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Spherical Fuzzy Aczel–Alsina Aggregation Operators with AHP Approach for Artificial Intelligence in Smart Agriculture

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ABSTRACT

This paper investigates multi-attribute decision-making (MADM) problems within the framework of spherical fuzzy (SF) sets for the evaluation and selection of artificial intelligence (AI) technologies in smart agriculture. The increasing complexity and uncertainty associated with agricultural decision-making, driven by environmental variability, technological advancements, and sustainability requirements, necessitate robust and flexible decision-support methodologies. To address this challenge, the Aczel–Alsina (AA) t-norm and t-conorm are incorporated into the spherical fuzzy environment to extend existing aggregation mechanisms. Accordingly, two novel aggregation operators, namely the Spherical Fuzzy Aczel–Alsina Geometric (SFAAG) operator and the Spherical Fuzzy Aczel–Alsina Averaging (SFAAA) operator, are proposed, and their fundamental mathematical properties, including idempotency, monotonicity, boundedness, and stability, are thoroughly investigated. Furthermore, a MADM framework based on the proposed operators is developed to support the selection of AI-based smart agriculture technologies under spherical fuzzy information. To demonstrate the applicability of the proposed approach, a case study involving the evaluation of five AI technologies, namely smart irrigation systems, crop disease identification systems, agricultural drone surveillance, smart harvesting robots, and AI-based weather forecasting systems, is conducted using productivity improvement, cost reduction, environmental sustainability, and technical reliability as evaluation criteria. Comparative analyses with existing aggregation-based decision-making methods confirm the effectiveness, robustness, and reliability of the proposed framework in handling complex and uncertain decision-making environments. The results indicate that the proposed approach provides an efficient and reliable decision-support tool for the assessment and selection of AI technologies in smart agriculture.

1. Introduction

Artificial Intelligence (AI) is a transformative technology that is reshaping conventional agriculture into a smarter, more efficient, and sustainable farming system. By enhancing productivity, reducing

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operational costs, optimizing resource utilization, and supporting data-driven decision-making, AI has become a key enabler of modern agricultural development. However, the selection of appropriate AI technologies for smart agriculture is a complex multi-attribute decision-making (MADM) problem due to the presence of uncertainty, imprecise information, and rapidly evolving technological environments. Consequently, advanced fuzzy decision-making frameworks have attracted considerable attention because of their ability to effectively model vagueness and uncertainty in real-world decision processes. In this study, spherical fuzzy Aczel–Alsina aggregation operators integrated with the Analytic Hierarchy Process (AHP) are proposed to develop an efficient MADM framework for evaluating AI technologies in smart agriculture. The incorporation of the Aczel–Alsina operational laws into the spherical fuzzy environment enhances the flexibility, reliability, and accuracy of the decision-making process, while AHP is employed to systematically determine the relative importance of the evaluation criteria. The applicability and effectiveness of the proposed framework are demonstrated through a case study involving the assessment of several AI-based agricultural technologies, including smart irrigation systems, crop disease detection systems, agricultural drone monitoring, smart harvesting robots, and AI-based weather forecasting systems.

Uncertainty and incompleteness are unavoidable characteristics of many real-world decision-making problems. Classical set theory is often inadequate for representing imprecise and ambiguous information encountered in practical applications. To address this limitation, Zadeh [1] introduced the concept of fuzzy sets (FSs) in 1965, which has since become a fundamental mathematical tool for modeling uncertainty in science and engineering. Subsequently, Atanassov [2] proposed intuitionistic fuzzy sets (IFSs), characterized by membership and non-membership degrees satisfying the condition $\mu + \nu \leq 1$. Although IFSs provide a more comprehensive representation of uncertainty than traditional fuzzy sets, they may be insufficient in situations where the sum of membership and non-membership degrees exceeds unity. To overcome this limitation, Yager [3] introduced Pythagorean fuzzy sets (PyFSs), in which the squared sum of membership and non-membership degrees satisfies $\mu^2 + \nu^2 \leq 1$. Compared with IFSs, PyFSs offer greater flexibility and improved capability for handling uncertainty.

To further generalize this concept, Yager [4] proposed q-rung orthopair fuzzy sets (q-ROFSs), which satisfy the condition $\mu^q + \nu^q \leq 1$, thereby providing decision-makers with greater freedom in expressing uncertain information. Nevertheless, certain real-world problems involve not only support and opposition but also neutral or abstention opinions. To address this issue, Cuong [5] introduced picture fuzzy sets (PFSs), which incorporate positive, neutral, and negative membership degrees. Although PFSs enhance the representation of uncertainty, they remain constrained by predefined relationships among these degrees. To provide greater flexibility and reduce information loss, Mahmood *et al.,* [6] proposed spherical fuzzy sets (SFSs), which are characterized by the condition $\mu^2 + \eta^2 + \nu^2 \leq 1$, where μ, η, and ν denote membership, abstinence, and non-membership degrees, respectively. Owing to their ability to independently represent these three components, SFSs offer a more general and flexible framework for modeling complex uncertainty and have become increasingly popular in contemporary decision-making applications.

2. Literature Review

Imran *et al.,* [7] introduced Aczel–Alsina (AA) Bonferroni mean aggregation operators for interval-valued intuitionistic fuzzy information and demonstrated their applicability to group decision-making problems. Garg [8] developed Pythagorean fuzzy aggregation operators based on neutrality operations and applied them to multiple-attribute group decision-making. Sarkar *et al.,* [9] proposed dual hesitant q-rung orthopair fuzzy Frank power partitioned Heronian mean aggregation operators for evaluating sustainable urban transportation solutions. Gál *et al.,* [10] investigated the use of Hamacher t-norms in fuzzy rule-based model extraction and parameter optimization. Qin *et al.,* [11] introduced picture fuzzy Archimedean power Maclaurin symmetric mean operators for multiple-attribute decision-making environments. Wang *et al.,* [12] proposed power aggregation operators based on Aczel–Alsina t-norms and t-conorms for intuitionistic hesitant fuzzy information and applied them to logistics service provider selection. Furthermore, Sarfraz [13] developed Maclaurin symmetric mean aggregation operators for spherical fuzzy numbers based on Schweizer–Sklar operations and demonstrated their applicability in artificial intelligence applications.

Subsequently, Chien *et al.,* [14] explored the integration of Aczel–Alsina operators with Grey DEMATEL–ISM techniques for analyzing industrial transformation challenges. Debnath *et al.,* [15] proposed an integrated MADM framework based on an extended MABAC method and an Aczel–Alsina generalized weighted Bonferroni mean operator. Liu *et al.,* [16] extended Aczel–Alsina aggregation operators to complex intuitionistic fuzzy environments and demonstrated their effectiveness in decision-making applications. Alhulwah *et al.,* [17] proposed prioritized aggregation operators for complex spherical fuzzy information and applied them to mobile e-tourism decision problems. Kara *et al.,* [18] developed Aczel–Alsina-based aggregation operators for type-2 neutrosophic numbers within multiple-attribute decision-making contexts. Mahmood *et al.,* [19] introduced geometric aggregation operators based on Aczel–Alsina t-norms and t-conorms in bipolar complex fuzzy environments and applied them to operating system selection problems. Moreover, Yalçın *et al.,* [20] developed a spherical fuzzy decision-support framework for evaluating sustainability-oriented activist advertising campaigns.

Beyond methodological developments, fuzzy decision-making approaches have been successfully applied in a variety of domains. Rocco and Colombo [21] employed input–output analysis to evaluate energy embodied in international trade activities. Suh [22] investigated physical input–output analysis for assessing land appropriation associated with international trade. Thiermann [23] discussed the impact of globalization and international trade on animal health management. Luo and Yuan [24] analyzed temporal patterns of multilateral spatial interactions using international trade data. Jan *et al.,* [25] addressed international trade decision problems under a complex cubic fuzzy environment. Genç *et al.,* [26] applied fuzzy social and economic network models to international trade analysis, whereas Wu *et al.,* [27] developed fuzzy-system-based approaches for stock trading evaluation. Makhazhanova *et al.,* [28] proposed a fuzzy logic framework for assessing the creditworthiness of trade and service enterprises. These studies collectively demonstrate the versatility and effectiveness of fuzzy methodologies in addressing complex decision-making problems under uncertainty.

Despite the significant progress achieved in the development of Aczel–Alsina-based aggregation operators and spherical fuzzy decision-making frameworks, the integration of Aczel–Alsina operational laws within the spherical fuzzy environment for evaluating artificial intelligence technologies in smart agriculture remains largely unexplored. Furthermore, existing studies have paid limited attention to combining spherical fuzzy Aczel–Alsina aggregation mechanisms with systematic weighting procedures such as the Analytic Hierarchy Process (AHP). Motivated by these research gaps, this study develops novel spherical fuzzy Aczel–Alsina aggregation operators and an integrated MADM framework for the assessment and selection of AI technologies in smart agriculture.

This study makes several important contributions to the development of spherical fuzzy decision-making methodologies and their application in smart agriculture. First, novel Spherical Fuzzy Aczel–Alsina Averaging (SFAAA) and Spherical Fuzzy Aczel–Alsina Geometric (SFAAG) aggregation operators are developed by incorporating Aczel–Alsina t-norms and t-conorms into the spherical fuzzy environment, thereby providing a more flexible and effective mechanism for aggregating uncertain information. Second, the fundamental mathematical properties of the proposed operators, including idempotency, monotonicity, boundedness, and stability, are rigorously established and analyzed to ensure their theoretical validity and applicability.

Furthermore, an integrated spherical fuzzy multi-attribute decision-making (MADM) framework is developed by combining the proposed aggregation operators with the Analytic Hierarchy Process (AHP) for determining the relative importance of evaluation criteria. The practical applicability of the proposed framework is demonstrated through a case study involving the assessment and selection of artificial intelligence technologies in smart agriculture, including smart irrigation systems, crop disease identification systems, agricultural drone surveillance, smart harvesting robots, and AI-based weather forecasting systems. Finally, comparative and sensitivity analyses are conducted to evaluate the performance of the proposed approach and to demonstrate its effectiveness, robustness, reliability, and superiority over several existing aggregation-based decision-making methods.

The remainder of this paper is organized as follows. Section 2 presents the fundamental concepts related to spherical fuzzy sets, Aczel–Alsina t-norms and t-conorms, and existing aggregation operators. Section 3 introduces the proposed Spherical Fuzzy Aczel–Alsina Averaging (SFAAA) and Spherical Fuzzy Aczel–Alsina Geometric (SFAAG) operators and investigates their mathematical properties. Section 4 develops the proposed MADM framework by integrating the aggregation operators with the Analytic Hierarchy Process (AHP). Section 5 presents a case study on the evaluation of artificial intelligence technologies in smart agriculture to demonstrate the applicability of the proposed approach. Section 6 provides comparative and sensitivity analyses to validate the effectiveness and robustness of the proposed framework. Finally, Section 7 concludes the paper and outlines potential directions for future research.

2. Preliminaries

This section presents the fundamental concepts and preliminaries necessary for the development of the proposed methodology. These concepts provide the theoretical foundation for understanding the proposed framework and its subsequent applications. In particular, the definitions and basic properties of spherical fuzzy sets (SFSs), Aczel–Alsina (AA) operational laws, including the Aczel–Alsina t-norm (TN) and t-conorm (TCN), are briefly reviewed.

Definition 1 [2]: Assuming that F is a universe of discourse, an SF in F is an expression ϱ that is provided by

$$\Xi = \{(\mathbb{x}, \mathcal{T}_{\Xi}(\mathbb{x}), \mathcal{J}_{\Xi}(\mathbb{x}), \mathcal{L}_{\Xi}(\mathbb{x}) : \mathbb{x} \in F)\}$$

Where the elements $\mathcal{T}_{\Xi}(\mathbb{x})$, $\mathcal{J}_{\Xi}(\mathbb{x})$ and $\mathcal{L}_{\Xi}(\mathbb{x})$are MD, AD and NMD function such that $\mathcal{T}_{\Xi}: F \to [0,1]$, $\mathcal{J}_{\Xi}: F \to [0,1]$ and $\mathcal{L}_{\Xi}: F \to [0,1]$ with $0 \leq \mathcal{T}_{\Xi}^2(\mathbb{x}) + \mathcal{J}_{\Xi}^2(\mathbb{x}) + \mathcal{L}_{\Xi}^2(\mathbb{x}) \leq 1$, and$\pi_{\Xi}(\mathbb{x}) = 1 - \mathcal{T}_{\Xi}^2(\mathbb{x}) - \mathcal{J}_{\Xi}^2(\mathbb{x}) - \mathcal{L}_{\Xi}^2(\mathbb{x}), \forall\, \mathbb{x} \in F$ is called hesitancy degree (HD) of $\mathbb{x}$ to Ξ. Further,$\left(\mathbb{x}, \mathcal{T}_{\Xi}^2(\mathbb{x}), \mathcal{J}_{\Xi}^2(\mathbb{x}), \mathcal{L}_{\Xi}^2(\mathbb{x})\right)$ are recognized by Spherical fuzzy value (SFV).

Definition 2 [28]: Assume $\Xi_1 = \left(\mathcal{T}_{\Xi_1}, \mathcal{J}_{\Xi_1}, \mathcal{L}_{\Xi_1}\right)$ be an SFV. Then

$$Sco(\Xi_1) = \frac{\left(1 + \mathcal{T}_{\Xi_1}^2 - \mathcal{J}_{\Xi_1}^2 - \mathcal{L}_{\Xi_1}^2\right)}{2} \quad (1)$$

Be the score value of Ξ_1.

Definition 3 [28]: Assume $\Xi_1 = \left(\mathcal{T}_{\Xi_1}, \mathcal{J}_{\Xi_1}, \mathcal{L}_{\Xi_1}\right)$ be an SFV. Then

$$Acc\ (\Xi_1) = \frac{\left(1 + \mathcal{T}_{\Xi_1}^2 + \mathcal{J}_{\Xi_1}^2 + \mathcal{L}_{\Xi_1}^2\right)}{2} \quad (2)$$

Be the degrees of accuracy of Ξ_1.

i. If $Sco(\Xi_1) < Sco(\Xi_2)$, then $\Xi_1$has less partiality than Ξ_2.
ii. If $Sco(\Xi_1) = Sco(\Xi_2)$, then $\Xi_1$and $\Xi_2$are the same.
iii. If $Acc\ (\Xi_1) < Acc\ (\Xi_2)$, then $\Xi_1$has less partiality than Ξ_2.
iv. If $Acc\ (\Xi_1) = Acc\ (\Xi_2)$, then $\Xi_1$and $\Xi_2$are the same.

Definition 4[29]: A function $\mathcal{L}$: $[0,1]^2 \longrightarrow [0,1]$ is called TN if for all $\mathcal{T}_1, \mathcal{J}_2, \mathcal{L}_3 \in [0,1]$, it satisfies the following axioms:

i. $\mathcal{L}(\mathcal{T}_{1,}\mathcal{J}_{2,}) = \mathcal{L}(\mathcal{J}_2, \mathcal{T}_1)$;

ii. $\mathcal{L}\left(\mathcal{T}_{1,} \perp (\mathcal{J}_{2,}\mathcal{L}_3)\right) = \mathcal{L}(\mathcal{L}(\mathcal{T}_{1,}\mathcal{J}_{2,}), \mathcal{L}_3)$;

iii. $\mathcal{L}(\mathcal{T}_{1,}\mathcal{J}_{2,}) \leq \mathcal{L}(\mathcal{T}_1, \mathcal{L}_3)\ if \leq \mathcal{J}_2 \leq \mathcal{L}_{3,}$

iv. $\mathcal{L}(\mathcal{T}_{1,}0) = \mathcal{U}_1$;

For all $\mathcal{T}_1, \mathcal{J}_2, \mathcal{L}_3 \in [0,1]$.

The following are examples of TCNs:

i. Minimum TCN: $\mathcal{L}(\mathcal{T}_{1,}\mathcal{J}_{2,}) = min(\mathcal{T}_1, \mathcal{J}_2)$

ii. Product TCN: $\mathcal{L}_p(\mathcal{T}_{1,}\mathcal{J}_{2,}) = \mathcal{T}_1\mathcal{J}_2$;

iii. Lukasiewicz TCN: $\mathcal{L}_*(\mathcal{T}_{1,}\mathcal{J}_{2,}) = max\ (\mathcal{J}_2 + \mathcal{T}_1, 1)$

Drastic TCN: $\mathcal{L}_D(\mathcal{T}_{1,}\mathcal{J}_{2,}) = \begin{cases} \mathcal{T}_{1,} & if\ \ \mathcal{J}_2 = 1 \\ \mathcal{J}_2 & if\ \ \mathcal{T}_1 = 1 \\ 0 & otherwise \end{cases}$

Definition 5 [30]: A TCN is a map S: $[0,1]^2 \longrightarrow [0,1]$ if it is satisfying the following features:

i. $S(\mathcal{T}_{1,}\mathcal{J}_{2,}) = S(v_2, \mathcal{T}_1)$;

ii. $S\left(\mathcal{T}_{1,} \perp (\mathcal{J}_{2,}\mathcal{L}_3)\right) = S(S(\mathcal{T}_{1,}\mathcal{J}_{2,}), \mathcal{L}_3)$;

iii. $S(\mathcal{T}_{1,}\mathcal{J}_{2,}) \leq S(\mathcal{T}_1, \mathcal{L}_3)\ if \leq \mathcal{J}_2 \leq \mathcal{L}_{3,}$

iv. $S(\mathcal{T}_{1,}0) = \mathcal{T}_1$;

For all $\mathcal{T}_1, \mathcal{J}_2, \mathcal{L}_3 \in [0,1]$

The following are examples of TCNs:

i. Maximum TCN: $S(\mathcal{T}_{1,}\mathcal{J}_{2,}) = max(\mathcal{J}_2, \mathcal{T}_1)$

ii. Probabilistic TCN: $S_p(\mathcal{T}_{1,}\mathcal{J}_{2,}) = (\mathcal{J}_2 + \mathcal{U}_1 - \mathcal{J}_2\mathcal{T}_1)$

iii. Lukasiewicz TCN: $S_*(\ell_{1,}\mathcal{J}_{2,}) = min\ (\mathcal{J}_2 + \mathcal{T}_1, 1)$

Drastic TCN: $S_D(\mathcal{T}_{1,}\mathcal{J}_{2,}) = \begin{cases} \mathcal{T}_{1,} & if\ \ \mathcal{J}_2 = 0 \\ \mathcal{J}_2 & if\ \ \mathcal{T}_1 = 0 \\ 1 & otherwise \end{cases}$

Definition 6 [30]: T-norms $(\mathcal{L}_A^{\Delta})_{\Delta\in[0,\infty]}$ for Aczel-Alsina, are defined as

$$(\mathcal{L}_A^{\Delta})_{(\ell,v)} = \begin{cases} \mathcal{L}_D(\ell, v) & \iota f\ \ \Delta = 0 \\ min(\ell, v) & \iota f\ \ \Delta = \infty \\ exp^{-((-ln)^{\Delta}+(-ln)^{\Delta})^{\frac{1}{\Delta}}} & otherwise \end{cases}$$

The t-conorms $(S_A^{\Delta})_{\Delta\in[0,\infty]}$ for Aczel-Alsina, is defined as

$$(S_A^{\Delta})_{(\ell,v)} = \begin{cases} S_D(\ell, v) & \iota f\ \ \Delta = 0 \\ max(\ell, v) & \iota f\ \ \Delta = \infty \\ 1 - exp^{-\left((-ln(1-\ell))^{\Delta}+(-ln(1-v))^{\Delta}\right)^{\frac{1}{\Delta}}} & otherwise \end{cases}$$

Where $\Delta \in [0, \infty]$.

3. Prioritized Aczel-Alsina Averaging Operators on SF

Definition 7: Assume $\Xi = (\mathcal{T}_{\Xi}, \mathcal{J}_{\Xi}, \mathcal{L}_{\Xi})$ $\Xi_1 = (\mathcal{T}_{\Xi_1}, \mathcal{J}_{\Xi_1}, \mathcal{L}_{\Xi_1})$ and $\Xi_2 = (\mathcal{T}_{\Xi_2}, \mathcal{J}_{\Xi_2}, \mathcal{L}_{\Xi_2})$ be three SFVs, $\Delta \geq 1$ and $\mathcal{R} \geq 0$. Next, the following are the definitions of the Aczel-Alsina TN and TCN operations of SFVs:

(i) $\Xi_1 \oplus \Xi_2 = \left(\begin{array}{c} \sqrt[2]{1 - exp^{-\left(\left(-ln\left(1-\mathcal{T}^2_{\Xi_1}\right)\right)^{\Delta} + \left(-ln\left(1-\mathcal{T}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \\ exp^{-\left(\left(-ln\left(\mathcal{J}^2_{\Xi_1}\right)\right)^{\Delta} + \left(-ln\left(\mathcal{J}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\left(-ln\left(\mathcal{L}^2_{\Xi_1}\right)\right)^{\Delta} + \left(-ln\left(\mathcal{L}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}} \end{array} \right),$

(ii) $\Xi_1 \otimes \Xi_2 =$

$$\left(\begin{array}{c} exp^{-\left(\left(-ln\left(\mathcal{T}^2_{\Xi_1}\right)\right)^{\Delta} + \left(-ln\left(\mathcal{T}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \sqrt[2]{1 - exp^{-\left(\left(-ln\left(1-\mathcal{T}^2_{\Xi_1}\right)\right)^{\Delta} + \left(-ln\left(1-\mathcal{J}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \sqrt[2]{1 - exp^{-\left(\left(-ln\left(1-\mathcal{L}^2_{\Xi_1}\right)\right)^{\Delta} + \left(-ln\left(1-\mathcal{L}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}} \end{array} \right),$$

(iii) $\mathcal{R}\Xi = \left(\sqrt[2]{1 - exp^{-\left(\mathcal{R}\left(-ln\left(1-\mathcal{T}^2_{\Xi}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}},\ exp^{-\left(\mathcal{R}\left(-ln\left(\mathcal{J}^2_{\Xi}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\mathcal{R}\left(-ln\left(\mathcal{L}^2_{\Xi}\right)\right)^{\Delta}\right)^{1/\Delta}} \right),$

(iv) $\Xi^{\mathcal{R}} =$

$$\left(exp^{-\left(\mathcal{R}\left(-ln\left(\mathcal{T}^2_{\Xi}\right)\right)^{\Delta}\right)^{1/\Delta}}, \sqrt[2]{1 - exp^{-\left(\mathcal{R}\left(-ln\left(1-\mathcal{J}^2_{\Xi}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \sqrt[2]{1 - exp^{-\left(\mathcal{R}\left(-ln\left(1-\mathcal{L}^2_{\Xi}\right)\right)^{\Delta}\right)^{1/\Delta}}} \right),$$

Theorem 1: Assume $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ be a set of SFVs. Then the aggregated value of Ξ_τ by developing the SFPAAA operator is also an SFV given by:

$$SFPAAA(\Xi_1, \Xi_2, \dots \Xi_\vartheta) = \left(\begin{array}{c} \sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}^2_{\Xi_\tau}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \\ exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}^2_{\Xi_\tau}\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}^2_{\Xi_\tau}\right)\right)^{\Delta}\right)^{1/\Delta}} \end{array} \right) \quad (3)$$

Proof:

The mathematical induction method can be used to show Theorem 1 in the following manner.

(I) Using SFVs' Aczel-Alsina operations, we obtain for $\vartheta = 2$,

$$\omega_1\Xi_1 = \left(\sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{T}^2_{\Xi_1}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, exp^{-\left(\omega_1\left(-ln\left(\mathcal{J}^2_{\Xi_1}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\omega_1\left(-ln\left(\mathcal{L}^2_{\Xi_1}\right)\right)^{\Delta}\right)^{1/\Delta}} \right)$$

$$\omega_2\Xi_2 = \left(\sqrt[2]{1 - exp^{-\left(\omega_2\left(-ln\left(1-\mathcal{T}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, exp^{-\left(\omega_2\left(-ln\left(\mathcal{J}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\omega_2\left(-ln\left(\mathcal{L}^2_{\Xi_2}\right)\right)^{\Delta}\right)^{1/\Delta}} \right)$$

$$SFPAAA(\Xi_1, \Xi_2) = \omega_1\Xi_1 \oplus \omega_2\Xi_2$$

$$= \left(\begin{array}{c} \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{T}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \\ exp^{-\left(\omega_1\left(-ln\left(\mathcal{J}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\omega_1\left(-ln\left(\mathcal{L}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \end{array} \right)$$

$$\oplus \left(\begin{array}{c} \sqrt[2]{1 - exp^{-\left(\omega_2\left(-ln\left(1-\mathcal{T}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \\ exp^{-\left(\omega_2\left(-ln\left(\mathcal{J}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\omega_2\left(-ln\left(\mathcal{L}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \end{array} \right)$$

$$= \left(\sqrt[2]{1 - exp^{-\left(\begin{array}{c}\omega_1\left(-ln\left(1-\mathcal{T}_{\Xi_1}^2\right)\right)^{\Delta} + \\ \omega_2\left(-ln\left(1-\mathcal{T}_{\Xi_2}^2\right)\right)^{\Delta}\end{array}\right)^{1/\Delta}}}, exp^{-\left(\begin{array}{c}\omega_1\left(-ln\left(\mathcal{J}_{\Xi_1}^2\right)\right)^{\Delta} + \\ \omega_2\left(-ln\left(\mathcal{J}_{\Xi_2}^2\right)\right)^{\Delta}\end{array}\right)^{1/\Delta}}, exp^{-\left(\begin{array}{c}\omega_1\left(-ln\left(\mathcal{L}_{\Xi_1}^2\right)\right)^{\Delta} + \\ \omega_2\left(-ln\left(\mathcal{L}_{\Xi_2}^2\right)\right)^{\Delta}\end{array}\right)^{1/\Delta}} \right)$$

$$= \left(\sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \right)$$

Advanced, Eq. (3) is correct for $\vartheta = 2$.

(II) Take that Eq. (3) is correct for $\vartheta = k$, formerly

$$SFPAAA(\Xi_1, \Xi_2, \dots \Xi_k) = \oplus_{\tau=1}^{k} \frac{\mathcal{L}_\tau}{\sum_{\tau=1}^{k} \mathcal{L}_\tau} (\Xi_\tau)$$

$$= \left(\sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{k} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, exp^{-\left(\sum_{\tau=1}^{k} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, exp^{-\left(\sum_{\tau=1}^{k} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \right)$$

Currently for $\vartheta = k + 1$, we develop

$$SFPAAA(\Xi_1, \Xi_2, \dots \Xi_{k+1}) = \oplus_{\tau=1}^{k} \omega_\tau(\Xi_\tau) \oplus \omega_{k+1}(\Xi_{k+1})$$

$$= \left(\left(\sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{k} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}}, exp^{-\left(\sum_{\tau=1}^{k} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\sum_{\tau=1}^{k} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \right) \right.$$

$$\left. \oplus \left(\sqrt[2]{1 - exp^{-\left(\omega_{k+1}\left(-ln\left(1-\mathcal{T}_{\Xi_{k+1}}^2\right)\right)^{\Delta}\right)^{1/\Delta}}}, exp^{-\left(\omega_{k+1}\left(-ln\left(1-\mathcal{J}_{\Xi_{k+1}}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\omega_{k+1}\left(-ln\left(1-\mathcal{L}_{\Xi_{k+1}}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \right) \right)$$

$$= \left(\sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{k+1} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}}, exp^{-\left(\sum_{\tau=1}^{k+1} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\sum_{\tau=1}^{k+1} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \right)$$

Thus, Eq. (3) is appropriate for $= k + 1$. As a consequence of forms (I) and (II), we conclude that Eq. (3) holds for any value of ϑ. The fundamental properties of the proposed SFPAAA operator are presented in the following theorems. In particular, Theorems 2–4 establish its essential characteristics.

Theorem 2: If all $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right) = (\mathcal{T}_\Xi, \mathcal{J}_\Xi, \mathcal{L}_\Xi) = \Xi$, that is $\Xi_\tau = \Xi$ for all τ. Then $SFPAAA(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) = \Xi$.

Proof:

Subsequently $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right) = (\mathcal{T}_\Xi, \mathcal{J}_\Xi, \mathcal{L}_\Xi) = \Xi$. Afterward, by Eq. (3) we get

$$SFPAAA(\Xi_1, \Xi_2, \dots \Xi_\vartheta) = \left(\begin{array}{c} 1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \end{array} \right)$$

$$= \left(1 - exp^{-\left(\left(-ln\left(1-\mathcal{T}_{\Xi}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\left(-ln\left(\mathcal{J}_{\Xi}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\left(-ln\left(\mathcal{L}_{\Xi}^2\right)\right)^{\Delta}\right)^{1/\Delta}} \right)$$

$$= \left(1 - exp^{-ln\left(1-\mathcal{T}_\Xi^2\right)}, exp^{ln\mathcal{J}_\Xi^2}, exp^{l2\mathcal{L}_\Xi^2}\right) = (\mathcal{T}_\Xi, \mathcal{J}_\Xi, \mathcal{L}_\Xi) = \Xi$$

Thus $SFPAAA\left(\Xi_1, \Xi_2, \dots, \Xi_\varrho\right) = \Xi$ holds.

Theorem 3: Assume $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ be a Set of T-SFVs. Assume $\Xi^- = min(\Xi_1, \Xi_2, \dots, \Xi_\vartheta)$ and $\Xi^+ = max(\Xi_1, \Xi_2, \dots, \Xi_\vartheta)$. Then $\Xi^- \le SFPAAA(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) \le \Xi^+$.

Proof:

Assume $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$be a Set of SFVs. Consider $\Xi^- = min(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) = (\mathcal{T}_\Xi^-, \mathcal{J}_\Xi^-, \mathcal{L}_\Xi^-)$ and $\Xi^+ = max(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) = (\mathcal{T}_\Xi^+, \mathcal{J}_\Xi^+, \mathcal{L}_\Xi^+)$. Thus, the following inequalities exist:

$$1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}_\Xi^{2-}\right)\right)^{\Delta}\right)^{1/\Delta}} \le 1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}$$

$$\le 1 - exp^{-\left(\sum_{\tau=1} \omega_\tau\left(-ln\left(1-\mathcal{T}_\Xi^{2+}\right)\right)^{\Delta}\right)^{1/\Delta}}$$

$$exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^{2+}\right)\right)^{\Delta}\right)^{1/\Delta}} \ge exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_\Xi^2\right)\right)^{\Delta}\right)^{1/\Delta}} \ge exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^{2-}\right)\right)^{\Delta}\right)^{1/\Delta}}$$

$$exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^{2+}\right)\right)^{\Delta}\right)^{1/\Delta}} \ge exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_\Xi^2\right)\right)^{\Delta}\right)^{1/\Delta}} \ge exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^{2-}\right)\right)^{\Delta}\right)^{1/\Delta}}$$

Therefore $\Xi^- \le SFPAAA(\Xi_1, \Xi_2, \dots, \Xi_\vartheta \le \Xi^+)$.

Theorem 4: Assume Ξ_τ and Ξ_τ' are two sets of SFVs. If $\Xi_\tau \le \Xi_\tau'$ for all τ. Then

$$SFPAAA(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) \le SFPAAA(\Xi_1', \Xi_2', \dots, \Xi_\vartheta')$$

Proof:

Simple and direct.

Definition 8: Assume that $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ is a set of SFVs, and let $SFPAAG: L^{*\vartheta} \to L^*$ if:

$$SFPAAG(\Xi_1, \Xi_2, \dots \Xi_\vartheta) = \otimes_{\tau=1}^{\vartheta} \Xi_\tau^{\omega_\tau}$$

Next, the SFPAAG operator is the function that is called SFPAAG. Where $\mathcal{L}_{\tau=} \prod_{k=1}^{\tau-1} S\ (\Xi_k)\ (\tau = 2, \dots \vartheta)$, $\mathcal{L}_1 = 1$ and $S\ (\Xi_k)$ is the score of SFVs Ξ_k.

Theorem 5: Assume that $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ is a set of SFVs. SFVs are then also obtained by applying the SFPAAG operator to the aggregated value.

$$SFPAAG(\Xi_1, \Xi_2, \ldots \Xi_\vartheta) = \left(\begin{array}{c} exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau \left(-ln\left(\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau \left(-ln\left(1-\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \\ \sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau \left(-ln\left(1-\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \end{array} \right) \quad (4)$$

Proof:
Using the mathematical induction method, the following is how Theorem 5 can be shown:
Using SFVs' Aczel-Alsina operations, we obtain for $\vartheta = 2$.

$$\Xi_1^{\omega_1}$$

$$= \left(exp^{-\left(\omega_1\left(-ln\left(\mathcal{T}_{\Psi_1}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{J}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{L}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \right)$$

$$\Xi_2^{\omega_2}$$

$$= \left(exp^{-\left(\omega_2\left(-ln\left(\mathcal{T}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}}, \sqrt[2]{1 - exp^{-\left(\omega_2\left(-ln\left(1-\mathcal{J}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}}}, \sqrt[2]{1 - exp^{-\left(\omega_2\left(-ln\left(1-\mathcal{L}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \right)$$

$$SFPAAG(\Xi_1, \Xi_2) = \Xi_1^{\omega_1} \otimes \Xi_2^{\omega_2}$$

$$= \left(\begin{array}{c} exp^{-\left(\omega_1\left(-ln\left(\mathcal{T}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{J}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{1/\Delta}}}, \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{L}_{\Xi_1}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \end{array} \right)$$

$$\otimes \left(\begin{array}{c} exp^{-\left(\omega_2\left(-ln\left(\mathcal{T}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \sqrt[2]{1 - exp^{-\left(\omega_2\left(-ln\left(1-\mathcal{J}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}}}, \sqrt[2]{1 - exp^{-\left(\omega_2\left(-ln\left(1-\mathcal{L}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \end{array} \right)$$

$$= \left(\begin{array}{c} , exp^{-\left(\omega_1\left(-ln\left(\mathcal{T}_{\Xi_1}^2\right)\right)^{\Delta} + \omega_2\left(-ln\left(\mathcal{T}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{J}_{\Xi_1}^2\right)\right)^{\Delta} + \omega_2\left(-ln\left(1-\mathcal{J}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \sqrt[2]{1 - exp^{-\left(\omega_1\left(-ln\left(1-\mathcal{L}_{\Xi_1}^2\right)\right)^{\Delta} + \omega_2\left(-ln\left(1-\mathcal{L}_{\Xi_2}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \end{array} \right)$$

$$= \left(exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau \left(-ln\left(\mathcal{T}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau \left(-ln\left(1-\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}}, \sqrt[2]{1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau \left(-ln\left(1-\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\Delta}\right)^{1/\Delta}}} \right)$$

Therefore, for $\vartheta = 2$, Eq. (4) is true.

(I) If $\vartheta = k$ and Eq. (4) is correct, then we have

$$SFPAAG(\Xi_1,\Xi_2,\dots\Xi_k)=\otimes_{\tau=1}^{k}\Xi_\tau^{\;\omega_\tau}$$

$$=\left(\begin{array}{c} exp^{-\left(\sum_{\tau=1}^{k}\omega_\tau\left(-ln(\mathcal{T}_{\Xi_\tau}^2)\right)^{\varDelta}\right)^{\frac{1}{\varDelta}}}, \\ \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{k}\omega_\tau\left(-ln\left(1-\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{\frac{1}{\varDelta}}}}, \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{k}\omega_\tau\left(-ln\left(1-\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}} \end{array}\right)$$

For $\vartheta = k+1$, we now obtain

$$SFPAAG(\Xi_1,\Xi_2,\dots\Xi_{k+1})=\otimes_{\tau=1}^{k}\Xi_\tau^{\;\omega_\tau}\otimes\Xi_{\mathrm{k+1}}^{\;\omega_{\mathrm{k+1}}}$$

$$\left(\begin{array}{c} exp^{-\left(\sum_{\tau=1}^{k}\omega_\tau\left(-ln(\mathcal{T}_{\Xi_\tau}^2)\right)^{\varDelta}\right)^{\frac{1}{\varDelta}}}, \\ \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{k}\omega_\tau\left(-ln\left(1-\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{\frac{1}{\varDelta}}}}, \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{k}\omega_\tau\left(-ln\left(1-\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{\frac{1}{\varDelta}}}} \end{array}\right)$$

$$\otimes\left(\begin{array}{c} exp^{-\left(\omega_{\mathrm{k+1}}\left(-ln\left(1-\mathcal{T}_{\Xi_{\mathrm{k+1}}}^2\right)\right)^{\varDelta}\right)^{\frac{1}{\varDelta}}}, \\ \sqrt[2]{1-exp^{-\left(\omega_{\mathrm{k+1}}\left(-ln\left(1-\mathcal{J}_{\Xi_{\mathrm{k+1}}}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}, \sqrt[2]{1-exp^{-\left(\omega_{\mathrm{k+1}}\left(-ln\left(1-\mathcal{L}_{\Xi_{\mathrm{k+1}}}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}} \end{array}\right)$$

$$\left(exp^{-\left(\sum_{\tau=1}^{k+1}\omega_\tau\left(-ln(\mathcal{T}_{\Xi_\tau}^2)\right)^{\varDelta}\right)^{1/\varDelta}}, \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{k+1}\omega_\tau\left(-ln\left(1-\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}, \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{k+1}\omega_\tau\left(-ln\left(1-\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}\right)$$

Hence, Eq. (4) holds for $\vartheta = k+1$. Therefore, by mathematical induction, Eq. (4) is valid for any positive integer value of ϑ. Based on the established result, the proposed SFPAAG operator satisfies several fundamental properties. These properties are formally presented and proved in Theorems 6-10.

Theorem 6: Assume that $\Xi_\tau=\left(\mathcal{T}_{\Xi_\tau},\mathcal{J}_{\Xi_\tau},\mathcal{L}_{\Xi_\tau}\right)$ are spherical fuzzy values (SFVs). Where $\mathcal{L}_{\tau=}\prod_{k=1}^{\tau-1}S\ (\Xi_k)\ (\tau=2,\dots\vartheta)$, $\mathcal{L}_1=1$ and $S\ (\Xi_k)$ is the SF $S\ (\Xi_k)$) score. If for every τ, $\Xi_\tau=\Xi$. So $SFPAAG(\Xi_1,\Xi_2,\dots,\Xi_\vartheta)=\ \Xi$.

Proof:

Since $\Xi_\tau=\left(\mathcal{T}_{\Xi_\tau},\mathcal{J}_{\Xi_\tau},\mathcal{L}_{\Xi_\tau}\right)=(\mathcal{T}_\Xi,\mathcal{J}_\Xi,\mathcal{L}_\Xi)=\Xi$. Next, using Equation (4), we have:

$SFPAAG(\Xi_1,\Xi_2,\dots\Xi_\vartheta)$

$$=\left(exp^{-\left(\sum_{\tau=1}^{\vartheta}\omega_\tau\left(-ln(\mathcal{T}_{\Xi_\tau}^2)\right)^{\varDelta}\right)^{1/\varDelta}}, \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{\vartheta}\omega_\tau\left(-ln\left(1-\mathcal{J}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}, \sqrt[2]{1-exp^{-\left(\sum_{\tau=1}^{\vartheta}\omega_\tau\left(-ln\left(1-\mathcal{L}_{\Xi_\tau}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}\right)$$

$$=\left(exp^{-\left(\left(-ln(\mathcal{T}_{\Xi}^2)\right)^{\varDelta}\right)^{1/\varDelta}}, \sqrt[2]{1-exp^{-\left(\left(-ln\left(1-\mathcal{J}_{\Xi}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}, \sqrt[2]{1-exp^{-\left(\left(-ln\left(1-\mathcal{L}_{\Xi}^2\right)\right)^{\varDelta}\right)^{1/\varDelta}}}\right)$$

$$=1-exp^{-ln\left(1-\mathcal{T}_{\Xi}^2\right)}, exp^{ln\mathcal{J}_{\Xi}^2}, exp^{ln\mathcal{L}_{\Xi}^2}=(\mathcal{T}_{\Xi}^2,\mathcal{J}_{\Xi}^2\mathcal{L}_{\Xi}^2)=\Xi$$

Hence $SFPAAG\left(\Xi_1,\Xi_2,\dots,\Xi_\varrho\right)=\Xi$ holds.

Theorem 7: Suppose that $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ is a set of SFVs. Assume $\Xi^- = min(\Xi_1, \Xi_2, \dots, \Xi_\vartheta)$ be equal to $\Xi^+ = max(\Xi_1, \beta_2, \dots, \Xi_\vartheta)$. Consequently $\Xi^- \leq SFPAAG(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) \leq \Xi^+$.

Proof:

Assume that $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ represents a set of SFVs. Assume that $\Xi^+ = max(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) = \left(\mathcal{T}_{\Xi}^{2+}, \mathcal{J}_{\Xi_\tau}^{2+}, \mathcal{L}_{\Xi_\tau}^{2+}\right)$ and $\Xi^- = min(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) = \left(\mathcal{T}_{\Xi}^{2-}, \mathcal{J}_{\Xi_\tau}^{2-}, \mathcal{L}_{\Xi_\tau}^{2+}\right)$. Then the following inequalities exist:

$$1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi}^{2-}\right)\right)^{\Delta}\right)^{1/\Delta}} \geq 1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi_\tau}^{2}\right)\right)^{\Delta}\right)^{1/\Delta}}$$
$$\geq 1 - exp^{-\left(\sum_{\tau=1} \omega_\tau\left(-ln\left(1-\mathcal{T}_{\Xi}^{2+}\right)\right)^{\Delta}\right)^{1/\Delta}}$$
$$exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^{2+}\right)\right)^{\Delta}\right)^{1/\Delta}} \leq exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi}^{2}\right)\right)^{\Delta}\right)^{1/\Delta}} \leq exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{J}_{\Xi_\tau}^{2-}\right)\right)^{\Delta}\right)^{1/\Delta}}$$
$$exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^{2+}\right)\right)^{\Delta}\right)^{1/\Delta}} \leq exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi}^{2}\right)\right)^{\Delta}\right)^{1/\Delta}} \leq exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau\left(-ln\left(\mathcal{L}_{\Xi_\tau}^{2-}\right)\right)^{\Delta}\right)^{1/\Delta}}$$

Hence, $\Xi^- \leq SFPAAG(\Xi_1, \Xi_2, \dots, \Xi_\vartheta \leq \Xi^+)$.

Theorem 8: Assume that $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ represent a set of SFVs. Where $\mathcal{L}_{\tau=} \prod_{k=1}^{\tau-1} S\,(\Xi_k)\,(\tau = 2, \dots \vartheta)$, When $\mathcal{L}_1 = 1$ and $S\,(\Xi_k)$ is the score of SFVs(Ξ_k), we have $S\,(\Xi_k)$. If α is SFVS on k and $\alpha = (\mathcal{T}_\alpha, \mathcal{J}_\alpha)$.

$$SFPAAG(\Xi_1 \otimes \alpha, \Xi_2 \otimes \alpha, \dots, \Xi_\vartheta \otimes \alpha) = SFPAAG(\Xi_1, \Xi_2, \dots, \Xi_\vartheta) \otimes \alpha$$

Proof:

The proof of Theorem 2 can also be applied to Theorem 3.8.

Theorem 9: Assume $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$ be a collection of SFVs. Where $\mathcal{L}_{\tau=} \prod_{k=1}^{\tau-1} S\,(\Xi_k)\,(\tau = 2, \dots \vartheta)$, $\mathcal{L}1 = 1$ and $S\,(\Xi_k)$ is the score of SFVs(Ξ_k). If $r > 0$, then:

$$SFPAAG(\Xi_1^r, \Xi_2^r, \dots, \Xi_\vartheta^r) = SFPAAG(\Xi_1, \Xi_2, \dots, \Xi_\vartheta)^r$$

Proof:

The proof of Theorem 3 can also be applied to Theorem 9.

Theorem 10: Assume that $\Xi_\tau = \left(\mathcal{T}_{\Xi_\tau}, \mathcal{J}_{\Xi_\tau}, \mathcal{L}_{\Xi_\tau}\right)$, represents a set of SFVs. Where $\mathcal{L}_{\tau=} \prod_{k=1}^{\tau-1} S\,(\Xi_k)\,(\tau = 2, \dots \vartheta)$When $\mathcal{L}_1 = 1$ and $S\,(\Xi_k)$ is the score of SFVs(Ξ_k), we have $S\,(\Xi_k)$. If $\alpha = (\mathcal{T}_\alpha, \mathcal{J}_\alpha, \mathcal{L}_\alpha)$ is SFV on k and $r > 0$, then:

$$SFPAAG(\Xi_1^r \otimes \alpha, \Xi_2^r \otimes \alpha, \dots \Xi_\vartheta^r \otimes \alpha) = SFPAAG(\Xi_1, \Xi_2, \dots, \Xi_\vartheta)^r \otimes \alpha$$

Proof:

The proof of Theorem 4 can also be applied to Theorem 10.

4. A MAGDM Framework Based on the Proposed Spherical Fuzzy Aggregation Operators

To demonstrate the applicability, effectiveness, and reliability of the proposed approach, this section develops a multi-attribute decision-making (MADM) methodology based on the proposed aggregation operators under a spherical fuzzy (SF) environment. The overall framework of the proposed methodology is illustrated in Figure 1.

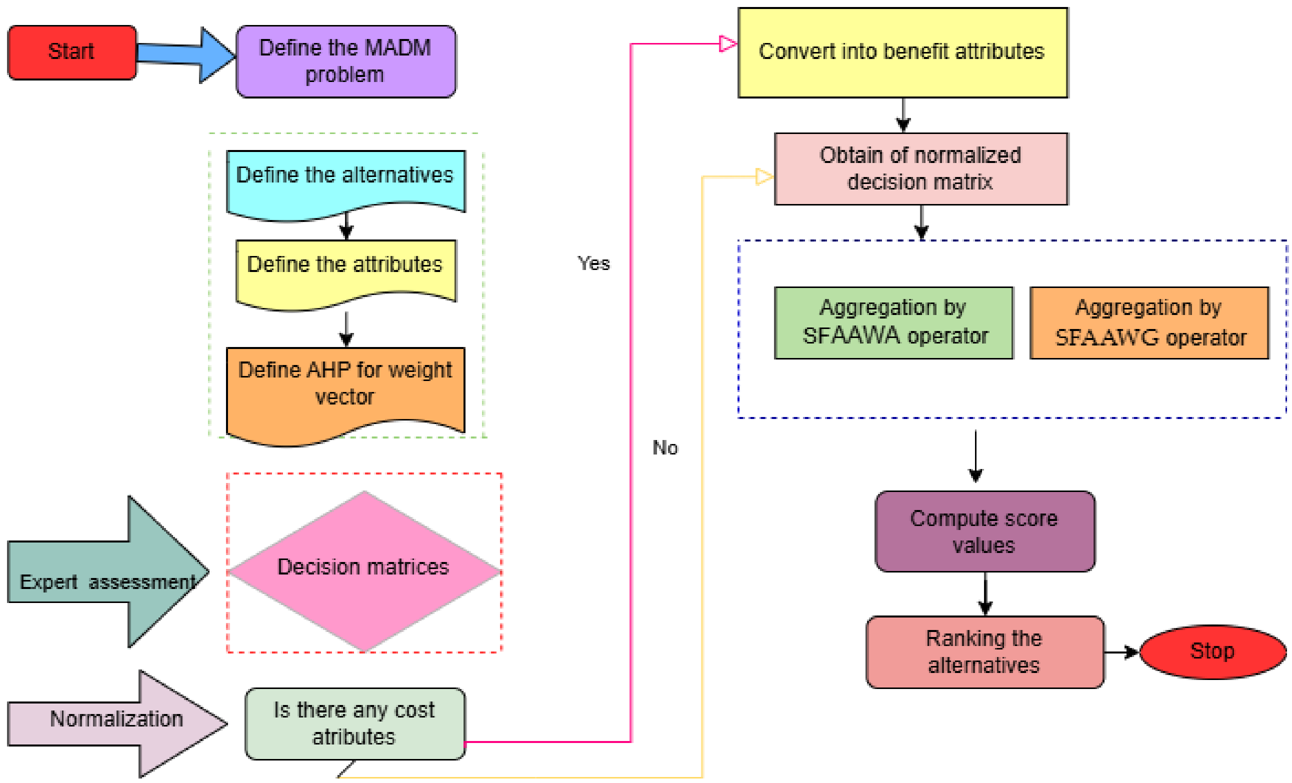

Fig. 1. Defines the Structure of the Article.

Consider $K^2 = (K^2_{\iota\tau})_{m\times n}$ as a spherical fuzzy (SF) decision matrix provided by the decision-maker DM$_2$, where $K^2_{\iota\tau} = (\mathcal{T}_{\iota\tau}, \mathcal{J}_{\iota\tau}, \mathcal{L}_{\iota\tau})$ denotes a spherical fuzzy value (SFV) corresponding to the evaluation of alternative k_ι with respect to criterion c_τ. Here, $(\mathcal{T}_{\iota\tau}, \mathcal{J}_{\iota\tau}, \mathcal{L}_{\iota\tau})$ represent the membership, abstinence, and non-membership degrees assigned by the decision-maker, respectively, satisfying 0 $(\mathcal{T}_{\iota\tau}, \mathcal{J}_{\iota\tau}, \mathcal{L}_{\iota\tau}) \subset (0,1)$, $(\mathcal{T}_{\iota\tau} + \mathcal{J}_{\iota\tau} + \mathcal{L}_{\iota\tau}) \le 1, \iota = 1,2,\dots m, \tau = 1,2,\dots n$.

If all criteria $c_\tau(\tau = 1,2,\dots,n)$ are of the same type, no normalization procedure is required. Otherwise, the original decision matrix $K^2 = (K^2_{\iota\tau})_{min}$ is transformed into the normalized spherical fuzzy decision matrix $R^2 = (r^2_{\iota\tau})_{min}$, where

$$r^2_{\iota\tau} = \begin{cases} k^2_{\iota\tau}, for\ benefit\ attribute\ c_\tau \\ k^2_{\iota\tau}, for\ cos\,t\ attribute\ c_\tau \end{cases}$$

Next, a multi-attribute decision-making (MADM) framework based on the proposed SFAAA operator is developed under a spherical fuzzy value (SFV) environment. The main steps of the proposed decision-making procedure are summarized as follows.

Step 1: Using the following equations $\mathcal{L}^2_{\iota\tau}, (2 = 1,2,\dots p)$, the corresponding values are calculated as follows.

$$\mathcal{L}_{\tau} = \prod_{k=1}^{\tau-1} S\,(r^2_{\iota\tau})\,(2 = 2,\dots p), \mathcal{L}^1_{\iota\tau} = 1$$

Step 2: Apply the proposed SFAAA operator to aggregate the spherical fuzzy evaluations. The aggregated values are obtained using the following expression:

$$r_{\iota\tau} = SFAAA\left(\Xi_{\iota\tau}{}^{1}, \Xi_{\iota\tau}{}^{2}, \dots, \Xi_{\iota\tau}{}^{p}{}_{\vartheta}\right) = \begin{pmatrix} 1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau{}^{(p)}\left(-ln\left(1-\mathcal{T}^2_{\Xi_\tau}\right)\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau{}^{(p)}\left(-ln\left(\mathcal{J}^2_{\Xi}\right)\right)^{\Delta}\right)^{1/\Delta}}, exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_\tau{}^{(p)}\left(-ln\left(\mathcal{L}^2_{\Xi}\right)\right)^{\Delta}\right)^{1/\Delta}} \end{pmatrix}$$

or the SFAAG operator:

$$r_{\iota\tau} = SFPAAG\left(\Xi_{\iota\tau}{}^{1}, \Xi_{\iota\tau}{}^{2}, \dots \Xi_{\iota\tau}{}^{p}{}_{\vartheta}\right) = \begin{pmatrix} exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_{\tau}{}^{(p)}\left(-ln(\mathcal{T}_{\Xi_\tau}^{2})\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ 1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_{\tau}{}^{(p)}\left(-ln(1-\mathcal{J}_{\Xi}^{2})\right)^{\Delta}\right)^{\frac{1}{\Delta}}}, \\ 1 - exp^{-\left(\sum_{\tau=1}^{\vartheta} \omega_{\tau}{}^{(p)}\left(-ln(1-\mathcal{L}_{\Xi}^{2})\right)^{\Delta}\right)^{1/\Delta}} \end{pmatrix}$$

Combine the individual spherical fuzzy decision matrices $R^2 = (r_{\iota\tau}^2)_{mx2}(2 = n, \dots p)$, provided by the decision-makers to obtain the collective spherical fuzzy decision matrix $R = (r_{\iota\tau})_{mx2}$.

Step 3: Determine $\mathcal{L}_{\iota\tau}, (\iota = 1, 2, \dots m, \tau = 1, 2, \dots n)$, solving the following equation:

$$\mathcal{L}_{\tau=} \prod_{k=1}^{\tau-1} S\ (r_{\iota\tau}^2)\ (2 = n, \dots p), \mathcal{L}_{\iota\tau}^1 = 1$$

Step 4: Using the score function presented in Section 2, the alternatives are ranked as follows:

$$Sco(\Xi_\iota) = (1 + \mathcal{T}_{\Xi_\tau}^2 - \mathcal{J}_{\Xi}^2 - \mathcal{L}_{\Xi}^2)/2 \quad \iota = 1, 2, \dots m$$

Therefore, the larger the score value of the overall spherical fuzzy value k_ι the more desirable the alternative $k_\iota(\iota = 1, 2, \dots m)$. Accordingly, the alternatives are ranked in descending order of their score values $S(r_\iota)$.

4.1 AHP Method (Analytic Hierarchy Process)

The Analytic Hierarchy Process (AHP), developed by Saaty, is one of the most widely used multi-attribute decision-making (MADM) techniques for evaluating and ranking alternatives with respect to multiple criteria. The method provides a structured and systematic framework for determining the relative importance of criteria and selecting the most appropriate alternative. Due to its simplicity, effectiveness, and ability to incorporate expert judgments, AHP has been extensively applied in various fields, including banking, supplier selection, risk assessment, project evaluation, and financial performance analysis.

Step 1: Problem Definition and Hierarchical Structure. The decision problem is structured hierarchically into three levels: the overall goal, the evaluation criteria, and the alternatives. The goal represents the selection of the most suitable AI technology for smart agriculture. The criteria constitute the evaluation dimensions, while the alternatives represent the candidate AI technologies under consideration.

Step 2: Construction of the Pairwise Comparison Matrix. The decision-maker performs pairwise comparisons of the criteria using Saaty's nine-point scale. The pairwise comparison matrix is defined as

$$A = (a_{ij})_{n\times n}, a_{ij} = \frac{C_j}{C_i}$$

where a_{ij} denotes the relative importance of criterion Ci compared with criterion C_j. The matrix satisfies the following properties:

$$a_{ij} = 1, a_{ij} = \frac{1}{a_{ij}}$$

Step 3: Normalization of the Pairwise Comparison Matrix. The elements of the comparison matrix are normalized as follows:

$$a_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_{ij}}$$

The normalized matrix is then used to derive the priority weights.

Step 4: Determination of Priority Weights. The priority weight associated with criterion C_i is calculated by averaging the normalized values in the corresponding row:

$$w_i = \frac{1}{n} = 1 \sum_{j=1}^{n} a_{ij}$$

where w_i denotes the weight of criterion C_i.

Step 5: Consistency Index Calculation. To verify the consistency of the pairwise comparisons, the maximum eigenvalue λmax of the comparison matrix is first determined. The Consistency Index (CI) is then calculated as

$$CI = \frac{\lambda max - n}{n-1}$$

Step 6: Consistency Ratio Evaluation. The Consistency Ratio (CR) is computed as

$$CR = \frac{CI}{RI}, CR < 0.1$$

where RI denotes the Random Index. The pairwise comparison matrix is considered consistent if CR < 0.10. Otherwise, the pairwise judgments should be revised.

5. Selection of Artificial Intelligence Technologies for Smart Agriculture

Artificial Intelligence (AI) is transforming modern agriculture by enhancing productivity, optimizing resource utilization, and promoting sustainable farming practices. AI-driven technologies enable farmers to monitor crop health, predict weather conditions, detect plant diseases, optimize irrigation systems, and automate agricultural operations. Through intelligent data analytics and advanced automation, AI facilitates efficient farm management, improves decision-making processes, and contributes to increased agricultural productivity while minimizing environmental impacts.

AI technologies have found widespread applications in smart agriculture. These applications include agricultural drones for crop monitoring and field assessment, smart irrigation systems for efficient water management, weather forecasting tools for agricultural planning, disease and pest detection systems based on image analysis, autonomous harvesting robots, soil monitoring systems for nutrient assessment, and AI-supported supply chain management solutions. Such technologies play a crucial role in improving the efficiency, sustainability, and resilience of modern agricultural systems.

To demonstrate the applicability of the proposed spherical fuzzy Aczel–Alsina aggregation framework, five AI-based smart agriculture alternatives are considered:

R_1: *Smart Irrigation System* – An AI-based irrigation management system that regulates water distribution according to soil moisture levels and climatic conditions.

R_2: *Crop Disease Identification System* – An intelligent system that employs image processing and machine learning techniques to detect and diagnose crop diseases at an early stage.

R_3: *Agricultural Drone Surveillance* – AI-powered drones utilized for crop monitoring, pesticide application, and field condition assessment.

R_4: *Smart Harvesting Robot* – An autonomous robotic system designed to harvest crops efficiently while reducing labor requirements and operational costs.

R_5: *AI-Based Weather Forecasting System* – An intelligent forecasting platform that predicts weather conditions and supports agricultural planning and risk management.

These alternatives represent advanced AI applications that contribute significantly to improving the productivity, sustainability, and operational efficiency of smart agriculture.

The alternatives are evaluated with respect to the following criteria:

C_1: *Productivity Improvement* – The capability of the AI technology to enhance agricultural productivity and operational performance.

C_2: *Cost Reduction* – The potential to reduce operational, maintenance, and labor costs associated with agricultural activities.

C_3: *Environmental Sustainability* – The contribution of the technology to minimizing environmental impacts and preserving natural resources.

C_4: *Technical Reliability* – The degree of accuracy, consistency, robustness, and dependability of the AI system under operational conditions.

The relative importance of the evaluation criteria is determined using the Analytic Hierarchy Process (AHP). The resulting criteria weight vector is given by w_j = (0.7897, 0.1302, 0.0288, 0.0514).

Subsequently, the proposed Spherical Fuzzy Aczel–Alsina Averaging (SFAAA) and Spherical Fuzzy Aczel–Alsina Geometric (SFAAG) operators are employed to aggregate the evaluation information and rank the alternatives. The main steps of the proposed decision-making procedure are presented below.

Step 1: Establish the spherical fuzzy value (SFV) decision matrix D_1 based on the evaluations provided by the decision-maker (Table 1).

Table 1
SFVs decision matrix D_1

	$\mathcal{C}_1$			$\mathcal{C}_2$			$\mathcal{C}_3$			$\mathcal{C}_4$		
$\mathcal{R}_1$	0.3	0.5	0.4	0.3	0.6	0.5	0.5	0.3	0.55	0.44	0.43	0.53
$\mathcal{R}_2$	0.7	0.2	0.3	0.25	0.7	0.44	0.4	0.4	0.56	0.32	0.45	0.21
$\mathcal{R}_3$	0.5	0.6	0.4	0.22	0.5	0.34	0.6	0.36	0.62	0.35	0.51	0.22
$\mathcal{R}_4$	0.6	0.2	0.4	0.43	0.45	0.52	0.5	0.44	0.52	0.41	0.43	0.35
$\mathcal{R}_5$	0.4	0.3	0.2	0.56	0.34	0.62	0.7	0.45	0.43	0.45	0.22	0.45

Step 2: Using the SFAAA operator, the individual spherical fuzzy decision matrices $R^2 = (r_{\iota\tau}^2)_{4x5}(2 = 1, 2,3)$ are aggregated to obtain the collective spherical fuzzy decision matrix $R = (r_{\iota\tau})_{5x4}$, as presented in Table 2.

Table 2
SFVs decision matrix R

Alt.	SFAAA			SFAAG		
$\mathcal{R}_1$	0.0256	0.6175	0.4092	0.1118	0.1011	0.0590
$\mathcal{R}_2$	0.2780	0.0115	0.1106	0.5267	0.1159	0.0341
$\mathcal{R}_3$	0.0845	0.7710	0.2808	0.3543	0.1569	0.0507
$\mathcal{R}_4$	0.1543	0.0107	0.3873	0.7363	0.0276	0.0556
$\mathcal{R}_5$	0.0825	0.0990	0.0115	0.4146	0.0204	0.0724

Step 3: To obtain the overall preference value r_ι for each alternative, the evaluation values $r_{\iota\tau}(\iota = 1, 2, \dots 5)$ corresponding to the criteria in the *i*th row of the collective spherical fuzzy decision matrix R are aggregated using the proposed SFAAA operator.

Table 3
Result of decision matrix

Alt.	SFAAA	SFAAG	Ranking
$\mathcal{R}_1$	0.3480	0.4001	4
$\mathcal{R}_2$	0.5101	0.6722	1
$\mathcal{R}_3$	0.2601	0.3422	5
$\mathcal{R}_4$	0.4728	0.4344	3
$\mathcal{R}_5$	0.4998	0.5354	2

Step 5: Determine the overall preference values $r_i(1,2,\ldots 5)$ and compute their corresponding score values. Subsequently, rank the alternatives according to the descending order of the obtained scores. The ranking results obtained using the SFAAA operator are as follows: $R_2 > R_5 > R_4 > R_1 > R_3$ (Figure 2).

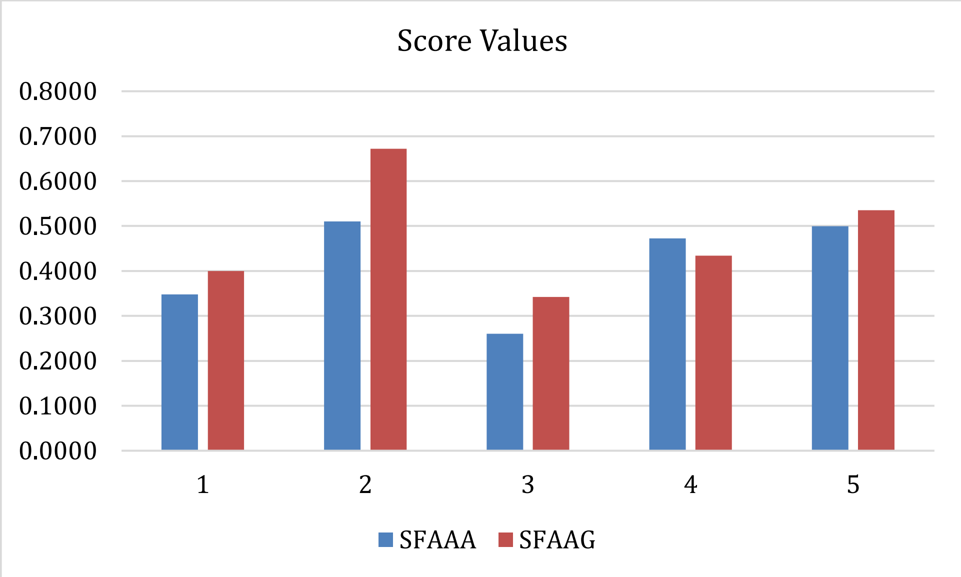

Fig. 2. Score values presented graphically

Therefore, R_2 (Crop Disease Identification System) is identified as the most desirable alternative and is selected as the optimal AI technology for smart agriculture under the proposed decision-making framework.

The same computational procedure is subsequently implemented using the proposed SFAAG operator to validate the consistency and robustness of the obtained results.

5. Comparative Study

In this section, the performance of the proposed SFPAAG and SFPAAA operators is compared with several existing spherical fuzzy aggregation operators reported in the literature, including those proposed by Wu *et al.,* [27], Riaz *et al.,* [28], Ju *et al.,* [29], and Sarfraz [13]. To evaluate the effectiveness and reliability of the proposed approach, the previously presented decision-making problem is reanalyzed using these aggregation operators. The obtained ranking results are summarized in Table 4.

Table 4
Comparative analysis

Operator	Ranking
$\boldsymbol{SFPAAA\ operator}$	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
$\boldsymbol{SFPAAG\ operator}$	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Wu *et al.* [27]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Wu *et al.* [27]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Riaz *et al.* [20]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Riaz *et al.*, [20]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Ju *et. al.* [29]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Ju *et. al.* [29]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Sarfraz [13]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$
Sarfraz [13]	$\mathcal{R}_2 > \mathcal{R}_4 > \mathcal{R}_3 > \mathcal{R}_5 > \mathcal{R}_1$

The comparison enables an assessment of the consistency, robustness, and practical applicability of the proposed operators relative to existing aggregation frameworks. According to the ranking results obtained using the proposed SFAAG and SFAAA operators, R_2 (Crop Disease Identification System) is identified as the most preferred alternative.

6. Conclusions

This study investigated Aczel–Alsina (AA)-based aggregation operators within the framework of spherical fuzzy values (SFVs) and the Analytic Hierarchy Process (AHP) for addressing multi-attribute decision-making (MADM) problems under uncertainty. To effectively aggregate spherical fuzzy information, two novel aggregation operators, namely the Spherical Fuzzy Aczel–Alsina Averaging (SFAAA) operator and the Spherical Fuzzy Aczel–Alsina Geometric (SFAAG) operator, were developed. Furthermore, the AHP method was incorporated to determine the relative importance of evaluation criteria in a systematic and reliable manner. Theoretical analyses were conducted to establish the fundamental properties of the proposed operators, including idempotency, monotonicity, and boundedness, thereby confirming their mathematical validity and applicability.

To demonstrate the practicality of the proposed framework, a case study involving the evaluation and selection of artificial intelligence (AI) technologies for smart agriculture was presented. Five AI-based agricultural technologies were assessed with respect to productivity improvement, cost reduction, environmental sustainability, and technical reliability. The obtained results indicated that the proposed aggregation operators effectively handled uncertain and imprecise information and generated consistent and reliable rankings of the considered alternatives. In particular, the Crop Disease Identification System (R_2) was identified as the most desirable alternative, highlighting the potential of AI-driven disease monitoring technologies to support sustainable agricultural development.

Despite its advantages, the proposed framework has several limitations. First, the computational complexity of the model may increase when dealing with large-scale agricultural datasets and a substantial number of evaluation criteria. Second, the final decision outcomes depend on expert judgments and the criteria weights derived through AHP, which may introduce a degree of subjectivity into the decision-making process. In addition, the proposed methodology is developed within a spherical fuzzy environment and may require further modifications when applied to other uncertainty representations.

Future research may focus on extending the proposed framework to more advanced fuzzy environments, such as interval-valued spherical fuzzy sets, T-spherical fuzzy sets, and q-rung orthopair fuzzy sets. Furthermore, the integration of machine learning techniques and optimization algorithms with the proposed MADM framework could enhance intelligent decision-support systems for smart agriculture. Another promising research direction involves the development of group decision-making models that incorporate the opinions of multiple experts operating under uncertain agricultural conditions. Beyond smart agriculture, the proposed methodology may also be applied to a wide range of practical domains, including smart healthcare, sustainable transportation systems, environmental management, and renewable energy planning.

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Conflicts of Interest
The authors declare no conflicts of interest.

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