



A Novel Computational Approach for Linear Fractional Programming Problems in a Neutrosophic Environment

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ABSTRACT

This paper introduces a novel computational approach for solving neutrosophic linear fractional programming (NLFP) problems in which all objective coefficients, technological coefficients, resource values, and decision variables are represented by triangular neutrosophic numbers. The proposed method addresses the uncertainty, inconsistency, and indeterminacy inherent in real-world optimization problems by employing a neutrosophic environment characterized by truth, indeterminacy, and falsity membership degrees. A systematic transformation procedure is developed to reformulate the original NLFP model into an equivalent neutrosophic linear programming model through a parametric representation of the fractional objective function. Subsequently, a specialized ranking function is employed to convert triangular neutrosophic numbers into crisp values, enabling the formulation of a solvable linear programming model while preserving the essential uncertainty information contained in the original problem. The proposed methodology simplifies the computational procedure, reduces model complexity, and provides an efficient mechanism for obtaining optimal solutions under neutrosophic uncertainty. A detailed solution algorithm is presented, and a numerical example is provided to demonstrate the applicability and effectiveness of the proposed method. The obtained results are compared with several existing approaches from the literature, demonstrating that the proposed method produces reliable and competitive solutions while maintaining computational efficiency and simplicity. Overall, the findings confirm the suitability of the proposed methodology for solving NLFP problems and highlight its potential for extension to more complex optimization models under uncertain and indeterminate decision-making environments.

1. Introduction

Linear fractional programming (LFP) is an extension of the classical linear programming (LP) in which the objective function is a ratio of two linear functions, instead of a single linear function in LP. This formulation is particularly useful for optimizing efficiency ratios like profit-to-cost, inventory-to-sales or output-per-employee ratios.

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In practice, decision-makers often face the situations that they cannot exactly identify all LFP coefficients due to the inherent data limitations and information uncertainty. The inaccuracy is for the real world where the complete information is not available or the measurements are not exact. Furthermore, there is an inherent ambiguity in determining exact aspiration levels for objective functions and problem parameters which creates additional decision-making challenges.

The proposed neutrosophic framework provides an effective mathematical model for accommodating these sorts of uncertainties as it can accommodate the simultaneous presence of the three types of indeterminacy, namely the truth, falsity and indeterminacy degrees of membership, which are the main characteristics of the ambiguous decision environments [1].

Smarandache [2] introduced neutrosophic logic as a significant extension of classical fuzzy logic and intuitionistic fuzzy sets (IFSs). The neutrosophic set framework is essentially defined by three different and independent membership functions, namely (1) truth-membership function (T) that stands for the degree of validity; (2) indeterminacy-membership function (I) that measures the level of uncertainty; and (3) falsity-membership function (F) that stands for the degree of invalidity. This three-component structure provides a broader mathematical representation of uncertain information than traditional fuzzy approaches.

In neutrosophic set theory, the decision maker tries to maximize the truth-membership degree (T) and minimize the indeterminacy (I) and falsity (F) membership degrees at the same time. The truth and falsity membership functions are inverses of each other, representing opposite sides of the evaluation. The indeterminacy membership function is in an intermediate position, it has some properties of the truth component and some of the falsity component. This structural relation shows that indeterminacy is an intermediate state between truth and falsity. Wang et al., [3] proposed a simplified but effective version of the neutrosophic framework in practical computational applications, named as the single-valued neutrosophic set (SVNS). The essential tripartite structure is kept, but the SVNS enables the development of more tractable solutions to real-world problems. Neutrosophic sets are very powerful tools for handling incomplete and indeterminate information [4,5].

The application of single-valued neutrosophic sets (SVNS) in practical domains has been investigated by several researchers. Important contributions include [6,7] in educational systems and social policy analysis; and Broumi and Smarandache [8] application of SVNS to many other real-world problems. These studies demonstrate how SVNS can effectively model uncertainty in diverse domains while retaining the essential (T, I, F) structure of neutrosophic theory. Smarandache [9] first laid the theoretical foundations of neutrosophic numbers and gave the crucial theoretical groundwork of neutrosophic mathematics. Building on this foundation. Several researchers have made significant contributions in neutrosophic linear programming. Abdel-Basset *et al.*, [1] introduced a new method for neutrosophic LP problem by considering trapezoidal neutrosophic numbers. A direct computation model for linear programming problems with triangular neutrosophic numbers was introduced by Edalatpanah [10], which is an efficient way to solve such problems under this type of uncertainty. Following this work, Ye *et al.*, [11] introduced a new optimization method to find optimal solutions of linear programming problems in general neutrosophic number environments. Banerjee and Pramanik [12] laid the foundation for solving single-objective linear programming problems in neutrosophic number (NN) environments, applying goal programming techniques in a novel way.

Pramanik and Dey [13] created an extensive mathematical approach to address bi-level linear programming issues utilizing neutrosophic numbers. Maiti *et al.*, [14] proposed a methodology for multi-level multi-objective linear programming problems utilizing neural networks. Neutrosophic geometric programming problems represent a novel domain within neutrosophic theory, established

by Huda E. Khalid [15] by the use of freshly formulated methodologies, definitions, and theorems. Das *et al.*, [16] suggest a novel approach for addressing NLFP issues by dissecting them into two neutrosophic linear programming (NLP) problems. The solution integrates the dual simplex approach with a specialized ranking function. Khalifa, H. and Pavan Kumar [17] propose an interval-valued fuzzy linear fractional programming (LFP) model. The goal function utilizes single-valued trapezoidal fuzzy neutrosophic coefficients, whereas the restrictions incorporate interval-valued fuzzy numbers. Das and Edalatpanah [18] provide a Neutrosophic Linear Fractional Programming Problem (NLFP) characterized by mixed equality and inequality constraints that incorporate neutrosophic triangular fuzzy numbers. Verma & Singh [19] describe a technique for addressing neutrosophic linear fractional optimization problems, utilizing neutrosophic numbers (triangular/trapezoidal) to denote objective costs, technological coefficients, and resources.

The remainder of this work is organized as follows: Section 2 deals with the preliminaries such as required terminology and mathematical operations on neutrosophic numbers. The approach of the suggested technique is described in Section 3. The implementation of the proposed system is presented in Section 4 via a numerical example to demonstrate its efficacy. Finally, Section 5 presents the conclusion.

2. Preliminaries

This section presents the essential definitions and notations utilized in this study.

Definition 1 [1]. Let X denote a universal set and $x \in X$. A neutrosophic set L can be characterized by three membership functions for truth, indeterminacy, and falsehood, denoted as $T_L(x)$, $I_L(x)$ and $F_L(x)$. These are genuine standard or genuine nonstandard subsets of $]0^-, 1^+[$. That is $T_L(x): X \rightarrow]0^-, 1^+[$, $I_L(x): X \rightarrow]0^-, 1^+[$, $F_L(x): X \rightarrow]0^-, 1^+[$. There are no limitations on the sum of $T_L(x)$, $I_L(x)$ and $F_L(x)$, so $0^- \leq \sup T_L(x) + \sup I_L(x) + \sup F_L(x) \leq 3^+$.

Definition 2 [20]. A triangular neutrosophic number (TNN) is denoted as (SVNS) L over X is defined as $L = \{x, T_L(x), I_L(x), F_L(x)\}$, where X represents a discourse space. The functions $T_L(x): X \rightarrow [0, 1]$, $I_L(x): X \rightarrow [0, 1]$, $F_L(x): X \rightarrow [0, 1]$ satisfying the condition $0 \leq T_L(x) + I_L(x) + F_L(x) \leq 3$, $\forall x \in X$.

Definition 3 [21]. A triangular neutrosophic number (TNNs) is denoted as $L = \langle (a_1^t, a_2^t, a_3^t); (q_1^t, q_2^t, q_3^t) \rangle$ representing an extension of the three membership functions for truth, indeterminacy, and falsity of x , which can be articulated as follows:

Truth-membership function:

$$T_L(x) = \begin{cases} \frac{x - a_1^t}{a_2^t - a_1^t}, & a_1^t \leq x < a_2^t, \\ 1, & x = a_2^t, \\ \frac{a_3^t - x}{a_3^t - a_2^t}, & a_2^t \leq x < a_3^t, \\ 0, & \text{otherwise.} \end{cases}$$

Indeterminacy-membership function:

$$I_L(x) = \begin{cases} \frac{x - a_1^i}{a_2^i - a_1^i}, & a_1^i \leq x < a_2^i, \\ 1, & x = a_2^i, \\ \frac{a_3^i - x}{a_3^i - a_2^i}, & a_2^i \leq x < a_3^i, \\ 0, & \text{otherwise.} \end{cases}$$

Falsity-membership function:

$$F_L(x) = \begin{cases} \frac{x - a_1^f}{a_2^f - a_1^f}, & a_1^f \leq x < a_2^f, \\ 1, & x = a_2^f, \\ \frac{a_3^f - x}{a_3^f - a_2^f}, & a_2^f \leq x < a_3^f, \\ 0, & \text{otherwise.} \end{cases}$$

Where, $0 \leq T_L(x) + I_L(x) + F_L(x) \leq 3, x \in L$ also, when $a_1^t \geq 0, L$ called a non negative TNN. Likewise, when $a_1^t < 0, L$ transforms into a negative TNN.

Definition 4 [1] (Arithmetic Operations). Let $L_1 = \langle (a_1^t, a_2^t, a_3^t); (q_1^t, q_2^t, q_3^t) \rangle$ and $L_2 = \langle (b_1^t, b_2^t, b_3^t); (r_1^t, r_2^t, r_3^t) \rangle$ represent two TNNs. Subsequently, the arithmetic relationships are delineated as:

- i $L_1 \oplus L_2 = \langle (a_1^t + b_1^t, a_2^t + b_2^t, a_3^t + b_3^t); (q_1^t \wedge r_1^t, q_2^t \vee r_2^t, q_3^t \vee r_3^t) \rangle$
- ii $L_1 - L_2 = \langle (a_1^t - b_1^t, a_2^t - b_2^t, a_3^t - b_3^t); (q_1^t \wedge r_1^t, q_2^t \vee r_2^t, q_3^t \vee r_3^t) \rangle$
- iii $L_1 \otimes L_2 = \langle (a_1^t b_1^t, a_2^t b_2^t, a_3^t b_3^t); (q_1^t \wedge r_1^t, q_2^t \vee r_2^t, q_3^t \wedge r_3^t) \rangle$, if $a_1^t > 0, b_1^t > 0$.
- iv $\lambda L_1 = \begin{cases} \langle (\lambda a_1^t, \lambda a_2^t, \lambda a_3^t); (q_1^t, q_2^t, q_3^t) \rangle, & \text{if } \lambda > 0 \\ \langle (\lambda a_3^t, \lambda a_2^t, \lambda a_1^t); (q_1^t, q_2^t, q_3^t) \rangle, & \text{if } \lambda < 0 \end{cases}$
- v $\frac{L_1}{L_2} = \begin{cases} \langle (\frac{a_1^t}{b_3^t}, \frac{a_2^t}{b_2^t}, \frac{a_3^t}{b_1^t}); q_1^t \wedge r_1^t, q_2^t \vee r_2^t, q_3^t \vee r_3^t \rangle, & a_3^t > 0, b_3^t > 0 \\ \langle (\frac{a_3^t}{b_3^t}, \frac{a_2^t}{b_2^t}, \frac{a_1^t}{b_1^t}); q_1^t \wedge r_1^t, q_2^t \vee r_2^t, q_3^t \vee r_3^t \rangle, & a_3^t < 0, b_3^t > 0 \\ \langle (\frac{a_3^t}{b_1^t}, \frac{a_2^t}{b_2^t}, \frac{a_1^t}{b_3^t}); q_1^t \wedge r_1^t, q_2^t \vee r_2^t, q_3^t \vee r_3^t \rangle, & a_3^t < 0, b_3^t < 0 \end{cases}$

Definition 5 [1]. Let L_1 and L_2 represent two TNNs. Then:

- i $L_1 \leq L_2$ if and only if $\Re(L_1) \leq \Re(L_2)$.
- ii $L_1 < L_2$ if and only if $\Re(L_1) < \Re(L_2)$.

Where $\Re(\cdot)$ denotes a ranking function.

Definition 6 [21]. The ranking function applied to triangular neutrosophic numbers $L = \langle (a_1^t, a_2^t, a_3^t); (q_1^t, q_2^t, q_3^t) \rangle$ is defined as:

$$\Re(L) = \frac{a_1^t + a_2^t + a_3^t}{3} \left(\frac{q_1^t + (1 - q_2^t) + (1 - q_3^t)}{3} \right).$$

3. Proposed Method

This study primarily contributes by extending conventional linear fractional programming (LFP) into the neutrosophic realm, so providing a neutrosophic linear fractional programming (NLFP) framework. The conventional crisp LFP problem is officially articulated as follows:

$$\begin{aligned} \text{Max } z(x) &= \frac{\sum c_j x_j + p}{\sum d_j x_j + q} = \frac{c^T x + p}{d^T x + q} = \frac{N(x)}{d(x)} \\ \text{Subject to} \end{aligned} \tag{1}$$

$$x \in S = \{x \in R^n: Ax \leq b, x \geq 0\}$$

Where $j = 1, 2, \dots, n, A \in R^{m \times n}, b \in R^m, c_j, d_j \in R^n$ and $p, q \in R$

The denominator is presumed to be positive for all viable solutions.

This section formulates a thorough methodology for addressing linear fractional programming issues in which all parameters are expressed as triangular neutrosophic numbers (TNNs).

Let us examine the NLFP issue:

$$\text{Max } z(x^{\sim n}) = \frac{\sum c_j^{\sim n} x_j + p^{\sim n}}{\sum d_j^{\sim n} x_j + q^{\sim n}}$$

Subject to

$$\sum a_{ij}^{\sim n} x_j \leq b_i^{\sim n}, i = 1, 2, \dots, m,$$

$$x_j \geq 0, j = 1, 2, \dots, n$$

We presume that $c_j^{\sim n}, d_j^{\sim n}, p^{\sim n}, q^{\sim n}, a_{ij}^{\sim n}$ and $b_i^{\sim n}$ are triangular neutrosophic numbers for each $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ hence, the problem can be articulated as:

$$\text{Max } z(x^{\sim n}) = \frac{\sum (c_{j1}, c_{j2}, c_{j3}; q_1^t, q_2^t, q_3^t) x_j + (p_1, p_2, p_3; q_1^t, q_2^t, q_3^t)}{\sum (d_{j1}, d_{j2}, d_{j3}; q_1^t, q_2^t, q_3^t) x_j + (q_1, q_2, q_3; q_1^t, q_2^t, q_3^t)}$$

Subject to

$$\sum (a_{ij1}, a_{ij2}, a_{ij3}; q_1^t, q_2^t, q_3^t) x_j \leq (b_{i1}, b_{i2}, b_{i3}; q_1^t, q_2^t, q_3^t), i = 1, 2, \dots, m$$

$$x_j \geq 0, j = 1, 2, \dots, n.$$

The truth-membership, indeterminacy, and falsity-membership functions of each neutrosophic number are denoted by $q_1^t, q_2^t, q_3^t \in [0, 1]$.

A methodological framework for converting problems with fractional objectives under neutrosophic uncertainty into equivalent linear formulations. The approach maintains the essential (T, I, F) characteristics while achieving computational tractability. We presume that the feasible region $S^{\sim n} = \{X \in R^n: A^{\sim n}X \leq b^{\sim n}, X \geq 0\}$ is non-empty and bounded, and that the denominator $(d^{\sim n}X + q^{\sim n}) > 0^{\sim n}$.

We may now transform the aforementioned LFP into an LP by assuming that $q^{\sim n} \neq 0^{\sim n}$.

The proposed conversion method systematically transforms the neutrosophic fractional objective into a linear form through the following rigorous steps:

Multiplying both the denominator and the numerator of $\max z^{\sim n} = \frac{c^{\sim n}X + p^{\sim n}}{d^{\sim n}X + q^{\sim n}}$ by $q^{\sim n}$, where $q^{\sim n} \neq 0^{\sim n}$.

We have

$$z^{\sim n} = \frac{c^{\sim n}Xq^{\sim n} + p^{\sim n}q^{\sim n}}{d^{\sim n}Xq^{\sim n} + q^{\sim n}q^{\sim n}} = \frac{c^{\sim n}Xq^{\sim n} + p^{\sim n}q^{\sim n}}{q^{\sim n}(d^{\sim n}X + q^{\sim n})}$$

$$= \frac{c^{\sim n}Xq^{\sim n} - d^{\sim n}Xp^{\sim n} + d^{\sim n}Xp^{\sim n} + p^{\sim n}q^{\sim n}}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} = \frac{(c^{\sim n}q^{\sim n} - d^{\sim n}p^{\sim n})X + (d^{\sim n}X + q^{\sim n})p^{\sim n}}{q^{\sim n}(d^{\sim n}X + q^{\sim n})}$$

$$= \left(c^{\sim n} - d^{\sim n} \frac{p^{\sim n}}{q^{\sim n}} \right) \frac{X}{d^{\sim n}X + q^{\sim n}} + \frac{p^{\sim n}}{q^{\sim n}} = k^{\sim n}Y^{\sim n} + g^{\sim n} \quad (4)$$

Where $k^{\sim n} = \left(c^{\sim n} - d^{\sim n} \frac{p^{\sim n}}{q^{\sim n}} \right), Y^{\sim n} = \frac{X}{d^{\sim n}X + q^{\sim n}}, g^{\sim n} = \frac{p^{\sim n}}{q^{\sim n}}$

Hence $F(Y) = k^{\sim n}Y^{\sim n} + g^{\sim n}$

For a linear fractional programming problem under neutrosophic uncertainty, the constraint transformation process involves:

$$A^{\sim n}X \leq b^{\sim n}, \text{ we have } A^{\sim n}X - b^{\sim n} \leq 0^{\sim n} \Rightarrow \frac{q^{\sim n}(A^{\sim n}X - b^{\sim n})}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} \leq 0^{\sim n}$$

$$\Rightarrow \frac{(A^{\sim n}q^{\sim n}X - b^{\sim n}q^{\sim n})}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} \leq 0^{\sim n} \Rightarrow \frac{(A^{\sim n}q^{\sim n}X + b^{\sim n}d^{\sim n}X - b^{\sim n}d^{\sim n}X - b^{\sim n}q^{\sim n})}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} \leq 0^{\sim n}$$

$$\Rightarrow \frac{(A^{\sim n}q^{\sim n}X + b^{\sim n}d^{\sim n}X)}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} - \frac{(b^{\sim n}d^{\sim n}X + b^{\sim n}q^{\sim n})}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} \leq 0^{\sim n}$$

$$\Rightarrow \frac{q^{\sim n}(A^{\sim n} + \frac{b^{\sim n}}{q^{\sim n}}d^{\sim n})X}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} - \frac{b^{\sim n}(d^{\sim n}X + q^{\sim n})}{q^{\sim n}(d^{\sim n}X + q^{\sim n})} \leq 0^{\sim n}$$

$$\Rightarrow \left(A^{\sim n} + \frac{b^{\sim n}}{q^{\sim n}}d^{\sim n} \right) \frac{X}{(d^{\sim n}X + q^{\sim n})} - \frac{b^{\sim n}}{q^{\sim n}} \leq 0^{\sim n}$$

$$\Rightarrow \left(A^{\sim n} + \frac{b^{\sim n}}{q^{\sim n}} d^{\sim n} \right) \frac{X}{(d^{\sim n} X + q^{\sim n})} \leq \frac{b^{\sim n}}{q^{\sim n}}$$

$$\Rightarrow G^{\sim n} Y^{\sim n} \leq h^{\sim n} \text{ where } G^{\sim n} = \left(A^{\sim n} + \frac{b^{\sim n}}{q^{\sim n}} d^{\sim n} \right), Y^{\sim n} = \frac{X}{(d^{\sim n} X + q^{\sim n})} \text{ and } h^{\sim n} = \frac{b^{\sim n}}{q^{\sim n}}.$$

Based on the above transformations, we derive the equivalent linear programming (LP) formulation of the original linear fractional programming (LFP) problem as follows:

$$\begin{aligned} \text{Max } F(Y) &= k^{\sim n} Y^{\sim n} + g^{\sim n} \\ \text{Subject to} \\ G^{\sim n} Y^{\sim n} &\leq h^{\sim n} \\ \text{and } Y^{\sim n} &\geq 0^{\sim n}. \end{aligned} \tag{5}$$

Algorithm

Step 1: Transform the objective function into a maximization problem, if needed.

Step 2: Transform the Neutrosophic Linear Fractional Programming Problem (NLFP) into a corresponding Neutrosophic Linear Programming Problem (NLPP).

Step 3: Transform the NLFP problem to a crisp LFP problem using the defined arithmetic operations and ranking function from the specified definitions.

Step 4: Convert the Linear Fractional Programming Problem (LFPP) into its corresponding parametric form.

Step 5: Compute the optimal solution for the converted crisp linear program using suitable algorithm.

Step 6: After obtaining the neutrosophic optimal solution $Y^* = (y_1^*, y_2^*, \dots, y_n^*)$ from the transformed problem, compute the original decision variables in the original problem $X^* = (x_1^*, x_2^*, \dots, x_n^*)$ using: $X = \frac{qY}{1-(dY)}$.

Step 7: Substitute the optimal decision variables $X^* = (x_1^*, x_2^*, \dots, x_n^*)$ into the original NLFP objective function to obtain the optimal solution.

Figure 1 shows the detailed mathematical and algorithmic steps flow of the proposed methodology.

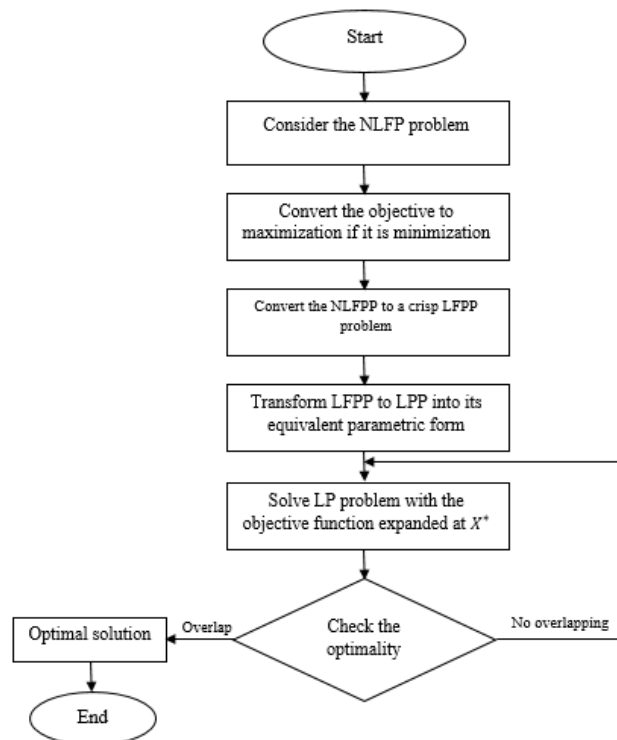


Fig. 1. Solution method flowchart

4. Numerical Example

A manufacturing company produces two products, A and B, with profits of around \$5 and around \$3 per unit respectively. The production costs are roughly \$5 per unit for product A and around \$2 per unit for product B, with an additional fixed cost of around \$1 due to production process requirements.

Each unit of product A requires around 3 pounds of raw material, while each unit of product B needs around 5 pounds. The total available raw material is limited to around 15 pounds. In terms of labor, manufacturing one unit of product A takes around 5 man-hours, and product B requires around 2 man-hours per unit. The total daily man-hour capacity is around 10 hours.

The goal is to determine the optimal production quantities of products A and B that will maximize the company's total profit under these conditions, solution, Neutrosophic Linear Fractional Programming Formulation:

$$\begin{aligned} \text{Max } z &= \frac{5^{\sim n}x_1 + 3^{\sim n}x_2}{5^{\sim n}x_1 + 2^{\sim n}x_2 + 1^{\sim n}} \\ \text{Subject to} & \\ 3^{\sim n}x_1 + 5^{\sim n}x_2 &\leq 15^{\sim n} \\ 5^{\sim n}x_1 + 2^{\sim n}x_2 &\leq 10^{\sim n} \\ x_1, x_2 &\geq 0. \end{aligned} \quad (6)$$

With $1^{\sim n} = (0.5, 1, 1.5; 0.8, 0.5, 0.2)$, $2^{\sim n} = (1, 2, 3; 0.8, 0.4, 0.2)$, $3^{\sim n} = (1, 3, 5; 0.7, 0.5, 0.3)$, $5^{\sim n} = (3, 5, 7; 0.2, 0.7, 0.4)$, $10^{\sim n} = (9, 10, 11; 0.75, 0.4, 0.25)$, $15^{\sim n} = (11, 15, 19; 0.8, 0.3, 0.2)$.

$$\begin{aligned} \text{That is } \text{max } z &= \frac{(3, 5, 7; 0.7, 0.4, 0.2)x_1 + (1, 3, 5; 0.8, 0.3, 0.1)x_2}{(3, 5, 7; 0.7, 0.4, 0.2)x_1 + (1, 2, 3; 0.8, 0.4, 0.2)x_2 + (0.5, 1, 1.5; 0.8, 0.5, 0.2)} \\ \text{Subject to} & \\ (1, 3, 5; 0.8, 0.3, 0.1)x_1 + (3, 5, 7; 0.7, 0.4, 0.2)x_2 &\leq (11, 15, 19; 0.8, 0.3, 0.2) \\ (3, 5, 7; 0.7, 0.4, 0.2)x_1 + (1, 2, 3; 0.8, 0.4, 0.2)x_2 &\leq (9, 10, 11; 0.75, 0.4, 0.25) \\ x_1, x_2 &\geq 0. \end{aligned} \quad (7)$$

Now transform the neutrosophic linear fractional programming problem (NLFP) into a crisp linear fractional programming (LFP) by applying a special ranking function defined in definition (6).

$$\begin{aligned} \text{max } z &= \frac{3.5x_1 + 2.4x_2}{3.5x_1 + 1.47x_2 + 0.7} \\ \text{Subject to} & \\ 2.4x_1 + 3.5x_2 &\leq 11.5 \\ 3.5x_1 + 1.47x_2 &\leq 7 \\ x_1, x_2 &\geq 0. \end{aligned} \quad (8)$$

Transform the Linear Fractional Programming Problem (LFPP) into a corresponding Linear Programming Problem (NLPP) utilizing the parametric form specified in (4) and (5):

$$\begin{aligned} \text{Max } z &= 3.5y_1 + 2.4y_2 \\ \text{Subject to} & \\ 59.9y_1 + 27.65y_2 &\leq 16.43 \\ 38.5y_1 + 16.17y_2 &\leq 10 \\ y_1, y_2 &\geq 0. \end{aligned} \quad (9)$$

Solve the problem using Lingo we have $y_1 = 0$ $y_2 = 0.59$

$\Rightarrow x_1 = 0$ $x_2 = 3.28$ and the objective function value in the original problem is $z = 1.30$

$$z(x^{\sim n}) = \frac{3^{\sim n}(3.28)}{2^{\sim n}(3.28) + 1^{\sim n}} = (0.87, 1.30, 1.45; 0.77, 0.54, 0.38)$$

As shown in Table 1, the comparison between the proposed method and existing methods demonstrates its reliability and effectiveness.

Table 1
Comparison with other methods

Method	Max z
Ebrahimnejad & Tavana method [22]	1.25
Pop & Stancu-Minasian [23]	1.28
S. Das & Mandal [24]	1.28
Loganathan & Ganesan [25]	1.29
Stanojevic-Stancu Minasian method [26]	1.30
Proposed method	1.30

5. Conclusion

In this paper, a new efficient method is proposed for solving neutrosophic linear fractional programming (NLFP) problems in which the objective function coefficients, resources, and technological coefficients are represented by triangular neutrosophic numbers. The proposed approach transforms the NLFP problem into a parametric neutrosophic linear programming (NLP) problem, which is subsequently converted into a crisp linear programming (LP) problem using a special ranking function. The comparative results presented in Table 1 demonstrate that the proposed method is reliable and effective for solving NLFP problems. Future research will extend this framework to multi-objective neutrosophic linear fractional programming problems (MONLFPPs) and explore its practical applications.

Conflicts of Interest

The authors declare no conflicts of interest.

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