathcal{D}}}(\eta), \nu_{\tilde{\mathcal{D}}}(\eta)) : \eta \in \mathscr{Y}\} \tag{8}$$

where $\mu_{\tilde{\mathcal{D}}}(\eta) : \mathscr{Y} \to [0,1]$ be the membership function and $\nu_{\tilde{\mathcal{D}}}(\eta) : \mathscr{Y} \to [0,1]$ be the non-membership function of $\tilde{\mathcal{D}}$ and satisfies $0 \leq \mu^2_{\tilde{\mathcal{D}}}(\eta) + \nu^2_{\tilde{\mathcal{D}}}(\eta) \leq 1$ for all $\eta \in \mathscr{Y}$.

Remark 4. ***Degree of hesitancy***

Let $\tilde{\mathcal{D}}$ be a Pythagorean fuzzy set on $\mathscr{Y}$, define in Definition 4, with its membership function $(\mu_{\tilde{\mathcal{D}}}(\eta))$ and non-membership function $(\nu_{\tilde{\mathcal{D}}}(\eta))$. Then the degree of hesitancy $(\pi_{\tilde{\mathcal{D}}}(\eta))$ of the element η on the Pythagorean fuzzy set $(\tilde{\mathcal{D}})$ is

$$\pi_{\tilde{\mathcal{D}}}(\eta) = \sqrt{1 - \mu^2_{\tilde{\mathcal{D}}}(\eta) - \nu^2_{\tilde{\mathcal{D}}}(\eta)} \tag{9}$$

Since, $\mu_{\tilde{\mathcal{D}}}(\eta)$ and $\nu_{\tilde{\mathcal{D}}}(\eta)$ always in $[0,1]$ for all $\eta \in \mathscr{Y}$, then $\pi_{\tilde{\mathcal{D}}}(\eta) \in [0,1]$ for all η.

Definition 5. ***Fermatean fuzzy set*** *[95]*

Assume $\mathscr{Y}$ to be a universal set of discourse. A Fermatean fuzzy set $(\tilde{\mathcal{E}})$ defined on $\mathscr{Y}$ and describes as

$$\tilde{\mathcal{E}} = \{(\eta; \mu_{\tilde{\mathcal{E}}}(\eta), \nu_{\tilde{\mathcal{E}}}(\eta)) : \eta \in \mathscr{Y}\} \tag{10}$$

where $\mu_{\tilde{\mathcal{E}}}(\eta) : \mathscr{Y} \to [0,1]$ be the membership function and $\nu_{\tilde{\mathcal{E}}}(\eta) : \mathscr{Y} \to [0,1]$ be the non-membership function of $\tilde{\mathcal{E}}$ and satisfies $0 \leq \mu^3_{\tilde{\mathcal{E}}}(\eta) + \nu^3_{\tilde{\mathcal{E}}}(\eta) \leq 1$ for all $\eta \in \mathscr{Y}$.

Remark 5. ***Degree of hesitancy***

Let $\tilde{\mathcal{E}}$ be a Fermatean fuzzy set on $\mathscr{Y}$, define in Definition 5, with its membership function $(\mu_{\tilde{\mathcal{E}}}(\eta))$ and non-membership function $(\nu_{\tilde{\mathcal{E}}}(\eta))$. Then the degree of hesitancy $(\pi_{\tilde{\mathcal{E}}}(\eta))$ of the element η on the Fermatean fuzzy set $(\tilde{\mathcal{E}})$ is

$$\pi_{\tilde{\mathcal{E}}}(\eta) = \sqrt[3]{1 - \mu^3_{\tilde{\mathcal{E}}}(\eta) - \nu^3_{\tilde{\mathcal{E}}}(\eta)} \tag{11}$$

Since, $\mu_{\tilde{\mathcal{E}}}(\eta)$ and $\nu_{\tilde{\mathcal{E}}}(\eta)$ always in $[0,1]$ for all $\eta \in \mathscr{Y}$, then $\pi_{\tilde{\mathcal{E}}}(\eta) \in [0,1]$ for all η.

4.3 q-rung orthopair fuzzy set

This section describes the q-rung orthopair fuzzy set (q-ROFSs) and its extension. Further, the operations on q-ROFSs and q-rung orthopair fuzzy numbers (q-ROFNs) are defined in detail. The q-ROFSs can be defined as follows:

Definition 6. ***q-rung orthopair fuzzy set*** *[5]*

Consider $\mathscr{X}$ to be a universal set and a q-rung orthopair fuzzy set $\tilde{\mathcal{F}}$ defined on it. Then the q-rung orthopair fuzzy set $\tilde{\mathcal{F}}$ represents as

$$\tilde{\mathcal{F}} = \{(\zeta; \mu_{\tilde{\mathcal{F}}}(\zeta), \nu_{\tilde{\mathcal{F}}}(\zeta)) : \zeta \in \mathscr{X}\} \tag{12}$$

where $\mu_{\tilde{\mathcal{F}}}(\zeta) : \mathscr{X} \to [0,1]$ be the membership function and $\nu_{\tilde{\mathcal{F}}}(\zeta) : \mathscr{X} \to [0,1]$ be the non-membership function of $\tilde{\mathcal{F}}$ and satisfies the property $0 \leq \mu^q_{\tilde{\mathcal{F}}}(\zeta) + \nu^q_{\tilde{\mathcal{F}}}(\zeta) \leq 1$ with $q \in \mathbb{N}$, for all $\zeta \in \mathscr{X}$.

Remark 6. ***Degree of hesitancy***

Let $\tilde{\mathcal{F}}$ be a q-rung orthopair fuzzy set on $\mathscr{X}$, define in Definition 6, with its membership function $(\mu_{\tilde{\mathcal{F}}}(\zeta))$ and non-membership function $(\nu_{\tilde{\mathcal{F}}}(\zeta))$. Then the degree of hesitancy $(\pi_{\tilde{\mathcal{F}}}(\zeta))$ of the element ζ on the q-rung orthopair fuzzy set $(\tilde{\mathcal{F}})$ is

$$\pi_{\tilde{\mathcal{F}}}(\zeta) = \sqrt[q]{1 - \mu^q_{\tilde{\mathcal{F}}}(\zeta) - \nu^q_{\tilde{\mathcal{F}}}(\zeta)} \tag{13}$$

where $q \in \mathbb{N}$.

Since, $\mu_{\tilde{\mathcal{F}}}(\zeta)$ and $\nu_{\tilde{\mathcal{F}}}(\zeta)$ always in $[0,1]$ for all $\eta \in \mathscr{X}$, then $\pi_{\tilde{\mathcal{F}}}(\zeta) \in [0,1]$ for all ζ.

Example 4. *Consider $\mathscr{X} = \{1,3,5,7,9\}$ to be a universal set of discourse and a q-rung orthopair fuzzy set $(\tilde{\mathcal{F}})$ defined on it. Then the $\tilde{\mathcal{F}}$ represents as*

$$\tilde{\mathcal{F}} = \{(1; 0.8, 0.3), (3; 0.7, 0.5), (5; 0.75, 0.45), (7; 0.9, 0.3), (9; 0.55, 0.35)\} \tag{14}$$

Here, the membership $(\mu_{\tilde{\mathcal{F}}})$ and non-membership $(\nu_{\tilde{\mathcal{F}}})$ values of the element 1 are 0.8 and 0.3, respectively and so on. The degree of hesitancy of the element 1 on the set $\tilde{\mathcal{F}}$ is $\pi_{\tilde{\mathcal{F}}}(1) = \sqrt[q]{1 - 0.8^q - 0.3^q}$ and if $q = 5$, then $\pi_{\tilde{\mathcal{F}}}(1) = \sqrt[5]{1 - 0.8^5 - 0.3^5} = 0.2.$

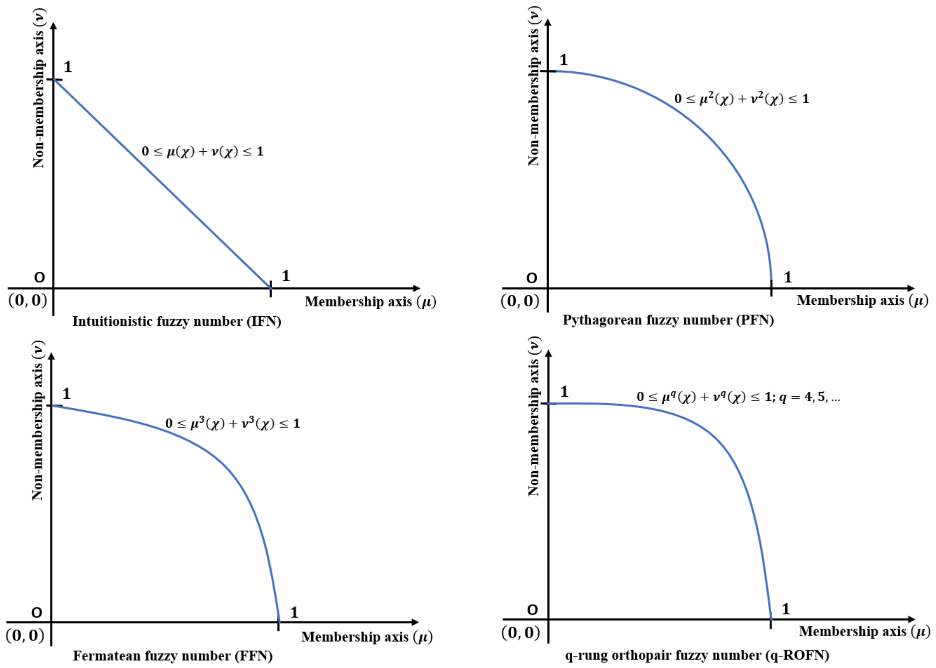

Fig. 2. Relation between membership (μ) and non-membership (ν) values of different fuzzy numbers

Table 3 represents the basic difference between IFS, PFS, FFS and q-ROFS from different views. Comparison between membership value (μ) and non-membership value (ν) of different fuzzy numbers are represents in Figure 2.

Table 3
Comparison between IFS, PFS, FFS and q-ROFS

Parameters	IFS [c]	PFS [c]	FFS [c]	q-ROFS [c]
Boundary condition	$\mu + \nu \le 1$	$\mu^2 + \nu^2 \le 1$	$\mu^3 + \nu^3 \le 1$	$\mu^q + \nu^q \le 1$
Exponent parameter (q)	$q = 1$	$q = 2$	$q = 3$	$q \in \mathbb{N}$
Degree of hesitancy (π)	$1 - \mu - \nu$	$\sqrt{1 - \mu^2 - \nu^2}$	$\sqrt[3]{1 - \mu^3 - \nu^3}$	$\sqrt[q]{1 - \mu^q - \nu^q}$
Geometric interpretation	Triangular region	Circular region	Cubic region	Generalized region (Approaches a full square as $q \to \infty$)
Flexibility of dataset	Lowest, highly restricted	Moderate, handles larger variations	High, handles highly asymmetrical data	Highest, dynamically adjusts to fit any boundary

Properties 1. ***Set operations on q-ROFSs:***

Assume $\mathscr{X}$ is a universal set and $\tilde{\mathcal{A}} = \{(\xi; \mu_{\tilde{\mathcal{A}}}(\xi), \nu_{\tilde{\mathcal{A}}}(\xi)) : \xi \in \mathscr{X}\}$ and $\tilde{\mathcal{B}} = \{(\zeta; \mu_{\tilde{\mathcal{B}}}(\zeta), \nu_{\tilde{\mathcal{B}}}(\zeta)) : \zeta \in \mathscr{X}\}$ are two q-ROFSs defined on it. Then the set operations on $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ are defined as follows:

1. *Union of two q-ROFSs:*

$$\begin{aligned}\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}} &= \{(\xi; \mu_{\tilde{\mathcal{A}}}(\xi), \nu_{\tilde{\mathcal{A}}}(\xi)) : \xi \in \mathscr{X}\} \cup \{(\zeta; \mu_{\tilde{\mathcal{B}}}(\zeta), \nu_{\tilde{\mathcal{B}}}(\zeta)) : \zeta \in \mathscr{X}\} \\ &= \{(\max\{\xi, \zeta\}; \max\{\mu_{\tilde{\mathcal{A}}}(\xi), \mu_{\tilde{\mathcal{B}}}(\zeta)\}, \min\{\mu_{\tilde{\mathcal{A}}}(\xi), \nu_{\tilde{\mathcal{B}}}(\zeta)\}) : \xi, \zeta \in \mathscr{X}\}\end{aligned} \tag{15}$$

2. *Union of two q-ROFSs:*

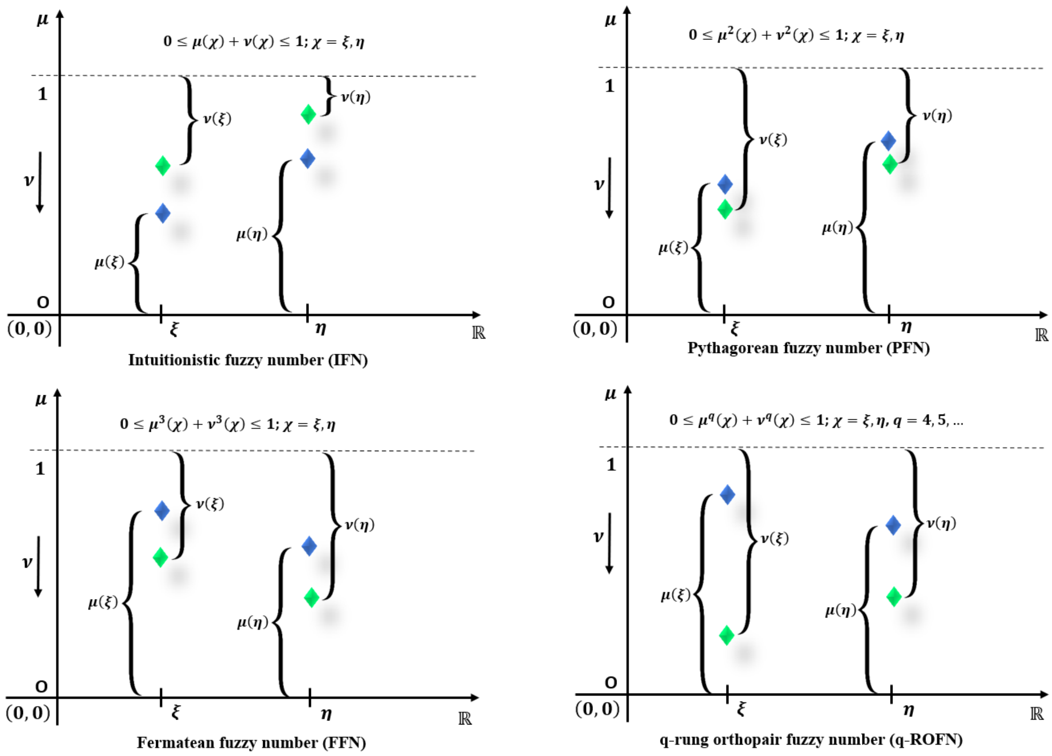

Fig. 3. Comparison among different fuzzy numbers

$$\begin{aligned}\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}} &= \{(\xi; \mu_{\tilde{\mathcal{A}}}(\xi), \nu_{\tilde{\mathcal{A}}}(\xi)) : \xi \in \mathscr{X}\} \cap \{(\zeta; \mu_{\tilde{\mathcal{B}}}(\zeta), \nu_{\tilde{\mathcal{B}}}(\zeta)) : \zeta \in \mathscr{X}\} \\ &= \{(\min\{\xi, \zeta\}; \min\{\mu_{\tilde{\mathcal{A}}}(\xi), \mu_{\tilde{\mathcal{B}}}(\zeta)\}, \max\{\mu_{\tilde{\mathcal{A}}}(\xi), \nu_{\tilde{\mathcal{B}}}(\zeta)\}) : \xi, \zeta \in \mathscr{X}\}\end{aligned} \tag{16}$$

3. *Compliment of q-ROFS:*

$$\begin{aligned}\tilde{\mathcal{A}}^C = \left(\tilde{\mathcal{A}}\right)^C &= \{(\xi; \mu_{\tilde{\mathcal{A}}}(\xi), \nu_{\tilde{\mathcal{A}}}(\xi)) : \xi \in \mathscr{X}\}^C \\ &= \{(\xi; \nu_{\tilde{\mathcal{A}}}(\xi), \mu_{\tilde{\mathcal{A}}}(\xi)) : \xi \in \mathscr{X}\}\end{aligned} \tag{17}$$

Theorem 1. *Consider $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ are two q-ROFSs, then the following operations are satisfied as*

1. $\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}} = \tilde{\mathcal{B}} \cup \tilde{\mathcal{A}}$,
2. $\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}} = \tilde{\mathcal{B}} \cap \tilde{\mathcal{A}}$,
3. $\left(\tilde{\mathcal{A}}^C\right)^C = \tilde{\mathcal{A}}$.

Definition 7. ***q-rung orthopair fuzzy number***

Consider the set of real numbers $(\mathbb{R})$ to be a universal set and a q-rung orthopair fuzzy set $\tilde{\mathcal{F}}$ defined on it. Then the q-rung orthopair fuzzy set $\tilde{\mathcal{F}}$ is called a q-rung orthopair fuzzy number if its membership and non-membership functions satisfy all the fuzzy numbers' properties (defined in Definition 2).

Graphical representation of the intuitionistic fuzzy numbers (IFNs), Pythagorean fuzzy numbers (PFNs), Fermatean fuzzy numbers (FFNs) and q-roung orthpair fuzzy numbers (q-ROFNs) are shown in Figure 3. The relation between membership value (μ) and non-membership value (ν) of different fuzzy numbers are shown here.

Remark 7. ***Reason for considering q-ROFNs as uncertainty tool***
The q-ROFNs are considered as uncertainty tools for this study, for their capability to capture uncertainty is very high and they are flexible for adoption in vague and unbiased space. The structural differences of other intuitionistic fuzzy numbers are discussed in detail in Table 3. In q-ROFNs, the value of q can be any natural number $(\mathbb{N})$ and it is utilized on the basis of the problem's requirements.

Properties 2. *Consider $\tilde{\mathcal{C}} = \{(\eta; \mu_{\tilde{\mathcal{C}}}(\eta), \nu_{\tilde{\mathcal{C}}}(\eta)) : \eta \in \mathbb{R}\}$ and $\tilde{\mathcal{D}} = \{(\zeta; \mu_{\tilde{\mathcal{D}}}(\zeta), \nu_{\tilde{\mathcal{D}}}(\zeta)) : \zeta \in \mathbb{R}\}$ are two q-ROFNs define on the set of real numbers $(\mathbb{R})$ and λ and γ be two scalars. Then the arithmetic operations on q-ROFNs are defined as*

A. *Addition of two q-ROFNs:*

$$\begin{aligned}\tilde{\mathcal{C}} \oplus \tilde{\mathcal{D}} &= \{(\eta; \mu_{\tilde{\mathcal{C}}}(\eta), \nu_{\tilde{\mathcal{C}}}(\eta)) : \eta \in \mathbb{R}\} \oplus \{(\zeta; \mu_{\tilde{\mathcal{D}}}(\zeta), \nu_{\tilde{\mathcal{D}}}(\zeta)) : \zeta \in \mathbb{R}\} \\ &= \left\{ \left(\eta + \zeta; \sqrt[q]{\mu^q_{\tilde{\mathcal{C}}}(\eta) + \mu^q_{\tilde{\mathcal{D}}}(\zeta) - \mu^q_{\tilde{\mathcal{C}}}(\eta)\mu^q_{\tilde{\mathcal{D}}}(\zeta)}, \nu_{\tilde{\mathcal{C}}}(\eta)\nu_{\tilde{\mathcal{D}}}(\zeta) \right) : \eta, \zeta \in \mathbb{R} \right\}\end{aligned} \tag{18}$$

B. *Scalar multiplication of q-ROFN:*

$$\begin{aligned}\lambda \times \tilde{\mathcal{C}} =& \lambda \times \{(\eta; \mu_{\tilde{\mathcal{C}}}(\eta), \nu_{\tilde{\mathcal{C}}}(\eta)) : \eta \in \mathbb{R}\} \\ =& \left\{ \left(\lambda \times \eta; \sqrt{1 - (1 - \mu_{\tilde{\mathcal{C}}}(\eta))^{\lambda}}, \nu_{\tilde{\mathcal{C}}}(\eta)^{\lambda} \right) : \eta \in \mathbb{R} \right\}\end{aligned} \tag{19}$$

where λ be a non-negative scalar.

C. *Multiplication of two q-ROFNs:*

$$\begin{aligned}\tilde{\mathcal{C}} \otimes \tilde{\mathcal{D}} &= \{(\eta; \mu_{\tilde{\mathcal{C}}}(\eta), \nu_{\tilde{\mathcal{C}}}(\eta)) : \eta \in \mathbb{R}\} \otimes \{(\zeta; \mu_{\tilde{\mathcal{D}}}(\zeta), \nu_{\tilde{\mathcal{D}}}(\zeta)) : \zeta \in \mathbb{R}\} \\ &= \left\{ \left(\eta\zeta; \mu_{\tilde{\mathcal{C}}}(\eta)\mu_{\tilde{\mathcal{D}}}(\zeta), \sqrt[q]{\nu^q_{\tilde{\mathcal{C}}}(\eta) + \nu^q_{\tilde{\mathcal{D}}}(\zeta) - \nu^q_{\tilde{\mathcal{C}}}(\eta)\nu^q_{\tilde{\mathcal{D}}}(\zeta)} \right) : \eta, \zeta \in \mathbb{R} \right\}\end{aligned} \tag{20}$$

D. *Scalar power of q-ROFN:*

$$\begin{aligned}\tilde{\mathcal{D}}^{\gamma} = \left(\tilde{\mathcal{D}}\right)^{\gamma} &= \{(\zeta; \mu_{\tilde{\mathcal{D}}}(\zeta), \nu_{\tilde{\mathcal{D}}}(\zeta)) : \zeta \in \mathbb{R}\}^{\gamma} \\ &= \left\{ \left(\zeta^{\gamma}; \mu_{\tilde{\mathcal{D}}}(\zeta)^{\gamma}, \sqrt{1 - (1 - \nu_{\tilde{\mathcal{D}}}(\zeta))^{\gamma}} \right) : \zeta \in \mathbb{R} \right\}\end{aligned} \tag{21}$$

where γ is an arbitrary positive integer.

Theorem 2. *Assume $\tilde{\mathcal{C}}$ & $\tilde{\mathcal{D}}$ are two q-ROFNs and λ & γ are two non-negative scalars, then the following operations are satisfied as*

1. $\tilde{\mathcal{C}} \oplus \tilde{\mathcal{D}} = \tilde{\mathcal{D}} \oplus \tilde{\mathcal{C}}$,
2. $\lambda \times \left(\tilde{\mathcal{C}} \oplus \tilde{\mathcal{D}}\right) = \left(\lambda \times \tilde{\mathcal{C}}\right) \oplus \left(\lambda \times \tilde{\mathcal{D}}\right)$,
3. $(\lambda + \gamma) \times \tilde{\mathcal{C}} = \left(\lambda \times \tilde{\mathcal{C}}\right) \oplus \left(\gamma \times \tilde{\mathcal{C}}\right)$,
4. $\tilde{\mathcal{C}} \otimes \tilde{\mathcal{D}} = \tilde{\mathcal{D}} \otimes \tilde{\mathcal{C}}$,
5. $\lambda \times \left(\tilde{\mathcal{C}} \otimes \tilde{\mathcal{D}}\right) = \left(\lambda \times \tilde{\mathcal{D}}\right) \otimes \tilde{\mathcal{C}} = \tilde{\mathcal{D}} \otimes \left(\lambda \times \tilde{\mathcal{C}}\right)$,

6. $\left(\tilde{\mathcal{C}} \otimes \tilde{\mathcal{D}}\right)^{\gamma} = \left(\tilde{\mathcal{C}}\right)^{\gamma} \otimes \left(\tilde{\mathcal{D}}\right)^{\gamma}$,

7. $\left(\tilde{\mathcal{D}}\right)^{\lambda+\gamma} = \left(\tilde{\mathcal{D}}\right)^{\lambda} \otimes \left(\tilde{\mathcal{D}}\right)^{\gamma}$.

4.4 Score and accuracy functions on q-ROFNs

This section discusses the score and accuracy functions [93] on the q-ROFNs. Since there are no ordered relations on the fuzzy numbers, we are unable to distinguish which is larger. To prioritize or define the order of a fuzzy number, many methods are considered. Like the de-fuzzification method [95], de-i-fuzzification method [96], score function [55] and accuracy function [93] are considered to define the order relation on fuzzy sets. In this study, we consider score and accuracy functions on q-ROFNs. There are several studies conducted on q-ROFNs, such as Ali, J. [67] applied on the MARCOS method in the waste management sectors, Xiao, L. et al. [30] utlized in the best-worst method for manufacturer departments and Mishra, A. R. et al. [55] applied in group decision-making methods in site selection problems. The proposed score and accuracy functions on q-ROFNs are defined as follows:

Definition 8. ***Proposed score function on q-ROFN:*** *[67]*
Assume $\tilde{\mathcal{E}} = \{(\chi; \mu_{\tilde{\mathcal{E}}}(\chi), \nu_{\tilde{\mathcal{E}}}(\chi)) : \chi \in \mathbb{R}\}$ is a q-ROFN and the proposed score function is defined as follows:

$$\mathfrak{S}\left(\tilde{\mathcal{E}}\right) = \chi\left(1 + \mu^q_{\tilde{\mathcal{E}}}(\chi) - \nu^q_{\tilde{\mathcal{E}}}(\chi) + \frac{\pi^q_{\tilde{\mathcal{E}}}(\chi)}{2}\right) \tag{22}$$

where $\pi_{\tilde{\mathcal{E}}}(\chi)$ be the degree of hesitancy of q-ROFN define in Equation (13).

Definition 9. ***Proposed accuracy function on q-ROFN:*** *[93]*
Assume $\tilde{\mathcal{E}} = \{(\chi; \mu_{\tilde{\mathcal{E}}}(\chi), \nu_{\tilde{\mathcal{E}}}(\chi)) : \chi \in \mathbb{R}\}$ is a q-ROFN and the proposed accuracy function is defined as follows:

$$\mathfrak{A}\left(\tilde{\mathcal{E}}\right) = \chi\left(1 + \mu^q_{\tilde{\mathcal{E}}}(\chi) + \nu^q_{\tilde{\mathcal{E}}}(\chi)\right) \tag{23}$$

Remark 8. *For numerical computation, we consider $q = 5$ for the degree of uncertainty. Further, computations are evaluated based on the initial consideration $q = 5$. Anyone can consider another value of q as the degree of uncertainty.*

Example 5. *From Example 4, $\tilde{\mathcal{F}}$ be a q-QOFNs on $\mathscr{X}$. Then the score and accuracy values of q-QOFNs are*

$$\begin{cases} \mathfrak{S}\left(\tilde{\mathcal{F}}\right) & = \mathfrak{S}\left(\{(1; 0.8, 0.3), (3; 0.7, 0.5), (5; 0.75, 0.45), (7; 0.9, 0.3), (9; 0.55, 0.35)\}\right) \\ & = \{1.660, 4.611, 7.955, 12.541, 13.656\} \\ \mathfrak{A}\left(\tilde{\mathcal{F}}\right) & = \mathfrak{A}\left(\{(1; 0.8, 0.3), (3; 0.7, 0.5), (5; 0.75, 0.45), (7; 0.9, 0.3), (9; 0.55, 0.35)\}\right) \\ & = \{1.330, 3.598, 6.279, 11.150, 9.500\} \end{cases} \tag{24}$$

To prioritize between two q-ROFNs, we first apply the score function to them; if they fail to distinguish, we then apply the accuracy function. Theorem 3 states in a logical way:

Theorem 3. *Consider $\tilde{\mathcal{C}}$ and $\tilde{\mathcal{D}}$ are two q-ROFNs defined on the set of real numbers $(\mathbb{R})$ and the score and accuracy functions defined on it. Then*

1. *if* $\mathfrak{S}\left(\tilde{\mathcal{C}}\right) < \mathfrak{S}\left(\tilde{\mathcal{D}}\right)$ *then* $\tilde{\mathcal{C}} < \tilde{\mathcal{D}}$,
2. *if* $\mathfrak{S}\left(\tilde{\mathcal{C}}\right) > \mathfrak{S}\left(\tilde{\mathcal{D}}\right)$ *then* $\tilde{\mathcal{C}} > \tilde{\mathcal{D}}$,
3. *if* $\mathfrak{S}\left(\tilde{\mathcal{C}}\right) = \mathfrak{S}\left(\tilde{\mathcal{D}}\right)$ *then*
   (a) *if* $\mathfrak{A}\left(\tilde{\mathcal{C}}\right) < \mathfrak{A}\left(\tilde{\mathcal{D}}\right)$ *then* $\tilde{\mathcal{C}} < \tilde{\mathcal{D}}$,
   (b) *if* $\mathfrak{A}\left(\tilde{\mathcal{C}}\right) > \mathfrak{A}\left(\tilde{\mathcal{D}}\right)$ *then* $\tilde{\mathcal{C}} > \tilde{\mathcal{D}}$,
   (c) *if* $\mathfrak{A}\left(\tilde{\mathcal{C}}\right) = \mathfrak{A}\left(\tilde{\mathcal{D}}\right)$ *then* $\tilde{\mathcal{C}} \simeq \tilde{\mathcal{D}}$.

5. Proposed Optimization Methodology

Multi-Criteria Decision Making (MCDM) [50] is a very effective technique for tackling complex decision-making problems. Further, these MCDM methods are mainly categorized into two groups, namely the criteria weight evaluation method [38] and the ranking alternatives method [45]. There are several methods utlized for these computations. However, in this study, we consider the two most popular MCDM methods for numerical evaluation. Here, we have taken the help of the MEthod based on the Removal Effects of Criteria (MEREC) method [52] for evaluating the criteria weight. After that, the Measurement of Alternatives and Ranking according to the Compromise Solution (MARCOS) method [71] is used to find the ranking of alternatives. The said methods are discussed in the subsections below.

5.1 *MEREC based weighted method*

MEthod based on the Removal Effects of Criteria (MEREC) method is an advanced weighted-based MCDM method which was introduced by Ghorabaee, M. K. et al. [97] in 2021. This method takes a slightly different approach from other weighted methods. Here, if a criterion is omitted, then its impact on the performance of alternatives will define the importance of that criterion. The opinions of DMs are essential to evaluate the weight of the criteria.

In this paper, we have considered Γ number of criteria as parameters along with Δ number of alternatives for the formation of the decision matrix. Here, data are collected from the Ω number of decision experts in linguistic terms. These linguistic terms are further converted to q-rung orthopair fuzzy numbers (q-ROFNs) using a conversion table. The steps of the MEREC methodology are briefly discussed as follows:

Step I **Formation of decision matrices** $(\tilde{\mathscr{D}}_{\omega})$**:**
Here, using the opinion of decision experts, a total of Ω number of $\Delta \times \Gamma$ order decision matrices are constructed in linguistic terms. The entries of the decision matrices are further converted to q-ROFNs using a conversion table. The decision matrix formulated by ω^{th} decision expert is

denoted by $\tilde{\mathscr{D}}_\omega$ and it is represented as

$$\tilde{\mathscr{D}}_\omega = \left[\left(\tilde{\mathscr{H}}_{\delta\gamma}\right)_\omega\right]_{\Delta\times\Gamma} = \begin{bmatrix} \left(\tilde{\mathscr{H}}_{11}\right)_\omega & \left(\tilde{\mathscr{H}}_{12}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{1\gamma}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{1\Gamma}\right)_\omega \\ \left(\tilde{\mathscr{H}}_{21}\right)_\omega & \left(\tilde{\mathscr{H}}_{22}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{2\gamma}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{2\Gamma}\right)_\omega \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \left(\tilde{\mathscr{H}}_{\delta 1}\right)_\omega & \left(\tilde{\mathscr{H}}_{\delta 2}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{\delta\gamma}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{\delta\Gamma}\right)_\omega \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \left(\tilde{\mathscr{H}}_{\Delta 1}\right)_\omega & \left(\tilde{\mathscr{H}}_{\Delta 2}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{\Delta\gamma}\right)_\omega & \ldots & \left(\tilde{\mathscr{H}}_{\Delta\Gamma}\right)_\omega \end{bmatrix}_{\Delta\times\Gamma} \tag{25}$$

where $\gamma = 1, 2, \ldots, \Gamma$, $\delta = 1, 2, \ldots, \Delta$ and $\omega = 1, 2, \ldots, \Omega$, respectively. Here, each element $(\tilde{\mathscr{H}}_{\delta\gamma})_\omega$ of decision matrices $(\tilde{\mathscr{D}}_\omega)$ are q-ROFNs and can be represented as

$$\begin{aligned} (\tilde{\mathscr{H}}_{\delta\gamma})_\omega &= \left\{\left(\eta_{\delta\gamma}; \mu_{\tilde{\mathscr{H}}_{\delta\gamma}}, \nu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right) : \eta_{\delta\gamma} \in \mathbb{R}\right\}_\omega \\ &= \left\{\left((\eta_{\delta\gamma})_\omega; \left(\mu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)_\omega, \left(\nu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)_\omega\right)\right\} \end{aligned} \tag{26}$$

Step II **Construction of aggregated decision matrix $(\tilde{\mathscr{D}})$:**
In this step, a single aggregated decision matrix $(\tilde{\mathscr{D}})$ is evaluated from Ω number of decision matrices $\tilde{\mathscr{D}}_\omega$ where $\omega = 1, 2, \ldots, \Omega$. The aggregated decision matrix $(\tilde{\mathscr{D}})$ is formed as

$$\tilde{\mathscr{D}} = \left[\tilde{\mathscr{H}}_{\delta\gamma}\right]_{\Delta\times\Gamma} = \begin{bmatrix} \tilde{\mathscr{H}}_{11} & \tilde{\mathscr{H}}_{12} & \ldots & \tilde{\mathscr{H}}_{1\gamma} & \ldots & \tilde{\mathscr{H}}_{1\Gamma} \\ \tilde{\mathscr{H}}_{21} & \tilde{\mathscr{H}}_{22} & \ldots & \tilde{\mathscr{H}}_{2\gamma} & \ldots & \tilde{\mathscr{H}}_{2\Gamma} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathscr{H}}_{\delta 1} & \tilde{\mathscr{H}}_{\delta 2} & \ldots & \tilde{\mathscr{H}}_{\delta\gamma} & \ldots & \tilde{\mathscr{H}}_{\delta\Gamma} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathscr{H}}_{\Delta 1} & \tilde{\mathscr{H}}_{\Delta 2} & \ldots & \tilde{\mathscr{H}}_{\Delta\gamma} & \ldots & \tilde{\mathscr{H}}_{\Delta\Gamma} \end{bmatrix}_{\Delta\times\Gamma} \tag{27}$$

Here, each entry $\tilde{\mathscr{H}}_{\delta\gamma}$ of aggregated decision matrix $(\tilde{\mathscr{D}})$ are q-ROFN and evaluated as

$$\begin{aligned} \tilde{\mathscr{H}}_{\delta\gamma} &= \left\{\left(\eta_{\delta\gamma}; \mu_{\tilde{\mathscr{H}}_{\delta\gamma}}, \nu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right) : \eta_{\delta\gamma} \in \mathbb{R}\right\} \\ &= \left\{\left(\frac{\sum_{\omega=1}^{\Omega} (\eta_{\delta\gamma})_\omega}{\Omega}; \min_{\omega=1,2,\ldots,\Omega}\left\{\left(\mu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)_\omega\right\}, \max_{\omega=1,2,\ldots,\Omega}\left\{\left(\nu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)_\omega\right\}\right)\right\} \end{aligned} \tag{28}$$

where $\gamma = 1, 2, \ldots, \Gamma$ and $\delta = 1, 2, \ldots, \Delta$.

Step III **Formulation of uniform aggregated fuzzy decision matrix $(\tilde{\mathscr{D}}^n)$:**
The uniform aggregated fuzzy decision matrix $(\tilde{\mathscr{D}}^n)$ is obtained through normalizing each entry of the aggregated decision matrix $(\tilde{\mathscr{D}})$. The uniform aggregated fuzzy decision matrix $(\tilde{\mathscr{D}}^n)$ is

calculated by Equation (29), as follows:

$$\begin{aligned}\tilde{\mathscr{D}}^n = \left[\tilde{\mathscr{H}}^n_{\delta\gamma}\right]_{\Delta\times\Gamma} &= \left[\left\{\left(\eta_{\delta\gamma}; \mu_{\tilde{\mathscr{H}}_{\delta\gamma}}, \nu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)\right\}^n\right]_{\Delta\times\Gamma} \\ &= \left[\left\{\left(\eta^n_{\delta\gamma}; \mu^n_{\tilde{\mathscr{H}}_{\delta\gamma}}, \nu^n_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)\right\}\right]_{\Delta\times\Gamma} \\ &= \left[\left\{\left(\frac{\eta_{\delta\gamma}}{\sqrt{\sum_{\delta=1}^{\Delta}\{\eta_{\delta\gamma}\}^2}}; \mu_{\tilde{\mathscr{H}}_{\delta\gamma}}, \nu_{\tilde{\mathscr{H}}_{\delta\gamma}}\right)\right\}\right]_{\Delta\times\Gamma}\end{aligned} \tag{29}$$

where $\gamma = 1, 2, \ldots, \Gamma$ and $\delta = 1, 2, \ldots, \Delta$. Here, $\tilde{\mathscr{H}}^n_{\delta\gamma}$ is the normalized value of the $\tilde{\mathscr{H}}_{\delta\gamma}$, determined by Equation (28).

Step IV Construction of score valued decision matrix $(\mathscr{D})$**:**
The score valued decision matrix $(\mathscr{D})$ is constructed in this step. Here, each entry of the uniform aggregated fuzzy decision matrix $(\tilde{\mathscr{D}}^n)$ are converted to a crisp value with the help of Equation (22). The score valued decision matrix $(\mathscr{D})$ can be formed as

$$\mathscr{D} = [\mathscr{H}_{\delta\gamma}]_{\Delta\times\Gamma} = \left[\mathfrak{S}\left(\tilde{\mathscr{H}}^n_{\delta\gamma}\right)\right]_{\Delta\times\Gamma} \tag{30}$$

where $\mathscr{H}_{\delta\gamma}$ be the score valued of the $\delta\gamma$th entry $\tilde{\mathscr{H}}^n_{\delta\gamma}$, for $\gamma = 1, 2, \ldots, \Gamma$ and $\delta = 1, 2, \ldots, \Delta$.

Step V Evaluating the overall performance $(\mathscr{P}_\delta)$ **of alternative:**
The process of obtaining the overall performance $(\mathscr{P}_\delta)$ of each alternative (δ) is discussed in this step. Here, the overall performance $(\mathscr{P}_\delta)$ of each alternative (δ) is obtained with the help of the following equation

$$\mathscr{P}_\delta = \frac{\sum\limits_{\gamma=1}^{\Gamma} \ln\left(1 - \mathscr{H}_{\delta\gamma}\right)}{\Gamma} \tag{31}$$

where $\mathscr{H}_{\delta\gamma}$ is the $\delta\gamma$th entry of score valued decision matrix $(\mathscr{D})$ and $\delta = 1, 2, \ldots, \Delta$.

Step VI Calculation of the alternatives' performance value $(\mathscr{Q}_{\delta\gamma})$ **by eliminating every criteria:**
The alternatives' performance value $(\mathscr{Q}_{\delta\gamma})$ is evaluated by eliminating every criteria (γ) from the score valued decision matrix $(\mathscr{D})$, by using Equation (32), as follows:

$$\mathscr{Q}_{\delta\gamma} = \frac{\sum\limits_{\gamma=1, \gamma\neq\gamma}^{\Gamma} \ln\left(1 - \mathscr{H}_{\delta\gamma}\right)}{\Gamma} \tag{32}$$

where $\mathscr{H}_{\delta\gamma}$ is the $\delta\gamma$th entry of score valued decision matrix $(\mathscr{D})$ with $\gamma = 1, 2, \ldots, \Gamma$ and $\delta = 1, 2, \ldots, \Delta$.

Step VII Evaluation of the aggregated absolute deviation value $(\mathscr{R}_\gamma)$**:**
The aggregated absolute deviation value $(\mathscr{R}_\gamma)$ of each criterion (γ) can be obtained from the overall performance value $(\mathscr{P}_\delta)$ of each alternative (δ) and the alternatives' performance value $(\mathscr{Q}_{\delta\gamma})$. The aggregated absolute deviation value $(\mathscr{R}_\gamma)$ evaluated by Equation (33), as

$$\mathscr{R}_\gamma = \sum_{\delta=1}^{\Delta} \left|\mathscr{Q}_{\delta\gamma} - \mathscr{P}_\delta\right| \tag{33}$$

where $\gamma = 1, 2, \ldots, \Gamma$.

Step VIII Determining criteria weight $(\mathscr{W}_\gamma)$:
Finally, the weight of each criterion $\mathscr{W}_\gamma$ is obtained from the aggregated absolute deviation value $(\mathscr{R}_\gamma)$ by using Equation (34), as

$$\mathscr{W}_\gamma = \frac{\mathscr{R}_\gamma}{\sum\limits_{\gamma=1}^{\Gamma} \mathscr{R}_\gamma} \tag{34}$$

where $\gamma = 1, 2, \ldots, \Gamma$.

Equation (34) provides the weights of the criteria $(\mathscr{W}_\gamma)$ by the MEREC method under q-ROFN environments. The steps of the MEREC method are graphically represented in the Figure 4.

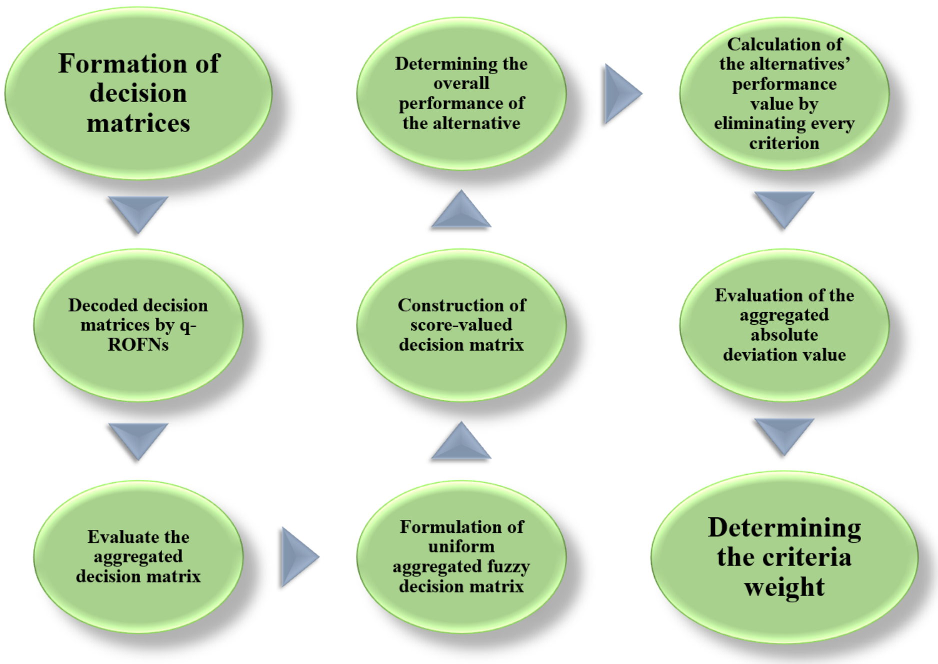

Fig. 4. Structural procedure of the MEREC method

5.2 *MARCOS based ranking method*

Measurement of Alternatives and Ranking according to the Compromise Solution (MARCOS) method is a widely used MCDM method known for its effectiveness in alternatives ranking. The MARCOS method was invented by Stević Željko [98] where both the ideal and the anti-ideal solutions are taken into account. Here, Γ number of criteria along with Δ number of alternatives are considered and the opinions of Ω number of decision experts are considered. The step-by-step detailed description of the MARCOS method is given in the following:

Step A Structure of decision matrix $(\tilde{\mathscr{D}}_\omega)$:
A total of Ω number of $\Delta \times \Gamma$ decision matrices $(\tilde{\mathscr{D}}_\omega)$ are formulated using the opinions of decision experts (ω) in the same way as shown in Step I. of the MEREC method.

Step B Aggregated decision matrix $(\tilde{\mathscr{D}})$ formulation:
All Ω number of decision matrices $(\tilde{\mathscr{D}}_\omega)$ transferred into a single aggregated decision matrix $(\tilde{\mathscr{D}}_\theta)$ with the help of Step II. of the MEREC method.

Step C Formulate extended decision matrix $(\tilde{\mathscr{D}}^{\mathfrak{E}})$**:**

An Ideal solution $(\tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{I}})$ and an Anti-Ideal solution $(\tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}})$ are included for extending the aggregated decision matrix $(\tilde{\mathscr{D}})$. The extended decision matrix $(\tilde{\mathscr{D}}^{\mathfrak{E}})$ represented as

$$\tilde{\mathscr{D}}^{\mathfrak{E}} = \begin{bmatrix} \tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}} \\ \tilde{\mathscr{H}}_{\delta\gamma} \\ \tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{I}} \end{bmatrix}_{(\Delta+2)\times\Gamma} = \begin{bmatrix} \tilde{\mathscr{H}}_{1}^{\mathscr{A}\mathscr{A}\mathscr{I}} & \tilde{\mathscr{H}}_{2}^{\mathscr{A}\mathscr{A}\mathscr{I}} & \dots & \tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}} & \dots & \tilde{\mathscr{H}}_{\Gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}} \\ \tilde{\mathscr{H}}_{11} & \tilde{\mathscr{H}}_{12} & \dots & \tilde{\mathscr{H}}_{1\gamma} & \dots & \tilde{\mathscr{H}}_{1\Gamma} \\ \tilde{\mathscr{H}}_{21} & \tilde{\mathscr{H}}_{22} & \dots & \tilde{\mathscr{H}}_{2\gamma} & \dots & \tilde{\mathscr{H}}_{2\Gamma} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathscr{H}}_{\delta 1} & \tilde{\mathscr{H}}_{\delta 2} & \dots & \tilde{\mathscr{H}}_{\delta\gamma} & \dots & \tilde{\mathscr{H}}_{\delta\Gamma} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathscr{H}}_{\Delta 1} & \tilde{\mathscr{H}}_{\Delta 2} & \dots & \tilde{\mathscr{H}}_{\Delta\gamma} & \dots & \tilde{\mathscr{H}}_{\Delta\Gamma} \\ \tilde{\mathscr{H}}_{1}^{\mathscr{A}\mathscr{I}} & \tilde{\mathscr{H}}_{2}^{\mathscr{A}\mathscr{I}} & \dots & \tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{I}} & \dots & \tilde{\mathscr{H}}_{\Gamma}^{\mathscr{A}\mathscr{I}} \end{bmatrix}_{(\Delta+2)\times\Gamma} \tag{35}$$

where $\gamma = 1, 2, \dots, \Gamma$ and $\delta = 1, 2, \dots, \Delta$.

Here, the ideal solution $(\tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{I}})$ is actually the best alternative among all alternatives and the anti-ideal solution $(\tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}})$ is the worst alternative among all alternatives. These values are evaluated with the help of Equation (36), as follows

$$\begin{cases} \tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{I}} = \begin{cases} \max\limits_{\delta=1,2,\dots,\Delta} \tilde{\mathscr{H}}_{\delta\gamma} \text{ ; if } \gamma \in \mathfrak{B} \\ \min\limits_{\delta=1,2,\dots,\Delta} \tilde{\mathscr{H}}_{\delta\gamma} \text{ ; if } \gamma \in \mathfrak{C} \end{cases} \\ \tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}} = \begin{cases} \min\limits_{\delta=1,2,\dots,\Delta} \tilde{\mathscr{H}}_{\delta\gamma} \text{ ; if } \gamma \in \mathfrak{B} \\ \max\limits_{\delta=1,2,\dots,\Delta} \tilde{\mathscr{H}}_{\delta\gamma} \text{ ; if } \gamma \in \mathfrak{C} \end{cases} \end{cases} \tag{36}$$

where the beneficial group of criteria is denoted as $\mathfrak{B}$ and the non-beneficiary criteria is denoted as $\mathfrak{C}$, respectively wth $\gamma = 1, 2, \dots, \Gamma$.

Step D Construction of score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}})$**:**

The extended decision matrix $(\tilde{\mathscr{D}}^{\mathfrak{E}})$ converted into the score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}})$ with the help of Equation (22). The score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}})$ is represented as

$$\mathscr{D}^{\mathfrak{E}} = \begin{bmatrix} \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}} \\ \mathscr{H}_{\delta\gamma} \\ \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}} \end{bmatrix}_{(\Delta+2)\times\Gamma} = \begin{bmatrix} \mathfrak{S}\left(\tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}}\right) \\ \mathfrak{S}\left(\tilde{\mathscr{H}}_{\delta\gamma}\right) \\ \mathfrak{S}\left(\tilde{\mathscr{H}}_{\gamma}^{\mathscr{A}\mathscr{I}}\right) \end{bmatrix}_{(\Delta+2)\times\Gamma} \tag{37}$$

where $\gamma = 1, 2, \dots, \Gamma$ and $\delta = 1, 2, \dots, \Delta$.

Step E Determine normalized score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}n})$**:**

The normalized score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}n})$ is evaluated from the score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}})$ by normalizing each entry. $\mathscr{D}^{\mathfrak{E}n}$ evaluated by Equation (29), as follows:

$$\mathscr{D}^{\mathfrak{E}n} = \begin{bmatrix} \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}n} \\ \mathscr{H}_{\delta\gamma}^{n} \\ \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}n} \end{bmatrix}_{(\Delta+2)\times\Gamma} = \begin{bmatrix} \begin{cases} \frac{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}}}{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}\text{; if } \gamma \in \mathfrak{B} \\ \frac{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}}}\text{; if } \gamma \in \mathfrak{C} \end{cases} \\ \begin{cases} \frac{\mathscr{H}_{\delta\gamma}}{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}\text{; if } \gamma \in \mathfrak{B} \\ \frac{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}{\mathscr{H}_{\delta\gamma}}\text{; if } \gamma \in \mathfrak{C} \end{cases} \\ \begin{cases} \frac{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}\text{; if } \gamma \in \mathfrak{B} \\ \frac{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}{\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}}}\text{; if } \gamma \in \mathfrak{C} \end{cases} \end{bmatrix}_{(\Delta+2)\times\Gamma} \tag{38}$$

where $\mathfrak{B}$ and $\mathfrak{C}$ be the sets of beneficial criteria and non-beneficial criteria, respectively, with $\gamma = 1, 2, \ldots, \Gamma$ and $\delta = 1, 2, \ldots, \Delta$.

Step F Evaluate weighted normalized decision matrix $(\mathscr{D}^{w})$:
The normalized score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}n})$ is transferred into the weighted normalized decision matrix $(\mathscr{D}^{w})$ with the help of the weight of the criteria $(\mathscr{W}_{\gamma})$ determined in the previous method. The weighted normalized decision matrix $(\mathscr{D}^{w})$ formed as

$$\mathscr{D}^{w} = \begin{bmatrix} \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}w} \\ \mathscr{H}_{\delta\gamma}^{w} \\ \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}w} \end{bmatrix}_{(\Delta+2)\times\Gamma} = \begin{bmatrix} \mathscr{W}_{\gamma} \times \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}n} \\ \mathscr{W}_{\gamma} \times \mathscr{H}_{\delta\gamma}^{n} \\ \mathscr{W}_{\gamma} \times \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}n} \end{bmatrix}_{(\Delta+2)\times\Gamma} \tag{39}$$

where $\gamma = 1, 2, \ldots, \Gamma$ and $\delta = 1, 2, \ldots, \Delta$.

Step F The utility degree evaluation:
The utility degree of alternatives are denoted as $\mathscr{U}_{\delta}^{+}$ and $\mathscr{U}_{\delta}^{-}$. These values are evaluated with the help of the following formula

$$\begin{cases} \mathscr{U}_{\delta}^{+} = \dfrac{\sum_{\gamma=1}^{\Gamma} \mathscr{H}_{\delta\gamma}^{w}}{\sum_{\gamma=1}^{\Gamma} \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}w}} \\ \mathscr{U}_{\delta}^{-} = \dfrac{\sum_{\gamma=1}^{\Gamma} \mathscr{H}_{\delta\gamma}^{w}}{\sum_{\gamma=1}^{\Gamma} \mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}w}} \end{cases} \tag{40}$$

where $\mathscr{H}_{\delta\gamma}^{w}$, $\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{I}w}$ and $\mathscr{H}_{\gamma}^{\mathscr{A}\mathscr{A}\mathscr{I}w}$ are consider from Equation (39) with $\delta = 1, 2, \ldots, \Delta$.

Step G Utility function $(\mathfrak{f}(\mathscr{U}_{\delta})$ construction:
The utility function $(\mathfrak{f}(\mathscr{U}_{\delta}))$ associated with each alternative (δ) is evaluated. It is calculated by using the following equation

$$\mathfrak{f}(\mathscr{U}_{\delta}) = \frac{(\mathscr{U}_{\delta}^{+} + \mathscr{U}_{\delta}^{-})}{1 + \dfrac{1 - \mathfrak{f}(\mathscr{U}_{\delta}^{+})}{\mathfrak{f}(\mathscr{U}_{\delta}^{+})} + \dfrac{1 - \mathfrak{f}(\mathscr{U}_{\delta}^{-})}{\mathfrak{f}(\mathscr{U}_{\delta}^{-})}} \tag{41}$$

where the utility function associated with the ideal solution and anti-ideal solution are denoted by $\mathfrak{f}(\mathscr{U}_{\delta}^{+})$ and $\mathfrak{f}(\mathscr{U}_{\delta}^{-})$, respectively and are evaluated as follows

$$\begin{cases} \mathfrak{f}(\mathscr{U}_{\delta}^{+}) = \dfrac{\mathscr{U}_{\delta}^{-}}{(\mathscr{U}_{\delta}^{-} + \mathscr{U}_{\delta}^{+})} \\ \mathfrak{f}(\mathscr{U}_{\delta}^{-}) = \dfrac{\mathscr{U}_{\delta}^{+}}{(\mathscr{U}_{\delta}^{-} + \mathscr{U}_{\delta}^{+})} \end{cases} \tag{42}$$

where $\mathscr{U}_{\delta}^{+}$ and $\mathscr{U}_{\delta}^{-}$ are utility degree of alternatives (δ) with $\delta = 1, 2, \ldots, \Delta$.

Step H Alternative ranking:
The ranking of alternatives is evaluated based on the values of utility functions $(\mathfrak{f}(\mathscr{U}_\delta))$ for $\delta = 1, 2, \ldots, \Delta$. The alternative that maximizes the utility function receives the optimal ranking; the alternative that has the second-highest utility value receives the second rank and so on. Figure 5 shows the graphical procedure of the MARCOS method.

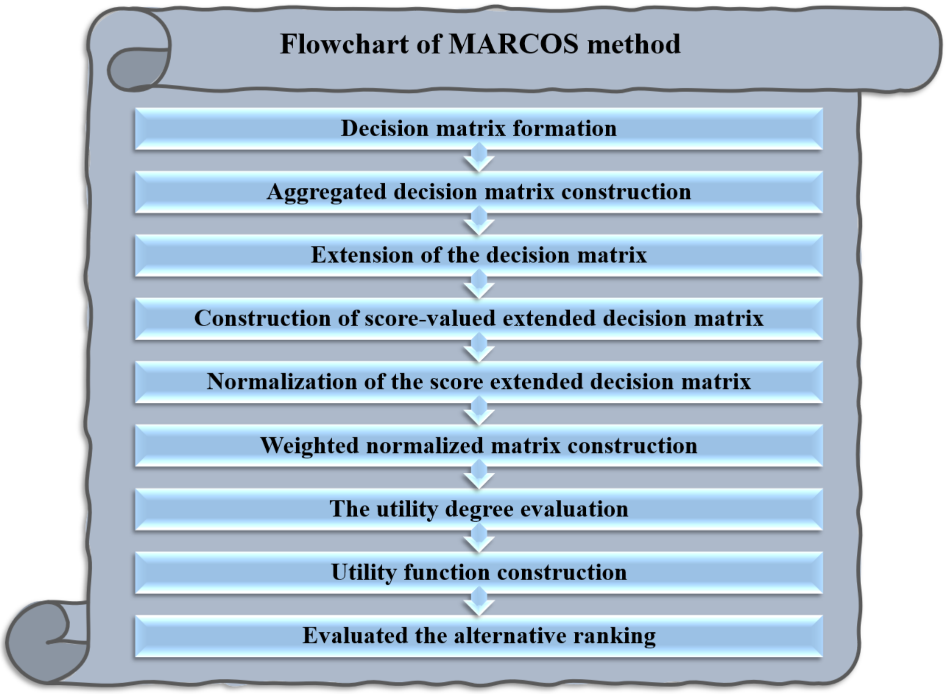

Fig. 5. Structural flowchart of the MARCOS method

6. Model Structured and Data Collection

In this section, we mainly focus on two tasks: firstly, model formulation and secondly, the process of data collection for numerical computations.

6.1 *Model structured*

In this study, our aim is to formulate an MCDM-based model for identifying the sustainable challenges of non-governmentally funded higher educational institutions. Here, we have included two MCDM methods, namely the MEREC and MARCOS methods, where the MEREC method is included to evaluate the criteria weights and the MARCOS method is applied to calculate the rank of alternatives. In this decision-making problem, 6 important factors are considered as criteria and 5 educational institutions are considered as alternatives. To maintain the privacy of the academic institutions, we consider the names of the academic institutions anonymous, such as Institute A (I_A), Institute B (I_B), Institute C (I_C), Institute D (I_D) and Institute E (I_E). Here, the opinions of 3 decision experts who are experienced and experts in their relevant field are considered. The DMs express their opinion in linguistic terms and we further convert them to q-rung orthopair fuzzy numbers (q-ROFNs) to capture uncertainty in the data set. Thus, a total of 3 decision matrices of order 5×6 are constructed and these decision matrices are used for further numerical calculations. Lastly, we have performed a total of 4 cases for the sensitivity analysis, which helps to verify the stability of the obtained result. In Figure 6, we have presented a hierarchical structure of our proposed model.

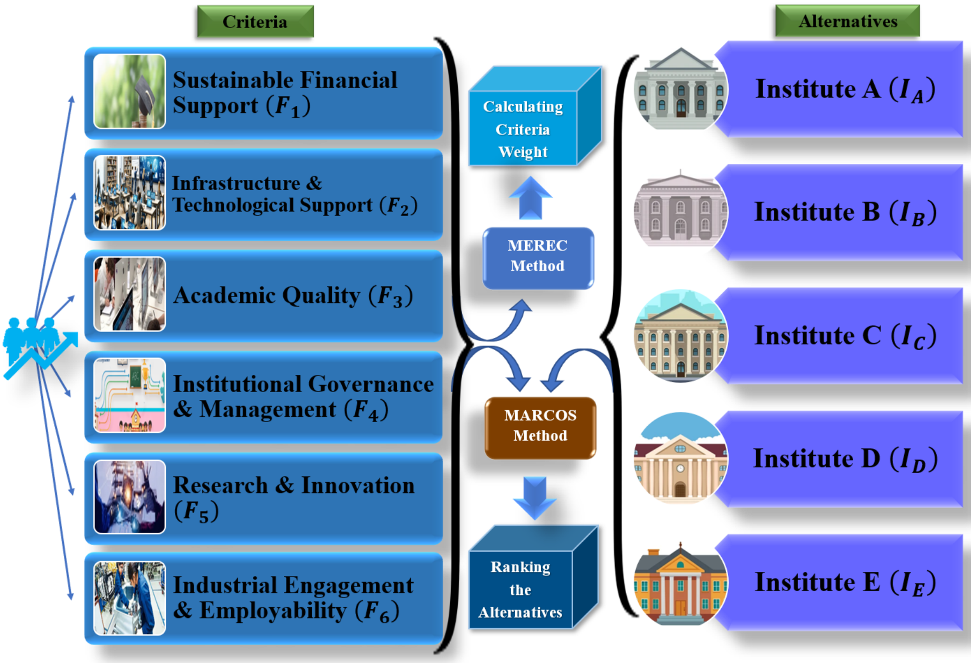

Fig. 6. Hierarchical structure of the proposed model

6.2 *Data collection*

In this section, the data collection process is briefly discussed. The opinions collected from different decision experts are given in linguistic form. For numerical evaluation, we have to convert them into equivalent q-ROFNs. Here, Table 4 is used to convert linguistic terms into q-ROFNs.

Table 4
Conversion table between linguistic terms and q-ROFNs

Linguistic terms	q-rung orthopair fuzzy numbers (q-ROFNs)	Score value	Accuracy value
Extremely Important (EI)	$\{(7; 0.95, 0.55)\}$	12.680	12.769
Very Important (VI)	$\{(6; 0.90, 0.55)\}$	10.319	9.845
Strongly Important (SI)	$\{(5; 0.90, 0.60)\}$	8.393	8.341
Average Important (AI)	$\{(4; 0.85, 0.60)\}$	6.421	6.086
Poorly Important (PI)	$\{(3; 0.85, 0.65)\}$	4.643	4.679
Weakly Important (WI)	$\{(2; 0.80, 0.65)\}$	2.980	2.887
Least Important (LI)	$\{(1; 0.80, 0.70)\}$	1.412	1.496

In this paper, we have collected the data from 3 decision experts who are knowledgeable, professional and unbiased in their opinions. These decision makers are experts in their respective fields. The details of the decision makers are given as follows:

DM1: A government officer from the Ministry of Education departments with more than 15 years of experience.

DM2: A senior professor from public institutions with more than 10 years of administrative experience.

DM3: An educational analyst from some non-profit NGOs with more than 15 years of experience.

These three DMs provided their decisions in linguistic terms using the Table 4. Thus, a total of 3 decision matrices are formulated in linguistic terms, presented in Table 5. These datasets are further considered in the numerical illustration section.

Table 5
Decision matrix in linguistic terms given by three DMs

	Criteria vs Alternatives	Sustainable Financial Support (F_1)	Infrastructure & Technological Support $(F2)$	Academic Quality (F_3)	Institutional Governance & Management (F_4)	Research & Innovation (F_5)	Industrial Engagement & Employability (F_6)
DM1	Institute A (I_A)	EI	VI	SI	AI	WI	VI
	Institute B (I_B)	SI	WI	SI	AI	VI	EI
	Institute C (I_C)	WI	SI	VI	PI	EI	VI
	Institute D (I_D)	SI	VI	VI	EI	SI	AI
	Institute E (I_E)	EI	VI	SI	EI	SI	LI
	Criteria vs Alternatives	F_1	F_2	F_3	F_4	F_5	F_6
DM2	Institute A (I_A)	SI	VI	AI	EI	VI	SI
	Institute B (I_B)	AI	SI	VI	EI	WI	VI
	Institute C (I_C)	VI	WI	SI	LI	SI	VI
	Institute D (I_D)	EI	VI	WI	AI	EI	SI
	Institute E (I_E)	VI	EI	AI	SI	VI	AI
	Criteria vs Alternatives	F_1	F_2	F_3	F_4	F_5	F_6
DM3	Institute A (I_A)	VI	SI	AI	EI	WI	LI
	Institute B (I_B)	EI	VI	EI	SI	EI	SI
	Institute C (I_C)	LI	SI	VI	WI	VI	VI
	Institute D (I_D)	VI	VI	SI	AI	VI	EI
	Institute E (I_E)	AI	SI	EI	VI	VI	LI

7. Numerical Illustration and Discussion

In this section, numerical illustration and discussion will be discussed elaborately. In this model, two MCDM methods, namely MEREC and MARCOS, are performed in an environment where the opinions of decision experts are further converted to their associated q-rung orthopair fuzzy numbers. The linguistic terms along with their corresponding fuzzy values are shown in Table 4. The data set, presented as a decision matrix by multiple decision experts, is shown in Table 5.

Firstly, we considered the MEREC-based weighted method, which is described in Section 5.1 under the q-ROFNs environments (see Section 4). The decision matrices are aggregated with the help of Equation (28) and build an aggregated decision matrix $(\tilde{\mathscr{D}})$. Then, transform the uniform aggregated fuzzy decision matrix $(\tilde{\mathscr{D}}^n)$ using Equation (29) and further evaluated the score valued decision matrix $(\mathscr{D})$ in Table 6 using Equation (22), respectively. After that, the overall performance value $(\mathscr{P}_\delta)$ of alternative (δ) is determine using Equation (31) and presented in Table 7 and the alternatives' performance value $(\mathscr{Q}_{\delta\gamma})$ by eliminating every criteria evaluating using Equation (32) and shown in Table

8, respectively. Lastly, the aggregated absolute deviation value $(\mathscr{R}_\gamma)$ of each criteria is calculated by Equation (33) and the final criteria weights are evaluated using Equation (34) and presented in Table 9. The Pi diagram of the criteria weights is shown graphically in Figure 7.

Table 6
Score valued decision matrix $(\mathscr{D})$

Criteria vs Alternatives	F_1	F_2	F_3	F_4	F_5	F_6
Institute A (I_A)	0.847	0.807	0.601	0.847	0.419	0.515
Institute B (I_B)	0.720	0.548	0.870	0.752	0.629	0.918
Institute C (I_C)	0.356	0.506	0.821	0.248	0.851	0.940
Institute D (I_D)	0.847	0.876	0.557	0.705	0.851	0.780
Institute E (I_E)	0.765	0.855	0.739	0.885	0.803	0.257

Table 7
Overall performance value $(\mathscr{P}_\delta)$ of the alternative (δ)

Alternative	Institute A (I_A)	Institute B (I_B)	Institute C (I_C)	Institute D (I_D)	Institute E (I_E)
$\mathscr{P}_\delta$ value	-1.2631	-1.4981	-1.3116	-1.5685	-1.4678

Table 8
Alternatives' performance value $(\mathscr{Q}_{\delta\gamma})$

Criteria vs Alternatives	F_1	F_2	F_3	F_4	F_5	F_6
Institute A (I_A)	-0.951	-0.989	-1.110	-0.951	-1.172	-1.143
Institute B (I_B)	-1.286	-1.366	-1.158	-1.265	-1.333	-1.082
Institute C (I_C)	-1.238	-1.194	-1.025	-1.264	-0.995	-0.842
Institute D (I_D)	-1.256	-1.221	-1.433	-1.365	-1.252	-1.316
Institute E (I_E)	-1.227	-1.146	-1.244	-1.107	-1.197	-1.418

Table 9
Criteria weight with associated data calculated

Criteria	$\mathscr{R}_\gamma$	**Weight** $(\mathscr{W}_\gamma)$
Sustainable Financial Support (F_1)	1.151	0.1619
Infrastructure & Technological Support $(F2)$	1.193	0.1678
Academic Quality (F_3)	1.140	0.1603
Institutional Governance & Management (F_4)	1.157	0.1628
Research & Innovation (F_5)	1.160	0.1632
Industrial Engagement & Employability (F_6)	1.308	0.1840

Remark 9. *Table 9 and Figure 7 represent the results of the MEREC method with associated data. From the evaluation results, the criteria Industrial Engagement & Employability (F_6) have the maximum weight and the criteria Infrastructure & Technological Support $(F2)$ have the second-highest weight. Further, the criteria Research & Innovation (F_5), Institutional Governance & Management (F_4) and Sustainable Financial Support (F_1) gets the third, fourth and fifth optimal weights. Lastly, the criteria Academic Quality (F_3) has the least weighted criteria by the MEREC method. All the weights are further considered for ranking academic institutions in the MARCOS methodology.*

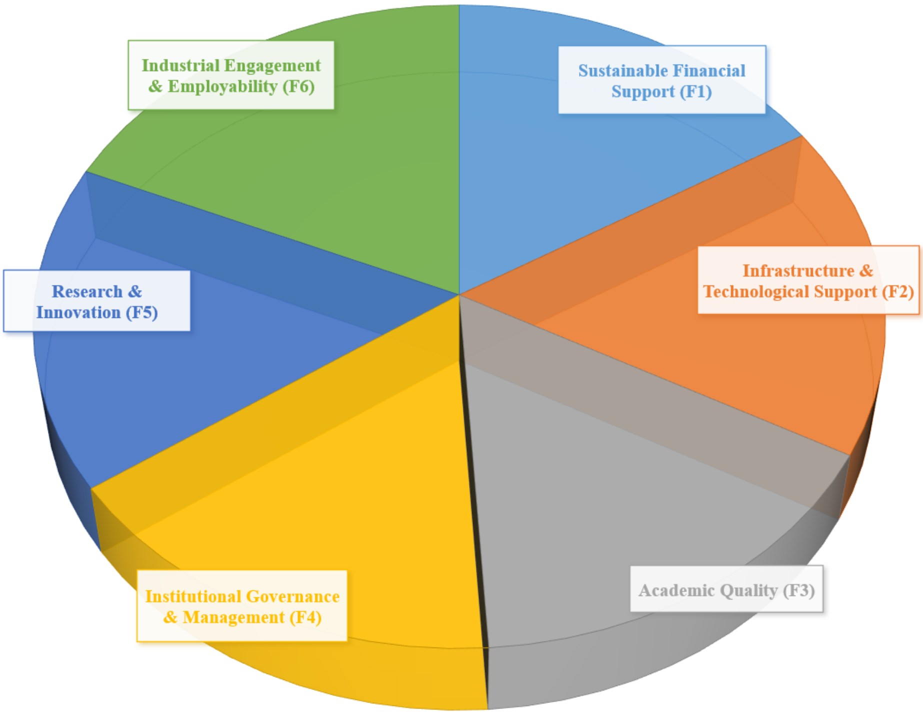

Fig. 7. Pi diagram of the criteria weight evaluated using the MEREC method

Now, we apply the MARCOS method for ranking the alternatives, which is elaborately discussed in Section 5.2. Up to the aggregation of the decision matrix is considered from the MEREC method in the previous steps. Then the extended decision matrix $(\tilde{\mathscr{D}}^{\mathfrak{E}})$ formulate with the help of Equation (36) and the score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}})$ calculate by using Equation (29) and presented in Table 10. Then the normalized score valued extended decision matrix $(\mathscr{D}^{\mathfrak{E}n})$ is evaluated by utilizing Equation (29) and displayed in Table 11. Further, the weighted normalized decision matrix $(\mathscr{D}^{w})$ is evaluated by Equation (39) and shown in Table 12. After that, the utility degrees and utility function values are calculated using Equation (42) and Equation (41), respectively and presented in Table 13. Finally, rank the alternatives on the basis of the utility function values in descending order and the ranks are shown in Table 13. The bar diagram of the institutes (alternatives) ranking are shown in Figure 8.

Table 10

Extended decision matrix $(\mathscr{D}^{\mathfrak{E}})$

C vs A	F_1	F_2	F_3	F_4	F_5	F_6
Anti-ideal solution	4.235	5.959	6.456	2.	.966	2.823
Institute A (I_A)	10.072	9.512	6.956	9.631	4.966	5.647
Institute B (I_B)	8.561	6.456	10.072	8.561	7.449	10.072
Institute C (I_C)	4.235	5.959	9.512	2.823	10.072	10.319
Institute D (I_D)	10.072	10.319	6.456	8.026	10.072	8.561
Institute E (I_E)	9.096	10.072	8.561	10.072	9.512	2.823
Ideal solution	10.072	10.319	10.072	10.072	10.072	10.319

Table 11
Normalization of decision matrix $(\mathscr{D}^{\mathfrak{E}n})$

C vs A	F_1	F_2	F_3	F_4	F_5	F_6
Anti-ideal solution	0.421	0.578	0.641	0.280	0.493	0.274
Institute A (I_A)	1.000	0.922	0.691	0.956	0.493	0.547
Institute B (I_B)	0.850	0.626	1.000	0.850	0.740	0.976
Institute C (I_C)	0.421	0.578	0.944	0.280	1.000	1.000
Institute D (I_D)	1.000	1.000	0.641	0.797	1.000	0.830
Institute E (I_E)	0.903	0.976	0.850	1.000	0.944	0.274
Ideal solution	1.000	1.000	1.000	1.000	1.000	1.000

Table 12
Weighted normalized decision matrix $(\mathscr{D}^{w})$

C vs A	F_1	F_2	F_3	F_4	F_5	F_6
Anti-ideal solution	0.068	0.097	0.103	0.046	0.080	0.050
Institute A (I_A)	0.162	0.155	0.111	0.156	0.080	0.101
Institute B (I_B)	0.138	0.105	0.160	0.138	0.121	0.180
Institute C (I_C)	0.068	0.097	0.151	0.046	0.163	0.184
Institute D (I_D)	0.162	0.168	0.103	0.130	0.163	0.153
Institute E (I_E)	0.146	0.164	0.136	0.163	0.154	0.050
Ideal solution	0.162	0.168	0.160	0.163	0.163	0.184

Table 13
Alternative ranking based on utility degrees and utility functions

Alternatives	$\mathscr{U}_\delta^+$	$\mathscr{U}_\delta^-$	$\mathscr{U}_\delta^+ + \mathscr{U}_\delta^-$	$\mathfrak{f}(\mathscr{U}_\delta^+)$	$\mathfrak{f}(\mathscr{U}_\delta^-)$	$\frac{1-\mathfrak{f}(\mathscr{U}_\delta^+)}{\mathfrak{f}(\mathscr{U}_\delta^+)}$	$\frac{1-\mathfrak{f}(\mathscr{U}_\delta^-)}{\mathfrak{f}(\mathscr{U}_\delta^-)}$	$\mathfrak{f}(\mathscr{U}_\delta)$	Rank
Institute A (I_A)	0.764	1.720	2.484	0.692	0.308	0.444	2.251	0.672	4
Institute B (I_B)	0.842	1.895	2.736	0.692	0.308	0.444	2.251	0.740	2
Institute C (I_C)	0.709	1.597	2.306	0.692	0.308	0.444	2.251	0.624	5
Institute D (I_D)	0.878	1.977	2.855	0.692	0.308	0.444	2.251	0.773	1
Institute E (I_E)	0.814	1.831	2.645	0.692	0.308	0.444	2.251	0.716	3

Remark 10. *Table 13 and Figure 8 are shown in the evaluated results by the MARCOS method of the ranking of academic institutions. From the determined results, Institute D (I_D) occupied the most prioritized institute for funding, followed by Institute B (I_B), Institute E (I_E), respectively. Furthermore, Institute A (I_A) and Institute C (I_C) are the second least and least optimal alternatives for funding an academic institution, respectively. All the institutions' names are anonymous for considering the privacy of the institutions.*

8. Sensitivity Analysis

This section presents the sensitivity analysis of this study to assess the stability and flexibility of the evaluated results. There are four cases considered for this sensitivity analysis and discussed as follows:

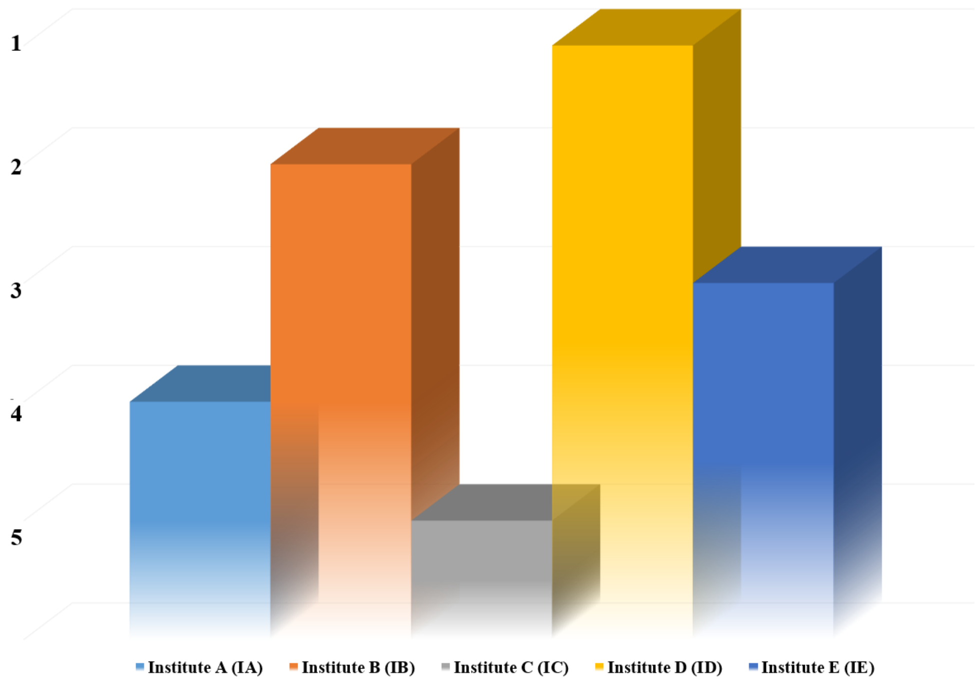

Fig. 8. Bar diagram of the alternative ranking calculated by the MARCOS method

Case 1. Remove criteria Sustainable Financial Support (F_1):
We removed an important criterion, Sustainable Financial Support (F_1), from the data set and evaluated the results using the modified model. The alternative rankings are presented in Table 14 and graphically depicted in Figure 9, respectively.

Case 2. Remove criteria Institutional Governance & Management (F_4):
In the second scenario, we removed the criterion Institutional Governance & Management (F_4) and evaluated the weights and ranked alternatives based on the updated model. The results evaluated using the MEREC and MARCOS methods are presented in Table 14 and visually compared in Figure 9.

Case 3. Remove criteria Research & Innovation (F_5):
In another scenario, we omit the criteria Research & Innovation (F_5) and determine the weight of the criteria and ranking alternatives by the modified model. All the results are shown in Table 14 and a graphical comparison of the main results is displayed in Figure 9.

Case 4. Remove criteria Industrial Engagement & Employability (F_6):
In the last case, we removed the optimality-weighted criteria Industrial Engagement & Employability (F_6) and determined the results of the modified systems. The evaluated results are presented in Table 14 and visually compared with existing results in Figure 9.

Table 14
Ranking of the institutes in different scenarios

Alternative	Case 1	Case 2	Case 3	Case 4	Main ranking
Institute A (I_A)	5	5	3	3	4
Institute A (I_A)	2	2	1	4	2
Institute A (I_A)	4	3	5	5	5
Institute A (I_A)	1	1	2	2	1
Institute A (I_A)	3	4	4	1	3

Table 14 and Figure 9 represent the results of the sensitivity analysis of four cases and compare with the main results. From the evaluated results of Case 1, Case 2 and Case 3 are more stable with the main results; however, Case 4 slightly fluctuates with the main results, since the highly weighted criteria is omitted from the model. Then, based on the sensitivity analysis, we conclude that the main results are stable, flexible and unbiased, and that our model is robust.

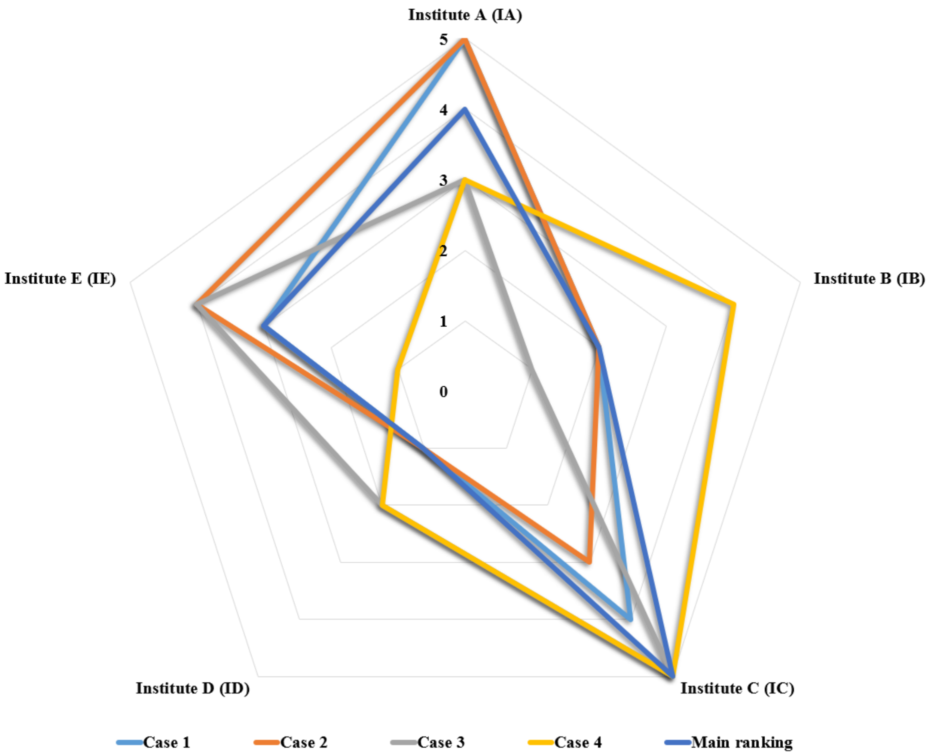

Fig. 9. Sensitivity analysis of the alternatives and comparison with the main results

9. Research Implication

In this paper, our purpose is to construct an MCDM-based framework to identify the sustainable challenges of non-governmentally funded higher educational institutions. Thus, the existing literature related to sustainability in higher educational institutions will be enriched by this paper. Here, we have proposed a score function and an accuracy function to convert q-ROFNs into crisp values. In this study, we have integrated q-ROFNs in the MEREC MARCOS-based model, which helps to capture uncertainty and ambiguity in the dataset. Thus, the result obtained from this model becomes more reliable and this concept can further be used in different complex decision-making problems. Here, we have selected 6 factors as criteria after going through a deep study on this topic and discussing it with DMs. Based on these criteria, we finally evaluate the ranking of 5 higher educational institutions. This model can further be applied to different fields, like finding the key challenges of universities, institutions, colleges and schools and further ranking them accordingly.

10. Conclusions and Future Research Scope

Education enlightens the path of life. To provide quality education to all in a society, it is necessary to develop the infrastructure and basic facilities related to deliver education. In the higher educational sector, funding plays a crucial role as these students not only pursue a degree but are also attached to research and innovation-related work. Thus, non-governmentally funded higher educational institutions faced various challenges. In this paper, we have developed a MEREC MARCOS-based MCDM model under the q-rung orthopair fuzzy environment for identifying the sustainable challenges of non-governmentally funded higher educational institutions. Here, we have considered 6 criteria and after applying the MEREC method, we have obtained that the criterion Industrial Engagement & Employability (F_6) acquired the highest weight, Infrastructure & Technological Support (F_2) obtained the 2^{nd} highest weight and so on. This means Industrial Engagement & Employability (F_6) is the most important challenge faced by non-governmentally funded higher educational institutions, Infrastructure & Technological Support (F_2) is the 2^{nd} most important challenge faced by non-governmentally funded higher educational institutions and so on. After that, we have applied the MARCOS method to find the ranking of 5 higher educational institutions, which are considered as alternatives. Finally, we have obtained that the Institute D (I_D) got the 1^{st} rank, Institute B (I_B) got the 2^{nd} rank, Institute E (I_E) got the 3^{rd} rank and so on. Lastly, we have considered 4 sensitivity analysis cases to assess the stability of the result.

There is some scope for extension in future research. This paper is based on a MEREC MARCOS-based approach, which can be altered in the future by other advanced MCDM techniques such as CRITIC, PROMETHEE, VIKOR, MABAC, CoCoSo, etc. One can implement some other newly developed fuzzy number that is capable of capturing uncertainty more than q-ROFNs. Although we have proposed a new score and accuracy function for this model, other newly developed score and accuracy functions can be implemented in the future. In this paper, we have considered only 6 challenges as criteria but one can include some other criteria such as Green Campus initiatives, sustainable resource utilization, Fluctuations in enrollment of student etc. Here, we have taken 5 higher educational institutions as alternatives; one can increase the number of alternatives to increase the complexity of this decision-making problem. Here, the data set is collected from only 3 DMs, which can further be increased so that the evaluated results can be more accurate and unbiased. Here, we have performed 4 cases of sensitivity analysis, where we have omitted one criterion in each case but one can perform sensitivity analysis by including cases where interchange of the weight between two criteria is conducted. This model can be further applied in different regions and institutional contexts.

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Conflicts of Interest
The authors declare no conflicts of interest.

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