Journal of Contemporary Decision Science Volume 2, Issue 1 (2026) 323-348

Journal of Contemporary Decision Science

Journal homepage: www.cds-journal.org
ISSN: 3104-3380

An Interval-Valued T-Spherical Fuzzy SWARA Approach with Sugeno–Weber Operators for Artificial Intelligence Selection

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ARTICLE INFO

Article history:
Received 14 February 2026
Received in revised form 28 March 2026
Accepted 15 May 2026
Available online 30 May 2026

Keywords:
T-spherical fuzzy set; Sugeno–Weber; T-SFS; Multi-Attribute Decision-Making; MADM; Artificial Intelligence.

ABSTRACT

The T-spherical fuzzy set (T-SFS) has emerged as a powerful and flexible framework for modelling uncertainty and ambiguity in decision-making processes. In this study, we examine the integration of Sugeno–Weber (SW) t-norms within an interval-valued T-spherical fuzzy (IVT-SF) environment. Based on this framework, a novel family of aggregation operators is developed, including the interval-valued T-spherical fuzzy Sugeno–Weber power averaging (IVT-SFSWPA), power geometric (IVT-SFSWPG), power-weighted averaging (IVT-SFSWPWA), and power-weighted geometric (IVT-SFSWPWG) operators. The proposed operators are analyzed in terms of their fundamental properties and special cases, demonstrating their flexibility and effectiveness in handling complex decision-making problems. In addition, a new multi-attribute decision-making (MADM) method is developed within the IVT-SF environment. Furthermore, a comparative analysis with existing approaches is conducted to highlight the superiority, robustness, and practicality of the proposed aggregation operators. The results indicate that the proposed method provides more consistent and reliable decision-making outcomes. This study advances fuzzy decision-making methodologies and offers a promising approach to addressing real-world problems in dynamic, complex environments.

1. Introduction

The concept of the fuzzy set (FS) was first proposed by Zadeh [1]. In this theory, an element's membership in a set is represented by a membership grade (MG). However, it was later observed that the applicability of FSs is limited because they consider only the MG of an element. To address this limitation, Atanassov [2] introduced the intuitionistic fuzzy set (IFS), which incorporates both membership grade (MG) and non-membership grade (NMG). The IFS framework has been successfully applied to many real-world problems and has attracted considerable attention from researchers. Nevertheless, the IFS model is constrained by the condition that the sum of MG and NMG must belong to the interval [0,1].

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To overcome this limitation, Yager [3] introduced the concept of the Pythagorean fuzzy set (PyFS), which extends the IFS framework to handle uncertain and inconsistent information more effectively. In PyFSs, the sum of the squares of MG and NMG is restricted to the interval [0,1].

In 2017, Yager [4] further generalized the theory of PyFSs by relaxing their structural constraints and proposed the concept of q-rung orthopair fuzzy sets (q-ROFSs). This extension provides a more flexible and comprehensive framework under the condition $0 \leq b^q + \phi^q \leq 1$, where $q \geq 1$.

To further address uncertainty in decision-making (MD), Cuong [5] introduced the concept of picture fuzzy sets (PFSs), which extend IFSs by incorporating MG b, abstinence grade (AG) p, and NMG ϕ, subject to the condition $0 \leq b + p + \phi \leq 1$, where $b, p, \phi \in [0,1]$. PFSs provide a more effective way of handling situations involving neutrality and hesitation.

Subsequently, Mahmood *et al.,* [6] proposed the spherical fuzzy set (SFS), which is considered a generalized form of the PFS. In the SFS and T-SFS framework, the condition $(MG^q + AG^q + NMG^q) \leq 1$, where $q \in \mathbb{Z}^+$, allows greater flexibility in representing uncertain information. This model significantly expands the domain and applicability of FS theory.

2. Literature Review

The increasing application of MADM approaches has attracted significant attention in the evaluation of electric vehicles. In [7], an MADM framework was applied for the assessment of electric vehicles. Similarly, electric passenger cars were selectively evaluated using MADM techniques in [8]. In [9], an MADM-based smart framework for electric vehicle charging and traction systems was proposed. Furthermore, [10] evaluated conventional fuel-powered vehicles with environmentally friendly characteristics using MADM methods. An improved MADM optimization and life-cycle assessment tool for passenger vehicles was introduced in [11]. In [12], vehicle emissions from oil-powered automobiles were investigated, while [13] focused on the long-term persistence of hydrocarbons in fuel emissions. Carson's life-cycle assessment for average passenger vehicles based on liquid and gaseous fuels was discussed in [14]. Moreover, [15] examined passenger vehicle evaluation using a scenario-based assessment framework. Sarfraz *et al.,* [16] proposed an approach based on IFSs for determining attribute priorities.

Zhou *et al.,* [17] introduced the concept of artificial intelligence (AI), while Shan *et al.* [18] developed a theoretical framework for AI based on technological advancements. Zhou *et al.* [19] applied AI-based software techniques, and Yang *et al.,* [20] further advanced AI software applications. Liu *et al.,* [21] contributed to the development of AI methodologies, whereas Wan et al., [22] explored the use of AI-based software systems.

Ghodousian *et al.,* [23] investigated the Sugeno–Weber (SW) t-norms using nonlinear optimization techniques. Kauers *et al.,* [24] developed the theoretical foundations of SW t-norms in the context of fuzzy set theory. Farahbod *et al.,* [25] conducted a comparative study between classical t-norms and SW t-norms, while Troiano *et al.,* [26] analyzed the statistical estimation of parametric models based on SW operators.

Furthermore, Ghodousian *et al.,* [27] introduced the concept of fuzzy connectivity symmetry using SW t-norms. Wang *et al.,* [28] developed SW-based differential anisotropic models for dominant t-norms. Senapati *et al.,* [29] proposed the application of SW t-norms within families of aggregation operators, whereas Sarfraz *et al.,* [30] investigated the role of SW operators in nonlinear fuzzy relational equations.

In addition, Senapati *et al.,* [31] presented an application of SW t-norms with constant factor terms in probabilistic metric spaces. Finally, Ullah *et al.,* [32] applied SW t-norms to unified aggregation operators, further demonstrating their flexibility and applicability in advanced fuzzy DM frameworks.

2.1 Motivation and Contribution

In this study, we developed several powerful mathematical frameworks by integrating three different theoretical concepts. Based on these theories, a novel family of aggregation operators (AOs) was proposed, including the IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG operators. These operators were investigated in detail, together with their important mathematical properties and special cases.

Furthermore, the applicability and effectiveness of the proposed AOs were evaluated through the developed mathematical strategies. An advanced MADM framework was also designed to manage uncertain, inconsistent, and redundant human opinion data more effectively. In addition, a numerical example was analyzed to demonstrate the practicality of the proposed approaches in identifying the most suitable renewable AI alternative under specific dominant criteria and characteristics.

The remainder of this paper is organized as follows. Section 2 presents the fundamental concepts of IVT-SFSs and the working principles of the SW TN. It also discusses the operational laws of SW TN in the IVT-SFS environment. Section 3 introduces several SW TN operational laws associated with IVT-SFSs and proposes a new family of aggregation operators designed for intelligent decision-making processes. Section 4 develops a novel MADM methodology (Figure 1) based on the proposed aggregation operators and provides a detailed numerical illustration to demonstrate the applicability of the proposed model. Moreover, a comparative analysis between the proposed and existing methods is conducted to validate the reliability, consistency, and superiority of the developed approach. Finally, Section 5 concludes the study by summarizing the major findings, contributions, and future research directions.

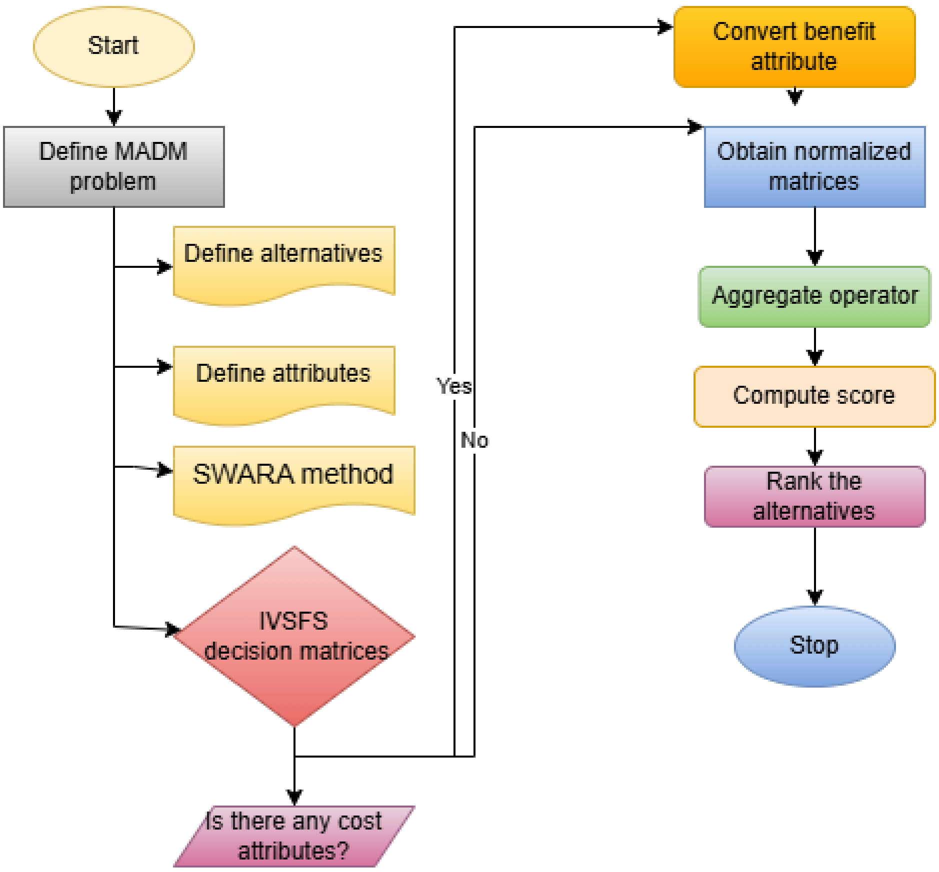

Fig. 1. Defines the Structure of the Article

2. Preliminaries

In this section, we examine the fundamental concepts of T-spherical fuzzy sets (T-SFSs) and Sugeno–Weber (SW) triangular norms, together with their operational laws, to establish the theoretical foundation of the proposed study.

Definition 1 [30]: A T-spherical fuzzy set α of the universe of discourse ϑ is given by:

$$\alpha = \{\langle \mathbb{j}, (\mu(\mathbb{j}), \nu(\mathbb{j}), \pi(\mathbb{j})) | \mathbb{j} \in \vartheta \rangle\}$$

Where $\mu(\mathbb{j}): \vartheta \to [0, \mathbb{1}], \nu(\mathbb{j}) : \vartheta \to [0, \mathbb{1}]$ and $\pi(\mathbb{j}) : \vartheta \to [0, \mathbb{1}]$ that defines the degrees of MG, NMG and hesitancy of respectively and

$$0 \leq \mu^{\Delta}(\mathbb{j}) + \pi^{\Delta}(\mathbb{j}) + \nu^{\Delta}(\mathbb{j}) \leq \mathbb{1}\ \forall \mathbb{j} \in \vartheta$$

We called $\alpha\ (\mu, \nu, \pi)$a SFN.

Definition 2 [31]: An interval-valued T-spherical fuzzy set α of the universe of discourse ϑ is defined as follows. $\alpha = \{\mathbb{j}, ([\mu^{\mathcal{G}}(\mathbb{j}), \mu^{u}(\mathbb{j})], [\nu^{\mathcal{G}}(\mathbb{j}), \nu^{u}(\mathbb{j})], [\pi^{\mathcal{G}}(\mathbb{j}), \pi^{u}(\mathbb{j})] \mathbb{j} \in \vartheta)\}$ and $0 \leq ((\mu)^{\Delta})^{u}(\mathbb{j}) + ((\nu)^{\Delta})^{u}(\mathbb{j}) + ((\pi)^{\Delta})^{u}(\mathbb{j}) \leq \mathbb{1}$. Here $\mu(\mathbb{j}) = [\mu^{\mathcal{G}}(\mathbb{j}), \mu^{u}(\mathbb{j})], \nu(\mathbb{j}) = [\nu^{\mathcal{G}}(\mathbb{j}), \nu(\mathbb{j})], \pi(\mathbb{j}) = [\pi^{\mathcal{G}}(\mathbb{j}), \pi^{u}(\mathbb{j})]$. the expression $r(\mathbb{j}) = [r^{\mathcal{G}}(\mathbb{j}), r^{u}(\mathbb{j})] = \left[\sqrt{\mathbb{1} - ((\mu)^{u})^{\Delta}(\mathbb{j}) + ((\nu)^{u})^{\Delta}(\mathbb{j}) + ((\pi)^{u})^{\Delta}(\mathbb{j})}, \sqrt{\mathbb{1} - ((\mu)^{\mathcal{G}})^{\Delta}(\mathbb{j}) + ((\nu)^{\mathcal{G}})^{\Delta}(\mathbb{j}) + ((\pi)^{\mathcal{G}})^{\Delta}(\mathbb{j})}\right]$ is termed as DR. We call $\alpha = (\mu, \nu, \pi) = ([\mu^{\mathcal{G}}(\mathbb{j}), \mu^{u}(\mathbb{j})], [\nu^{\mathcal{G}}(\mathbb{j}), \nu^{u}(\mathbb{j})], [\pi^{\mathcal{G}}(\mathbb{j}), \pi^{u}(\mathbb{j})])$ an IVT-SF number. Operation on interval-valued T-spherical fuzzy sets:

Definition 3 [33]: Consider $\Delta_{\mathbb{1}} = ([\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \mu_{2}^{u}(\mathbb{j})], [\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \pi_{2}^{u}(\mathbb{j})], [\nu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \nu_{2}^{u}(\mathbb{j})])$, be an IVT-SFV. Then

$$Sco(\Delta_{\mathbb{1}}) = \frac{\mu_{\mathbb{1}}^{\mathcal{G}^{\Delta}} + \mu_{2}^{\mathcal{G}^{\Delta}} - \pi_{\mathbb{1}}^{\mathcal{G}^{\Delta}} - \pi_{2}^{\mathcal{G}^{\Delta}} - \nu_{\mathbb{1}}^{\mathcal{G}^{\Delta}} - \nu_{2}^{\mathcal{G}^{\Delta}}}{3} \quad (1)$$

Be the score value of $\Delta_{\mathbb{1}}$.

Definition 4 [32]: Consider $\Delta_{\mathbb{1}} = (\gamma_{\beta_{\mathbb{1}}}, \delta_{\beta_{\mathbb{1}}})$ be an IVT-SFV. Then

$$Acc\ (\Delta_{\mathbb{1}}) = \frac{\mu_{\mathbb{1}}^{\mathcal{G}^{\Delta}} + \mu_{2}^{\mathcal{G}^{\Delta}} + \pi_{\mathbb{1}}^{\mathcal{G}^{\Delta}} + \pi_{2}^{\mathcal{G}^{\Delta}} + \nu_{\mathbb{1}}^{\mathcal{G}^{\Delta}} + \nu_{2}^{\mathcal{G}^{\Delta}}}{3} \quad (2)$$

Be the degrees of accuracy of $\Delta_{\mathbb{1}}$.

Clearly, see that $Sco(\Delta_{\mathbb{1}}) \in [-\mathbb{1}, \mathbb{1}]$ and $Acc\ (\Delta_{\mathbb{1}}) \in [0, \mathbb{1}]$ for all SFVs.

We derive some limitations, such as we will be considered $\Delta_{\mathbb{1}} > \Delta_2$ when we get $Sco\Delta_{\mathbb{1}} > Sco\Delta_2$: We will be considered $\Delta_{\mathbb{1}} < \Delta_2$ when we get $Sco\Delta_{\mathbb{1}} < Sco\Delta_2$: But if we get $Sco\Delta_{\mathbb{1}} = Sco\Delta_2$ such that we will be considered $\Delta_{\mathbb{1}} > \Delta_2$ when we get $Sco\Delta_{\mathbb{1}} > Sco\Delta_{\mathbb{1}}$: We will be considered $\Delta_{\mathbb{1}} < \Delta_2$ when we get $Sco\Delta_{\mathbb{1}} < Sco\Delta_2$.

Definition 5 [32]: The SW t-norms $(T_A^{\eta})_{\eta \epsilon [0, \beth]}$ are respectively expressed as follows.

$$(T_A^{\eta})_{(\ell, v)} = \begin{cases} T_D(\ell, v) \quad ef\ \eta = -\mathbb{1} \\ T_D(\ell, v)\ \ ef\ \eta = \infty \\ max\left(0, \dfrac{\ell + v - \mathbb{1} + \eta \ell v}{\mathbb{1} + \eta}\right) ef\ \ -\mathbb{1} < \eta < \infty \end{cases}$$

The SW t-norms $(T_A^{\eta})_{\eta \epsilon [0, \beth]}$ are respectively expressed as follows.

$$(S_A^{\eta})_{(\ell, v)} = \begin{cases} S_D(\ell, v) \quad ef\ \eta = -\mathbb{1} \\ S_D(\ell, v)\ \ ef\ \eta = \infty \\ me\psi\left(\mathbb{1}, \ell + v - \dfrac{\mathbb{1} + \eta \ell v}{\mathbb{1} + \eta}\right) ef\ \ -\mathbb{1} < \eta < \infty \end{cases}$$

Limiting values: $T_A^{\beth} = men$, $T_A^{0} = T_D$, $T_A^{\mathbb{1}} = T_p$, $S_A^{\beth} = max$, $S_A^{0} = S_D$, $S_A^{\mathbb{1}} = S_p$. The t-norm T_A^{η} and t-conorm S_A^{η} are dual about each other for all $\eta \epsilon [0, \beth]$. Both the SW t-norms family and the SW t-conorms family show strictly expanding and strictly reducing trends, respectively. Furthermore, the

SW t-norms family is the only ones that truly satisfy the equivalence $T(\ell^{ב}, v^{ב}) = T(\ell, v)^{ב}$ for any $ב > 0$ and $\ell, v \in [0, \mathbb{1}]$.

Definition 6 [32]: For these T-SFS $\alpha = \left(\mu_\alpha(\mathbb{j}), v_\alpha(\mathbb{j}), \pi_\alpha(\mathbb{j})\right)$ and $\beta = \left(\mu_\beta(\mathbb{j}), v_\beta(\mathbb{j}), \pi_\beta(\mathbb{j})\right)$.The following holds under the condition $ב_{\mathbb{1}}, ב_2, ב_3 > 0$

i. $\alpha \oplus \beta = \beta \oplus \alpha$
ii. $\alpha \otimes \beta = \beta \otimes \alpha$
iii. $\boldsymbol{\lambda}(\alpha \oplus \beta) = \boldsymbol{\lambda}\alpha \oplus ב\beta$
iv. $\boldsymbol{\lambda_1}\alpha \oplus ב_2\alpha = (\boldsymbol{\lambda_1} \oplus ב_2)\alpha$
v. $(\alpha \otimes \beta)^{ב} = \alpha^{\lambda} \otimes \beta^{ב}$
vi. $\alpha^{\lambda_{\mathbb{1}}} \otimes \alpha^{\lambda_{\triangle}} = \alpha^{\lambda_{\mathbb{1}+\lambda_2}}$

Definition 7 [32]: Let $\alpha = \langle[\mu^{\mathcal{G}}(\mathbb{j}), \mu^{u}(\mathbb{j})], [v^{\mathcal{G}}(\mathbb{j}), v^{u}(\mathbb{j})], , [\pi^{\mathcal{G}}(\mathbb{j}), \pi^{u}(\mathbb{j})]\rangle$ $\alpha_{\mathbb{1}} = \langle[\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \mu_{\mathbb{1}}^{u}(\mathbb{j})], [v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), v_{\mathbb{1}}^{u}(\mathbb{j})], , [\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \pi_{\mathbb{1}}^{u}(\mathbb{j})]\rangle$ and $\alpha_2 = \langle[\mu_2^{\mathcal{G}}(\mathbb{j}), \mu_2^{u}(\mathbb{j})], [v_2^{\mathcal{G}}(\mathbb{j}), v_2^{u}(\mathbb{j})], , [\pi_2^{\mathcal{G}}(\mathbb{j}), \pi_2^{u}(\mathbb{j})]\rangle$ be three IVT-SFS then.

$$\alpha_{\mathbb{1}} \cup \alpha_2 = \left\{\begin{matrix} \left[max\{\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \mu_2^{\mathcal{G}}(\mathbb{j})\}, max\{\mu_{\mathbb{1}}^{u}(\mathbb{j}), \mu_2^{u}(\mathbb{j})\}\right], \\ \left[men\{v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), v_2^{\mathcal{G}}(\mathbb{j})\}, men\{v_{\mathbb{1}}^{u}(\mathbb{j}), v_2^{u}(\mathbb{j})\}\right], \\ \left[men\{\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \pi_2^{\mathcal{G}}(\mathbb{j})\}, men\{\pi_{\mathbb{1}}^{U}(\mathbb{j}), \pi_2^{U}(\mathbb{j})\}\right] \end{matrix}\right\}$$

$$\alpha_{\mathbb{1}} \cap \alpha_2 = \left\{\begin{matrix} \left[men\{\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \mu_2^{\mathcal{G}}(\mathbb{j})\}, men\{\mu_{\mathbb{1}}^{u}(\mathbb{j}), \mu_2^{u}(\mathbb{j})\}\right], \\ \left[max\{v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), v_2^{\mathcal{G}}(\mathbb{j})\}, max\{v_{\mathbb{1}}^{u}(\mathbb{j}), v_2^{u}(\mathbb{j})\}\right], \\ \left[men\{\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}), \pi_2^{\mathcal{G}}(\mathbb{j})\}, men\{\pi_{\mathbb{1}}^{u}(\mathbb{j}), \pi_2^{u}(\mathbb{j})\}\right] \end{matrix}\right\}$$

$$\alpha_{\mathbb{1}} \oplus \alpha_2 = \left(\begin{matrix} \left[\begin{matrix} \sqrt{\left(\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} + \left(\mu_2^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} - \left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} . \left(\mu_2^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\right)}, \\ \sqrt{\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle} + \left(\mu_2^{u}(\mathbb{j})\right)^{\triangle} - \left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle} . \left(\mu_2^{u}(\mathbb{j})\right)^{\triangle}} \end{matrix}\right], \\ \left[v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}) v_2^{\mathcal{G}}(\mathbb{j}), v_{\mathbb{1}}^{u}(\mathbb{j}) v_2^{u}(\mathbb{j})\right], \\ \left[\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}) \pi_2^{\mathcal{G}}(\mathbb{j}), \pi_{\mathbb{1}}^{u}(\mathbb{j}) \pi_2^{u}(\mathbb{j})\right] \end{matrix}\right)$$

$$\alpha_{\mathbb{1}} \otimes \alpha_2 = \left(\begin{matrix} \left[\left[\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}) \mu_2^{\mathcal{G}}(\mathbb{j}), \mu_{\mathbb{1}}^{U}(\mathbb{j}) \mu_2^{U}(\mathbb{j})\right]\right], \\ \left[\begin{matrix} \sqrt{\left(\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} + \left(v_2^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} - \left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} . \left(v_2^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\right)}, \\ \sqrt{\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle} + \left(v_2^{u}(\mathbb{j})\right)^{\triangle} - \left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle} . \left(v_2^{u}(\mathbb{j})\right)^{\triangle}} \end{matrix}\right] \\ \left[\begin{matrix} \sqrt{\left(\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} + \left(\pi_2^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} - \left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle} . \left(\pi_2^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\right)}, \\ \sqrt{\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle} + \left(\pi_2^{u}(\mathbb{j})\right)^{\triangle} - \left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle} . \left(\pi_2^{u}(\mathbb{j})\right)^{\triangle}} \end{matrix}\right] \end{matrix}\right)$$

$$ב . \alpha = \left(\begin{matrix} \sqrt{\mathbb{1} - \left(\mathbb{1} - \left(\mu^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\right)^{ב}} . \sqrt{\mathbb{1} - \left(\mathbb{1} - \left(\mu^{u}(\mathbb{j})\right)^{\triangle}\right)^{ב}}, \\ \left[\left(v^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}, \left(v^{u}(\mathbb{j})\right)^{\triangle}\right], \left[\left(\pi^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}, \left(\pi^{u}(\mathbb{j})\right)^{\triangle}\right] \end{matrix}\right)$$

$ב th$ Power of α; $ב. > 0$

$$\alpha^{ב} = \left\{\begin{array}{c}\left[\left(\mu^{\mathcal{G}}(\mathbb{j})\right)^{ב}, \left(\mu^{u}(\mathbb{j})\right)^{ב}\right], \left[\sqrt{\mathbb{1} - \left(\mathbb{1} - \left(v^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\right)^{ב}}, \sqrt{\left(\mathbb{1} - \left(v^{u}(\mathbb{j})\right)^{\Delta}\right)^{ב}}\right], \\ \left[\sqrt{\mathbb{1} - \left(\mathbb{1} - \left(\pi^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\right)^{ב}}, \sqrt{\left(\mathbb{1} - \left(\pi^{u}(\mathbb{j})\right)^{\Delta}\right)^{ב}}\right]\end{array}\right\}$$

Definition 8 [32]: Let $\alpha = \langle[\mu_\sigma^{\mathcal{G}}(\mathbb{j}), \mu_\sigma^{u}(\mathbb{j})], [v_\sigma^{\mathcal{G}}(\mathbb{j}), v_\sigma^{u}(\mathbb{j})],, [\pi_\sigma^{\mathcal{G}}(\mathbb{j}), \pi_\sigma^{u}(\mathbb{j})]\rangle$ be a collection of Interval-Valued T-Spherical Weighted Arithmetic Mean (IVT-SWAM) with respect to $\vartheta_\sigma = (\vartheta_{\mathbb{1}}, \vartheta_2, \dots \vartheta_\psi)$; $\vartheta_\sigma \in [0, \mathbb{1}]$ and $\sum_{\sigma=\mathbb{1}}^{\psi} \vartheta_\sigma = \mathbb{1}$ is defined as

$$\mathrm{IVT-SWAM}(\alpha_{\mathbb{1}}, \alpha_{\Delta}, \dots \dots, \alpha_\psi) = \vartheta_{\mathbb{1}}.\alpha_{\mathbb{1}} \oplus \vartheta_2.\alpha_2 \oplus \dots \dots \oplus \vartheta_2.\alpha_\psi$$

$$= \left\{\begin{array}{c}\left[\left(\mathbb{1} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(\mu_\sigma^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{\Delta}}, \left(\mathbb{1} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(\mu_\sigma^{u}(\mathbb{j})\right)^{\Delta}\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}\right], \\ \left[\prod_{\sigma=\mathbb{1}}^{\psi}\left(v_\sigma^{\mathcal{G}}(\mathbb{j})\right)^{\vartheta_\sigma}, \prod_{\sigma=\mathbb{1}}^{\psi}\left(v_\sigma^{u}(\mathbb{j})\right)^{\vartheta_\sigma}\right] \\ \otimes\left[\begin{array}{c}\left(\mathbb{1} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(\mu_\sigma^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\right)^{\vartheta_\sigma} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(\mu_\sigma^{\mathcal{G}}(\mathbb{j})\right)^{\Delta} - \left(\pi_\sigma^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}, \\ \left(\mathbb{1} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(\mu_\sigma^{u}(\mathbb{j})\right)^{\Delta}\right)^{\vartheta_\sigma} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(\mu_\sigma^{u}(\mathbb{j})\right)^{\Delta} - \left(\pi_\sigma^{u}(\mathbb{j})\right)^{\Delta}\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}\end{array}\right]\end{array}\right\}$$

Definition 9 [32]: Consider $\alpha = \langle[\mu_\sigma^{\mathcal{G}}(\mathbb{j}), \mu_\sigma^{u}(\mathbb{j})], [v_\sigma^{\mathcal{G}}(\mathbb{j}), v_\sigma^{u}(\mathbb{j})],, [\pi_\sigma^{\mathcal{G}}(\mathbb{j}), \pi_\sigma^{u}(\mathbb{j})]\rangle$ be a collection of Interval-Valued T-Spherical Weighted Arithmetic Mean (IVT-SWAM) with respect to, $\vartheta_\sigma = (\vartheta_{\mathbb{1}}, \vartheta_{\Delta}, \dots \vartheta_\psi)$; $\vartheta_\sigma \in [0, \mathbb{1}]$ and $\sum_{\sigma=\mathbb{1}}^{\psi} \vartheta_\sigma = \mathbb{1}$ is defined as

$$\mathrm{IVT-SWAM}_w(\alpha_{\mathbb{1}}, \alpha_2, \dots \dots, \alpha_\psi) = \vartheta_{\mathbb{1}}.\alpha_{\mathbb{1}} \oplus \vartheta_2.\alpha_2 \oplus \dots \dots \oplus \vartheta_2.\alpha_\psi$$

$$= \left\{\begin{array}{c}\left[\prod_{\sigma=\mathbb{1}}^{\psi} \mu_\sigma^{\mathcal{G}}(\Xi)^{\vartheta_\sigma}, \prod_{\sigma=\mathbb{1}}^{\psi} \mu_\sigma^{U}(\mathbb{j})^{\vartheta_\sigma}\right], \left[\left(\mathbb{1} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(v_\sigma^{\mathcal{G}}(\mathbb{j})\right)^2\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}, \left(\mathbb{1} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(v_\sigma^{u}(\mathbb{j})\right)^2\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}\right], \\ , \left[\begin{array}{c}\left(\prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(v_\sigma^{\mathcal{G}}(\mathbb{j})\right)^2\right)^{\vartheta_\sigma} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(v_\sigma^{\mathcal{G}}(\mathbb{j})\right)^2 - \left(\pi_\sigma^{\mathcal{G}}(\mathbb{j})\right)^2\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}, \\ \left(\prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(v_\sigma^{u}(\mathbb{j})\right)^2\right)^{\vartheta_\sigma} - \prod_{\sigma=\mathbb{1}}^{\psi}\left(\mathbb{1} - \left(v_\sigma^{u}(\mathbb{j})\right)^2 - \left(\pi_\sigma^{u}(\mathbb{j})\right)^2\right)^{\vartheta_\sigma}\right)^{\frac{\mathbb{1}}{2}}\end{array}\right]\end{array}\right\}$$

3. Operation on SW Triangular Norm Based on IVT-SF Information

The primary contribution of this section is the development of the IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG operators, together with the investigation of their fundamental properties and practical applicability.

Definition 10: For three IVT-SFVs= $([(\mu^{\mathcal{G}}(\mathbb{j}),\mu(\mathbb{j})^{u}],[\pi(\mathbb{j})^{\mathcal{G}},\pi(\mathbb{j})^{u}],[v(\mathbb{j})^{\mathcal{G}},v(\mathbb{j})^{u}]),\theta_{\mathbb{1}} = ([(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}),\mu_{\mathbb{1}}^{u}(\mathbb{j})],[\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}),\pi_{\mathbb{1}}^{u}(\mathbb{j})],[v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}),v_{\mathbb{1}}^{u}(\mathbb{j})]),\ \theta_2 = ([(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}),\mu_{\mathbb{1}}^{u}(\mathbb{j})],[\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}),\pi_{\mathbb{1}}^{u}(\mathbb{j})],[v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j}),v_{\mathbb{1}}^{u}(\mathbb{j})])$

$$\theta_{\mathbb{1}}\oplus\theta_2 = \left(\begin{array}{l}\left[\left(\sqrt{\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}-\frac{ב}{\mathbb{1}+ב}\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}.\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}\right),\left(\sqrt{\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\left(\mu_{2}^{u}(\mathbb{j})\right)^{⊿}-\frac{ב}{\mathbb{1}+ב}\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}.\left(\mu_{2}^{u}(\mathbb{j})\right)^{⊿}}\right)\right],\\ \left[\left(\sqrt{\frac{\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}-\mathbb{1}+ב\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}.\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}}\right),\left(\sqrt{\frac{\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\left(\pi_{2}^{u}(\mathbb{j})\right)^{⊿}-\mathbb{1}+ב\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}.\left(\pi_{2}^{u}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}}\right)\right],\\ \left[\left(\sqrt{\frac{\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}-\mathbb{1}+ב\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}.\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}}\right),\left(\sqrt{\frac{\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\left(v_{2}^{u}(\mathbb{j})\right)^{⊿}-\mathbb{1}+ב\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}.\left(v_{2}^{u}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}}\right)\right]\end{array}\right)$$

$$\theta_{\mathbb{1}}\otimes\theta_2 = \left(\begin{array}{l}\left[\left(\sqrt{\frac{\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}-\mathbb{1}+ב\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}.\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}}\right),\left(\sqrt{\frac{\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\left(\mu_{2}^{u}(\mathbb{j})\right)^{⊿}-\mathbb{1}+ב\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}.\left(\mu_{2}^{u}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}}\right)\right],\\ \left[\left(\sqrt{\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}-\frac{ב}{\mathbb{1}+ב}\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}.\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}\right),\left(\sqrt{\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\left(\pi_{2}^{u}(\mathbb{j})\right)^{⊿}-\frac{ב}{\mathbb{1}+ב}\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}.\left(\pi_{2}^{u}(\mathbb{j})\right)^{⊿}}\right)\right],\\ \left[\left(\sqrt{\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}-\frac{ב}{\mathbb{1}+ב}\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}.\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}\right),\left(\sqrt{\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\left(v_{2}^{u}(\mathbb{j})\right)^{⊿}-\frac{ב}{\mathbb{1}+ב}\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}.\left(v_{2}^{u}(\mathbb{j})\right)^{⊿}}\right)\right]\end{array}\right)$$

$$\Delta\theta = \left(\begin{array}{l}\left[\sqrt{\frac{\mathbb{1}+ב}{ב}(\mathbb{1}-\left(\mathbb{1}-\left(\mu^{\mathcal{G}}(\mathbb{j})\right)^{⊿}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\Delta}}\right],\left[\sqrt{\frac{\mathbb{1}+ב}{ב}(\mathbb{1}-\left(\mathbb{1}-\left(\mu^{u}(\mathbb{j})\right)^{⊿}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\Delta}}\right]\\ \left[\sqrt{(\mathbb{1}+ב)\left(\mathbb{1}-\left(\frac{\mathbb{1}+ב\left(\pi^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}\right)^{\Delta}-\mathbb{1}\right)\frac{\mathbb{1}}{ב}}\right],\left[\sqrt{(\mathbb{1}+ב)\left(\mathbb{1}-\left(\frac{ב\left(\pi^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\Delta}-\mathbb{1}\right)\frac{\mathbb{1}}{ב}}\right]\\ \left[\sqrt{(\mathbb{1}+ב)\left(\mathbb{1}-\left(\frac{\mathbb{1}+ב\left(v^{\mathcal{G}}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}\right)^{\Delta}-\mathbb{1}\right)\frac{\mathbb{1}}{ב}}\right],\left[\sqrt{(\mathbb{1}+ב)\left(\mathbb{1}-\left(\frac{\mathbb{1}+ב\left(v^{u}(\mathbb{j})\right)^{⊿}}{\mathbb{1}+ב}\right)^{\Delta}-\mathbb{1}\right)\frac{\mathbb{1}}{ב}}\right]\end{array}\right)$$

$$\theta^{\Delta}=\left(\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}}{\beth}(\mathbb{1}+\beth)\left(\mathbb{1}-\left(\frac{\beth\left(\mu^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\Delta}-\mathbb{1}\right)}\right],\left[\sqrt{\frac{\mathbb{1}}{\beth}(\mathbb{1}+\beth)\left(\mathbb{1}-\left(\frac{\beth\left(\mu^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\Delta}-\mathbb{1}\right)}\right]\\ \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\pi^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\Delta}\right.}\right],\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\pi^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\Delta}\right.}\right]\\ \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(v^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\Delta}\right.}\right],\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(v^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\Delta}\right.}\right]\end{array}\right)$$

Definition 11: Consider a class of a IVT-SFVs$\Delta_e=$ $\left(\left[\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})\right],\left[\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})\right],\left[v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})\right]\right), e=\mathbb{1},2,\ldots,\psi$ the IVT-SFSWPA operator is characterized as follows:

$$\mathrm{IVT-SFSWPA}\left(\Delta_{\mathbb{1}}.\Delta_2,\ldots,\Delta_\psi\right)=\oplus_{e=\mathbb{1}}^{\psi}\wp_e\Delta_e$$

Where$\wp_e=\frac{(\mathbb{1}+A(\Delta_e))}{\sum_{e=\mathbb{1}}^{\psi}(\mathbb{1}+A(\Delta_e))}$and $\mathrm{A}(\Delta_e)=\sum_{e=\mathbb{1}}^{\psi}supp\left(\Delta_e,\Delta_{\mathbb{j}}\right)$

Theorem 1: Consider a class of an IVT-SFV*s* $\Delta_e=$ $\left(\left[\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})\right],\left[\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})\right],\left[v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})\right]\right), e=\mathbb{1},2,\ldots,\psi$. The accumulated value of the ISFSWPA operator, so we have the following expression:

$$\mathrm{IVT-SFSWPA}\left(\Delta_{\mathbb{1}}.\Delta_2,\ldots,\Delta_\psi\right)$$

$$=\left(\begin{array}{c}\left[\left(\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left(u_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\right)\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}}\right),\left(\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left(u_e^{u}(\mathbb{j})\right)^{\Delta}\right)\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}}\right)\right]\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(\pi_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(\pi_e^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)}\right]\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(v_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(v_e^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)}\right]\end{array}\right)$$

Proof:

We can prove the above expression by using the principle of mathematical induction. For $\psi=2$, we can write:

$$\mathcal{p}_{\mathbb{1}}\Delta_{\mathbb{1}}=\left(\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\ddot{\jmath})\right)^{\vartriangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\mathcal{p}_{\mathbb{1}}}}, \sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{u}(\ddot{\jmath})\right)^{\vartriangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\mathcal{p}_{\mathbb{1}}}}\right],\\ \left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}, \sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{u}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]\\ \left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{\mathcal{G}}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}, \sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{u}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]\end{array}\right)$$

$$\mathcal{p}_2\Delta_2=\left(\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{\mathcal{G}}(\ddot{\jmath})\right)^{\vartriangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\mathcal{p}_{\vartriangle}}}, \sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{u}(\ddot{\jmath})\right)^{\vartriangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\mathcal{p}_{2}}}\right],\\ \left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{\mathcal{G}}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{\vartriangle}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}, \sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{u}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]\\ \left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{\mathcal{G}}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{\vartriangle}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}, \sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{u}(\ddot{\jmath})\right)^{\vartriangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathcal{p}_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]\end{array}\right)$$

$$IVT-SFSWPA(\Delta_{\mathbb{1}},\Delta_2)=\bigoplus_{e=\mathbb{1}}^{2}\mathcal{p}_e\Delta_e$$

$$
\begin{aligned}
&\mathrm{IVT}-\mathrm{SFSWPA}(\Delta_{\mathbb{1}},\Delta_{⊿}) \\
&=\left(\begin{array}{l}
\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)}^{\,p_{\mathbb{1}}},\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)}^{\,p_{\mathbb{1}}}\right]\oplus \\
\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)}^{\,p_{2}},\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{u}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)}^{\,p_{2}}\right], \\
\left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}},\sqrt{\left((\mathbb{1}+\beth))\left(\frac{\beth\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right] \\
\left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}},\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]\oplus \\
\left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}},\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]\oplus \\
\left[\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}.\sqrt{\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{p_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}}\right]
\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{IVT}-\mathrm{SFSWPA}(\Delta_{\mathbb{1}},\Delta_{2}) \\
&=\Bigg(\Bigg[\sqrt{\begin{array}{l}\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{\mathbb{1}}}+\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{2}} \\ -\frac{\beth}{\mathbb{1}+\beth}\left(\left(\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{\mathbb{1}}}\right).\left(\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{2}}\right)\right)\end{array}}\Bigg], \\
&\Bigg[\sqrt{\begin{array}{l}\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{\mathbb{1}}}+\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{u}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{2}} \\ -\frac{\beth}{\mathbb{1}+\beth}\left(\left(\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{\mathbb{1}}}\right).\left(\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{2}^{u}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)^{\wp_{2}}\right)\right)\end{array}}\Bigg] \\
&\Bigg[\sqrt{\begin{array}{l}\frac{\beth}{\mathbb{1}+\beth}\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}+\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}-\mathbb{1} \\ +\beth\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right).\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right)\right)\end{array}}\Bigg], \\
&\Bigg[\sqrt{\begin{array}{l}\frac{\beth}{\mathbb{1}+\beth}\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}+\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}-\mathbb{1} \\ +\beth\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right).\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{2}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right)\right)\end{array}}\Bigg] \\
&\Bigg[\sqrt{\begin{array}{l}\frac{\beth}{\mathbb{1}+\beth}\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}+\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}-\mathbb{1} \\ +\beth\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right).\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{\mathcal{G}}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right)\right)\end{array}}\Bigg], \\
&\Bigg[\sqrt{\begin{array}{l}\frac{\beth}{\mathbb{1}+\beth}\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}+\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}-\mathbb{1} \\ +\beth\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{\mathbb{1}}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{\mathbb{1}}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right).\left(\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v_{2}^{u}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{2}}-\mathbb{1}\right)\frac{\mathbb{1}}{\beth}\right)\right)\end{array}}\Bigg]\Bigg)
\end{aligned}
$$

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_2)$$
$$=\left(\begin{array}{l}\left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{⊿}\left(\mathbb{1}-\left(\mu_e^{\mathcal{G}}(\mathbb{j})\right)^{⊿}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)},\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{⊿}\left(\mathbb{1}-\left(\mu_e^{u}(\mathbb{j})\right)^{⊿}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{⊿}\left(\frac{ב\left(\pi_e^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{⊿}\left(\frac{ב\left(\pi_e^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{⊿}\left(\frac{ב\left(v_e^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{⊿}\left(\frac{ב\left(v_e^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right]\end{array}\right)$$

Assume that the above expression is true for $\psi = k$.

$$IVT-SFSWPA(\Delta_{\mathbb{1}},\Delta_2)=\oplus_{e=\mathbb{1}}^{k}\wp_e\Delta_e$$

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_k)$$
$$=\left(\begin{array}{l}\left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{k}\left(\mathbb{1}-\left(\mu_e^{\mathcal{G}}(\mathbb{j})\right)^{⊿}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)},\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{k}\left(\mathbb{1}-\left(\mu_e^{u}(\mathbb{j})\right)^{⊿}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{k}\left(\frac{ב\left(\pi_e^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right]\left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{k}\left(\frac{ב\left(\pi_e^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right]\\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{k}\left(\frac{ב\left(v_e^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right]\left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{k}\left(\frac{ב\left(v_e^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right]\end{array}\right)$$

Thus, we prove the expression for $\psi = k + 1$.

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_k)=\oplus_{e=\mathbb{1}}^{k}\wp_k\Delta_k\oplus\wp_{k+\mathbb{1}}\Delta_{k+\mathbb{1}}$$

$$= \left(\begin{array}{l} \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{k}\left(\mathbb{1}-\left(\mu_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}\right)} \right], \\ \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{k}\left(\mathbb{1}-\left(\mu_e^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}\right)} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k}\left(\frac{\beth\left(\pi_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k}\left(\frac{\beth\left(\pi_e^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right] \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k}\left(\frac{\beth\left(v_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k}\left(\frac{\beth\left(v_e^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right] \end{array} \right) \oplus \left(\begin{array}{l} \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{k+\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_{k+\mathbb{1}}}\right)} \right], \\ \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu_{k+\mathbb{1}}^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_{k+\mathbb{1}}}\right)} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}(\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{k+\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{k+\mathbb{1}}}-\mathbb{1}} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}(\mathbb{1}+\beth)\left(\frac{\beth\left(\pi_{k+\mathbb{1}}^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{k+\mathbb{1}}}-\mathbb{1}} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}(\mathbb{1}+\beth)\left(\frac{\beth\left(v_{k+\mathbb{1}}^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{k+\mathbb{1}}}-\mathbb{1}} \right], \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}(\mathbb{1}+\beth)\left(\frac{\beth\left(v_{k+\mathbb{1}}^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_{k+\mathbb{1}}}-\mathbb{1}} \right] \end{array} \right)$$

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_2,\ldots,\Delta_{k+\mathbb{1}})$$

$$= \left(\begin{array}{l} \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{k+\mathbb{1}}\left(\mathbb{1}-\left(\mu_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}\right)} \right], \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{k+\mathbb{1}}\left(\mathbb{1}-\left(\mu_e^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}\right)} \right] \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k+\mathbb{1}}\left(\frac{\beth\left(\pi_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right], \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k+\mathbb{1}}\left(\frac{\beth\left(\pi_e^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right] \\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k+\mathbb{1}}\left(\frac{\beth\left(v_e^{\mathcal{G}}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right], \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{k+\mathbb{1}}\left(\frac{\beth\left(v_e^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)} \right] \end{array} \right)$$

Hence proved.

Theorem 2: Consider a class of IVT-SFVs $\Delta_e = ([\mu_e^L(j), \mu_e^U(j)], [\pi_e^L(j), \pi_e^U(j)], [v_e^L(j), v_e^U(j)]), e = 1,2, \ldots, \psi$. If all IVT-SFVs are identical, that is, $\Delta_e = \Delta$for all $e = 1,2, \ldots, \psi$, then we have:

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}}, \Delta_2, \ldots, \Delta_{\psi}) = \Delta$$

Proof:

Since all IVT-SFVs $\Delta_e = ([\mu_e^L(j), \mu_e^U(j)], [\pi_e^L(j), \pi_e^U(j)], [v_e^L(j), v_e^U(j)]), e = 1,2, \ldots, \psi,$ are identical, that is, $\Delta_e = \Delta$for all $e = 1,2, \ldots, \psi$, then we have:

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_{2},\dots,\Delta_{\psi})$$

$$=\left(\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left(\mu_{e}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}\right)},\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left(\mu_{e}^{u}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\wp_e}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(\pi_{e}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(\pi_{e}^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(v_{e}^{\mathcal{G}}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(v_{e}^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\wp_e}-\mathbb{1}\right)}\right]\end{array}\right)$$

$$=\left[\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu^{\iota}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\sum_{\mathrm{i}=\mathbb{1}}^{\psi}\wp_e}\right)},\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu^{u}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\sum_{\mathrm{i}=\mathbb{1}}^{\psi}\wp_e}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi^{\iota}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\sum_{\mathrm{i}=\mathbb{1}}^{\psi}\vartheta_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\sum_{\mathrm{i}=\mathbb{1}}^{\psi}\vartheta_e}-\mathbb{1}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v^{\iota}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\sum_{\mathrm{i}=\mathbb{1}}^{\psi}\vartheta_e}-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\sum_{\mathrm{i}=\mathbb{1}}^{\psi}\vartheta_e}-\mathbb{1}\right)}\right]\end{array}\right],$$

$$\left(\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu^{\iota}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)},\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\left(\mathbb{1}-\left(\mu^{u}(\mathbb{j})\right)^{⊿}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi^{\iota}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)-\mathbb{1}\right)},\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(\pi^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)-\mathbb{1}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v^{\iota}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)-\mathbb{1}\right)}\right]\left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\left(\frac{\beth\left(v^{u}(\mathbb{j})\right)^{⊿}+\mathbb{1}}{\mathbb{1}+\beth}\right)-\mathbb{1}\right)}\right]\end{array}\right)$$

$$= \left(\begin{array}{c} \left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\mathbb{1}+\left(\mu^{\iota}(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)}\right], \left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\mathbb{1}+\left(\mu^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)}\right], \\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(\pi^{\iota}(\mathbb{j})\right)^{\Delta}+\mathbb{1}-\mathbb{1}\right)}\right], \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(\pi^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}-\mathbb{1}\right)}\right], \\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(v^{\iota}(\mathbb{j})\right)^{\Delta}+\mathbb{1}-\mathbb{1}\right)}\right], \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(v^{u}(\mathbb{j})\right)^{\Delta}+\mathbb{1}-\mathbb{1}\right)}\right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\left(\mu^{\iota}(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)}\right], \left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\left(\mu^{u}(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)}\right] \\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(\pi^{\iota}(\mathbb{j})\right)^{\Delta}\right)}\right], \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(\pi^{u}(\mathbb{j})\right)^{\Delta}\right)}\right] \\ \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(v^{\iota}(\mathbb{j})\right)^{\Delta}\right)}\right], \left[\sqrt{\frac{\mathbb{1}}{ב}\left(ב\left(v^{u}(\mathbb{j})\right)^{\Delta}\right)}\right] \end{array} \right)$$

$$= \left(\mu(\mathbb{j}), \pi(\mathbb{j}), v(\mathbb{j})\right) = \Delta$$

Hence, the theorem is proved.

Theorem 3: Consider a collection of IVT-SFVs

$\Delta_e = ([\mu_e^L(j), \mu_e^U(j)], [\pi_e^L(j), \pi_e^U(j)], [v_e^L(j), v_e^U(j)]), e = 1,2, \dots, \psi$, such that all the IVT-SFVs are identical, i.e.,$\Delta_e = \Delta, \forall e = 1,2, \dots, \psi$. Then, we obtain:

$$I\text{VT} - \text{SFWPOA}\left(\Delta_{\mathbb{1}}, \Delta_2, \dots, \Delta_{\psi}\right) = \Delta$$

Theorem 4: Consider any two sets of IVT-SFVs

Δ_e= $([\grave{}\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^{u}(\mathbb{j})], [\grave{}\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^{u}(\mathbb{j})], [\grave{}v_e^{\mathcal{G}}(\mathbb{j}), v_e^{u}(\mathbb{j})])$ and

$\Delta'_e = ([\grave{}\mu_e'^{\mathcal{G}}(\mathbb{j}), \mu_e'^{u}(\mathbb{j})], [\grave{}\pi_e'^{\mathcal{G}}(\mathbb{j}), \pi_e'^{u}(\mathbb{j})], [\grave{}v_e'^{\mathcal{G}}(\mathbb{j}), v_e'^{u}(\mathbb{j})]),\ e = \mathbb{1}, 2 \dots \dots . \psi$. If $\Delta_e \leq \Delta'_e$, then we get $I\text{VT} - \text{SFWPOA}\left(\Delta_{\mathbb{1}}, \Delta_2, \dots, \Delta_{\psi}\right) \leq q - I\text{VT} - \text{SFWPOA}\left(\Delta'_{\mathbb{1}}, \Delta'_2, \dots, \Delta'_{\psi}\right)$.

Definition 12: For a class of IVT-SFVs, Δ_e= $\left(\left[\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^{u}(\mathbb{j})\right], \left[\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^{u}(\mathbb{j})\right], \left[v_e^{\mathcal{G}}(\mathbb{j}), v_e^{u}(\mathbb{j})\right]\right), e = \mathbb{1}, 2, \dots, \psi$ the IVT-SFSWOPA operator is characterized as follows:

$$I\text{VT} - \text{SFWPOA}\left(\Delta_{\mathbb{1}}. \Delta_2, \dots, \Delta_{\psi}\right) = \oplus_{e=\mathbb{1}}^{\psi} \wp_e \Delta_{\rho(e)}$$

Where $\wp_e = \frac{(\mathbb{1}+A(\Delta_e))}{\sum_{e=\mathbb{1}}^{\psi}(\mathbb{1}+A(\Delta_e))}$ and $A(\Delta_e) = \sum_{\substack{e=\mathbb{1} \\ e \neq \mathbb{j}}}^{\psi} supp\left(\Delta_e, \Delta_{\mathbb{j}}\right)$ and let $\rho(\mathbb{1}), \rho(2), \dots, \rho(\psi)$ a set of permutations of $e = \mathbb{1}, 2, \dots, \psi \Delta\rho_{(e-\mathbb{1})} \geq \Delta_e$.

Definition 13: For a class of an IVT-SFVs Δ_e= $\left(\left[\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^{u}(\mathbb{j})\right], \left[\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^{u}(\mathbb{j})\right], \left[v_e^{\mathcal{G}}(\mathbb{j}), v_e^{u}(\mathbb{j})\right]\right), e = \mathbb{1}, 2, \dots, \psi$ the $I\text{VT} - \text{SFSWPG}$ operator is characterized as follows:

$$I\text{VT} - \text{SFSWPG}\left(\Delta_{\mathbb{1}}. \Delta_2, \dots, \Delta_{\psi}\right) = \otimes_{e=\mathbb{1}}^{\psi} \Delta_e^{\wp_e}$$

Where $\wp_e = \frac{(\mathbb{1}+A(\Delta_e))}{\sum_{e=\mathbb{1}}^{\psi}(\mathbb{1}+A(\Delta_e))}$ and $\text{A}(\Delta_e) = \sum_{\substack{e=\mathbb{1} \\ e \neq \mathbb{j}}}^{\psi} supp\left(\Delta_e, \Delta_{\mathbb{j}}\right)$.

Theorem 5: Consider a class of an IVT-SFVs

Δ_e= $\left(\left[\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^{u}(\mathbb{j})\right], \left[\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^{u}(\mathbb{j})\right], \left[v_e^{\mathcal{G}}(\mathbb{j}), v_e^{u}(\mathbb{j})\right]\right), e = \mathbb{1}, 2, \dots, \psi$. By applying the accumulated value of the IVT-SFSWOPA operator, we obtain the following result:

Journal of Contemporary Decision Science
Volume 2, Issue 1 (2026) 323-348

$$IVT - \mathrm{SFSWOPA}\left(\Delta_{\mathbb{1}},\Delta_2,\ldots,\Delta_{\psi}\right)$$
$$=\left(\begin{array}{l}\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left(\mu^{\mathcal{G}}_{\rho(e)}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\mathscr{p}_e}\right)}\right],\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left(\mu^{u}_{\rho(e)}(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\mathscr{p}_e}\right)}\right]\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(\pi^{\mathcal{G}}_{\rho(e)}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathscr{p}_e}-\mathbb{1}\right)}\right],\left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(\pi^{u}_{\rho(e)}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathscr{p}_e}-\mathbb{1}\right)}\right]\\ \left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(v^{\mathcal{G}}_{\rho(e)}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathscr{p}_e}-\mathbb{1}\right)}\right],\left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left(v^{u}_{\rho(e)}(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathscr{p}_e}-\mathbb{1}\right)}\right]\end{array}\right)$$

Where $\mathscr{p}_e=\frac{(\mathbb{1}+A(\Delta_e))}{\sum_{e=\mathbb{1}}^{\psi}(\mathbb{1}+A(\Delta_e))}$ and $A(\Delta_e)=\sum_{\substack{e=\mathbb{1}\\ e\neq\mathbb{j}}}^{\psi} supp\left(\Delta_e,\Delta_{\mathbb{j}}\right)$ and $\rho(\mathbb{1}),\rho(2),\ldots,\rho(\psi)$ be set of the permutation of $e=\mathbb{1},2,\ldots,\psi\Delta\rho_{(e-\mathbb{1})}\geq\Delta_e$.

Theorem 6: Consider a class of IVT-SFVs $\Delta_e=\left(\left[\mu^{\mathcal{G}}_e(\mathbb{j}),\mu^{u}_e(\mathbb{j})\right],\left[\pi^{\mathcal{G}}_e(\mathbb{j}),\pi^{u}_e(\mathbb{j})\right],\left[v^{\mathcal{G}}_e(\mathbb{j}),v^{u}_e(\mathbb{j})\right]\right), e=\mathbb{1},2,\ldots,\psi$. The accumulated value of the IVT-SFSWPG operator, so we get the following:

$$IVT - \mathrm{SFSWPG}(\Delta_{\mathbb{1}},\Delta_2\ldots,\Delta_e)$$
$$=\left(\begin{array}{l}\left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left((\mu^{\mathcal{G}}_e)(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathscr{p}_e}-\mathbb{1}\right)}\right],\left[\sqrt{\frac{\mathbb{1}}{\beth}\left((\mathbb{1}+\beth)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{\beth\left((\mu^{u}_e)(\mathbb{j})\right)^{\triangle}+\mathbb{1}}{\mathbb{1}+\beth}\right)^{\mathscr{p}_e}-\mathbb{1}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((\pi^{\mathcal{G}}_e)(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\mathscr{p}_e}\right)}\right],\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((\pi^{u}_e)(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\mathscr{p}_e}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((v^{\mathcal{G}}_e)(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\mathscr{p}_e}\right)}\right],\left[\sqrt{\frac{\mathbb{1}+\beth}{\beth}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((v^{u}_e)(\mathbb{j})\right)^{\triangle}\left(\frac{\beth}{\mathbb{1}+\beth}\right)\right)^{\mathscr{p}_e}\right)}\right]\end{array}\right)$$

Theorem 7. Consider a class of IVT-SFVs

$\Delta_e=([`\mu^{\mathcal{G}}_e(\mathbb{j}),\mu^{u}_e(\mathbb{j})],[`\pi^{\mathcal{G}}_e(\mathbb{j}),\pi^{u}_e(\mathbb{j})],[`v^{\mathcal{G}}_e(\mathbb{j}),v^{u}_e(\mathbb{j})]), e=\mathbb{1},2\ldots\ldots\psi$, which implies that $\Delta_e=\Delta$. Then we obtain

$$IVT - \mathrm{SFSWPG}\left(\Delta_{\mathbb{1}},\Delta_2,\ldots,\Delta_{\psi}\right)=\Delta$$

Theorem 8: Consider two sets of IVT-SFVs

$\Delta_e=([`\mu^{\mathcal{G}}_e(\mathbb{j}),\mu^{u}_e(\mathbb{j})],[`\pi^{\mathcal{G}}_e(\mathbb{j}),\pi^{u}_e(\mathbb{j})],[`v^{\mathcal{G}}_e(\mathbb{j}),v^{u}_e(\mathbb{j})])$ and $\Delta'_e=([`\mu'^{\mathcal{G}}_e(\mathbb{j}),\mu'^{u}_e(\mathbb{j})],[`\pi'^{\mathcal{G}}_e(\mathbb{j}),\pi'^{u}_e(\mathbb{j})],[`v'^{\mathcal{G}}_e(\mathbb{j}),v'^{u}_e(\mathbb{j})]), e=\mathbb{1},2\ldots\ldots\psi$. If $\Delta_e\leq\Delta'_e$. Then we get:

$$\mathrm{IVT} - \mathrm{SFSWPG}\left(\Delta_{\mathbb{1}},\Delta_2,\ldots,\Delta_{\psi}\right)\leq\mathrm{IVT} - \mathrm{SFSWPG}\left(\Delta'_{\mathbb{1}},\Delta'_2,\ldots,\Delta'_{\psi}\right).$$

Definition 14: For the class of IVT-SFVs $\Delta_e=\left(\left[\mu^{\mathcal{G}}_e(\mathbb{j}),\mu^{u}_e(\mathbb{j})\right],\left[\pi^{\mathcal{G}}_e(\mathbb{j}),\pi^{u}_e(\mathbb{j})\right],\left[v^{\mathcal{G}}_e(\mathbb{j}),v^{u}_e(\mathbb{j})\right]\right), e=\mathbb{1},2,\ldots,\psi$ The IVT-SFSWPOG operator is characterized as follows:

$$\mathrm{IVT-SFSWPOG}(\Delta_{\mathbb{1}}.\Delta_2,\dots,\Delta_\psi)=\oplus_{e=\mathbb{1}}^{\psi}\Delta_{\rho(e)}^{\wp_e}$$

Where $\wp_e=\frac{(\mathbb{1}+A(\Delta_e))}{\sum_{e=\mathbb{1}}^{\psi}(\mathbb{1}+A(\Delta_e))}$ and $A(\Delta_e)=\sum_{\substack{e=\mathbb{1}\\ e\neq \mathbb{j}}}^{\psi} supp(\Delta_e,\Delta_{\mathbb{j}})$ and let $\rho(\mathbb{1}),\rho(2),\dots,\rho(\psi)$ be set of permutations of $e=\mathbb{1},2,\dots,\psi\Delta\rho_{(e-\mathbb{1})}\geq\Delta_e$.

Theorem 9: Consider a class of IVT-SFVs $\Delta_e=\left(\left[\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})\right],\left[\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})\right],\left[v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})\right]\right),e=\mathbb{1},2,\dots,\psi$. The accumulated value of the $\mathrm{IVT-SFSWPOG}$ operator. So, we get the following:

$$\mathrm{IVT-SFSWPOG}(\Delta_{\mathbb{1}},\Delta_{\Delta}\dots,\Delta_e)$$
$$=\left(\begin{array}{c}\left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{ב\left((\mu_e^{\mathcal{G}})(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right],\left[\sqrt{\frac{\mathbb{1}}{ב}\left((\mathbb{1}+ב)\prod_{e=\mathbb{1}}^{\psi}\left(\frac{ב\left((\mu_e^{u})(\mathbb{j})\right)^{\Delta}+\mathbb{1}}{\mathbb{1}+ב}\right)^{\wp_e}-\mathbb{1}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((\pi_e^{\mathcal{G}})(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)}\right],\left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((\pi_e^{u})(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)}\right],\\ \left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((v_e^{\mathcal{G}})(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)}\right],\left[\sqrt{\frac{\mathbb{1}+ב}{ב}\left(\mathbb{1}-\prod_{e=\mathbb{1}}^{\psi}\left(\mathbb{1}-\left((v_e^{u})(\mathbb{j})\right)^{\Delta}\left(\frac{ב}{\mathbb{1}+ב}\right)\right)^{\wp_e}\right)}\right]\end{array}\right)$$

Theorem 10. Consider a class of IVT-SFVs
$\Delta_e=([`\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})],[`\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})],[`v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})]),e=\mathbb{1},2\dots\dots.\psi$, which implies that$\Delta_e=\Delta$.
Then we obtain

$$\mathrm{IVT-SFSWPOG}\left(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_\psi\right)=\Delta$$

Theorem 11: Consider any two sets of IVT-SFVs
$\Delta_e=([`\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})],[`\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})],[`v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})])$and$\Delta'_e=([`\mu_e'^{\mathcal{G}}(\mathbb{j}),\mu'^{u}_e(\mathbb{j})],[`\pi_e'^{\mathcal{G}}(\mathbb{j}),\pi'^{u}_e(\mathbb{j})],[`v'^{\mathcal{G}}_e(\mathbb{j}),v_e'^{u}(\mathbb{j})]),\ e=\mathbb{1},2\dots\dots.\psi$. If$\Delta_e\leq\Delta'_e$.Then we get:

$$\mathrm{IVT-SFSWPOG}(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_\psi)\leq q-\mathrm{IVT-SFSWPOG}(\Delta'_{\mathbb{1}},\Delta'_2,\dots,\Delta'_\psi).$$

Theorem 12: For any two sets of IVT-SFVs $\Delta_e=([`\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})],[`\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})],[`v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})])$ and$\Delta'_e=([`\mu_e'^{\mathcal{G}}(\mathbb{j}),\mu'^{u}_e(\mathbb{j})],[`\pi_e'^{\mathcal{G}}(\mathbb{j}),\pi'^{u}_e(\mathbb{j})],[`v'^{\mathcal{G}}_e(\mathbb{j}),v_e'^{u}(\mathbb{j})]),\ e=\mathbb{1},2\dots\dots.\psi$. If$\Delta_e\leq\Delta'_e$, so we get:

$$\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_\psi)\leq q-\mathrm{IVT-SFSWPA}(\Delta'_{\mathbb{1}},\Delta'_2,\dots,\Delta'_\psi).$$

Theorem 13. Consider a class of IVT-SFVs
$\Delta_e=([`\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})],[`\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})],[`v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})]),e=\mathbb{1},2\dots\dots.\psi$. If
$\Delta^-=\left(me\psi\{\mu_e^{\mathcal{G}},\mu_e^{\mathcal{G}}\},max\{\pi_e^{\mathcal{G}},\pi_e^{u}\},max\{v_e^{\mathcal{G}},v_e^{u}\}\right)$ and
$\Delta^+=\left(max\{\mu_e^{\mathcal{G}},\mu_e^{\mathcal{G}}\},min\{\pi_e^{\mathcal{G}},\pi_e^{u}\},min\{v_e^{\mathcal{G}},v_e^{u}\}\right)$, then we get

$$\Delta^-\leq\mathrm{IVT-SFSWPA}(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_\psi)\leq\Delta^+$$

Theorem 14. Consider a class of IVT-SFVs
$\Delta_e=([`\mu_e^{\mathcal{G}}(\mathbb{j}),\mu_e^{u}(\mathbb{j})],[`\pi_e^{\mathcal{G}}(\mathbb{j}),\pi_e^{u}(\mathbb{j})],[`v_e^{\mathcal{G}}(\mathbb{j}),v_e^{u}(\mathbb{j})]),e=\mathbb{1},2\dots\dots.\psi$. If
$\Delta^-=\left(min\{\mu_e^{\mathcal{G}},\mu_e^{\mathcal{G}}\},max\{\pi_e^{\mathcal{G}},\pi_e^{u}\},max\{v_e^{\mathcal{G}},v_e^{u}\}\right)$ and
$\Delta^+=\left(max\{\mu_e^{\mathcal{G}},\mu_e^{\mathcal{G}}\},min\{\pi_e^{\mathcal{G}},\pi_e^{u}\},min\{v_e^{\mathcal{G}},v_e^{u}\}\right)$, then we get

$$\Delta^-\leq\mathrm{IVT-SFSWPOA}(\Delta_{\mathbb{1}},\Delta_2,\dots,\Delta_\psi)\leq\Delta^+$$

Theorem 15. Consider a class of IVT-SFVs $\Delta_e = ([\grave{}\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^u(\mathbb{j})], [\grave{}\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^u(\mathbb{j})], [\grave{}v_e^{\mathcal{G}}(\mathbb{j}), v_e^u(\mathbb{j})]), e = \mathbb{1}, 2 \dots\dots.\psi$. If $\Delta^- = \left(min\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, max\{\pi_e^{\mathcal{G}}, \pi_e^u\}, max\{v_e^{\mathcal{G}}, v_e^u\}\right)$ and $\Delta^+ = \left(max\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, min\{\pi_e^{\mathcal{G}}, \pi_e^u\}, me\psi\{v_e^{\mathcal{G}}, v_e^u\}\right)$, then we get

$$\Delta^- \leq \text{IVT} - \text{SFSWPG}(\Delta_{\mathbb{1}}, \Delta_2, \dots, \Delta_\psi) \leq \Delta^+$$

Theorem 16. Consider a class of IVT-SFVs $\Delta_e = ([\grave{}\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^u(\mathbb{j})], [\grave{}\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^u(\mathbb{j})], [\grave{}v_e^{\mathcal{G}}(\mathbb{j}), v_e^u(\mathbb{j})]), e = \mathbb{1}, 2 \dots\dots.\psi$. If $\Delta^- = \left(min\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, max\{\pi_e^{\mathcal{G}}, \pi_e^u\}, max\{v_e^{\mathcal{G}}, v_e^u\}\right)$ and $\Delta^+ = \left(max\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, min\{\pi_e^{\mathcal{G}}, \pi_e^u\}, min\{v_e^{\mathcal{G}}, v_e^u\}\right)$, then we get

$$\Delta^- \leq \text{IVT} - \text{SFSWPOG}(\Delta_{\mathbb{1}}, \Delta_2, \dots, \Delta_\psi) \leq \Delta^+$$

Theorem 17. Consider a class of IVT-SFVs $\Delta_e = ([\grave{}\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^u(\mathbb{j})], [\grave{}\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^u(\mathbb{j})], [\grave{}v_e^{\mathcal{G}}(\mathbb{j}), v_e^u(\mathbb{j})]), e = \mathbb{1}, 2 \dots\dots.\psi$. If $\Delta^- = \left(min\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, max\{\pi_e^{\mathcal{G}}, \pi_e^u\}, max\{v_e^{\mathcal{G}}, v_e^u\}\right)$ and $\Delta^+ = \left(max\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, min\{\pi_e^{\mathcal{G}}, \pi_e^u\}, min\{v_e^{\mathcal{G}}, v_e^u\}\right)$, then we get

$$\Delta^- \leq \text{IVSFSWPWA}(\Delta_{\mathbb{1}}, \Delta_2, \dots, \Delta_\psi) \leq \Delta^+$$

Theorem 18. Consider a class of IVT-SFVs $\Delta_e = ([\grave{}\mu_e^{\mathcal{G}}(\mathbb{j}), \mu_e^u(\mathbb{j})], [\grave{}\pi_e^{\mathcal{G}}(\mathbb{j}), \pi_e^u(\mathbb{j})], [\grave{}v_e^{\mathcal{G}}(\mathbb{j}), v_e^u(\mathbb{j})]), e = \mathbb{1}, 2 \dots.\psi$. If $\Delta^- = \left(min\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, max\{\pi_e^{\mathcal{G}}, \pi_e^u\}, max\{v_e^{\mathcal{G}}, v_e^u\}\right)$ and $\Delta^+ = \left(max\{\mu_e^{\mathcal{G}}, \mu_e^{\mathcal{G}}\}, min\{\pi_e^{\mathcal{G}}, \pi_e^u\}, min\{v_e^{\mathcal{G}}, v_e^u\}\right)$, then we get

$$\Delta^- \leq \text{IVT} - \text{SFSWPWG}(\Delta_{\mathbb{1}}, \Delta_2, \dots, \Delta_\psi) \leq \Delta^+$$

4. Calculation of Problems Based on Aggregation Operators

This section evaluates the uncertainty in human assessments using the IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG operators. T-spherical fuzzy (T-SF) information provides comprehensive details regarding the membership grade (MG), non-membership grade (NMG), and abstinence/hesitancy degree (AH) of an object.

Consider a finite set of alternatives denoted by $\partial = (\partial_1, \partial_2, \dots, \partial_\psi)$ and a finite set of attributes represented by $\mathcal{C} = (\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_m)$. Assume that each attribute is associated with a weight vector $F = (F_1, F_2, \dots, F_\psi)$, such that $F_e \geq 0$ and $\sum_{e=1}^{\psi} F_e = 1$.

The decision-maker expresses the evaluation information in the form of T-spherical fuzzy information satisfying the condition $0 \leq \mu^\Delta(j) + \pi^\Delta(j) + \nu^\Delta(j) \leq 1, \forall j \in \vartheta$. The assessment information is represented as $\Delta_e = ([\mu_e^L(j), \mu_e^U(j)], [\pi_e^L(j), \pi_e^U(j)], [\nu_e^L(j), \nu_e^U(j)]), e = 1,2, \dots, \psi$. These values are arranged into a standard decision matrix $\partial = (\mathcal{C}_{\sigma e})$.

Based on this framework, an innovative algorithm for the multi-attribute decision-making (MADM) process is proposed in the next section.

4.1. SWARA Method for Evaluating the Criteria Weights

The SWARA process begins with ranking the criteria according to their relative importance based on expert knowledge and experience. After ranking, each criterion is compared with the previous one to determine its relative significance. Subsequently, the comparative importance coefficients are calculated, and the corresponding weights of the criteria are determined. The SWARA method is widely used for estimating the weights of criteria in MADM problems due to its simplicity and effectiveness. The procedure of the SWARA method can be summarized through the following steps:

Step 1: Calculate the score values. The score values of $S^*(\eta_{Kj})$obtained by SWARA are computed using Equation (b), according to *Definition 3*.

Step 2: Evaluate the comparative importance of the criteria. The relative importance of each criterion is assessed based on expert judgments.

Step 3: Determine the sequential comparative importance of each criterion by comparing criterion $j$with the preceding criterion $j-1$.

Step 4: Calculate the coefficient K_j, which is defined as follows:

$$K_j = \begin{cases} 1, & j = 1 \\ S_j + 1, & j > 1 \end{cases}$$

where S_j represents the proportional significance of the score value.

Step 5: Estimate the recalculated weight. The recalculated weight $P_j$is determined as follows:

$$P_j = \begin{cases} 1, & j = 1 \\ \dfrac{K_j - 1}{K_j}, & j > 1 \end{cases}$$

Step 6: Compute the normalized criteria weights. The criteria weights are calculated as follows:

$$W_j = \frac{P_j}{\sum_{j=1}^{n} P_j}$$

4.2. Algorithm

Step 1: The decision-maker collects information about the real-world alternatives in the form of T-spherical fuzzy (T-SF) data and represents it as a preference decision matrix.

Step 2: The collected data are classified into benefit-type and non-benefit-type attributes. If the given information contains different types of attributes, the original decision matrix is transformed into a normalized decision matrix. Otherwise, this normalization process is not required.

Step 3: Compute the support degree by using the following expression:

$$supp(\Delta_{e\sigma}, \Delta_{\sigma k}) = \mathbb{1} - D(\Delta_{e\sigma}, \Delta_{\sigma k})$$

And $D(\Delta_e, \Delta_{\mathbb{j}}) = \frac{\mathbb{1}}{6}\Big(\left|\left(\mu_e^{\mathcal{G}}\right)^{\varDelta} - \left(\mu_k^{\mathcal{G}}\right)^{\varDelta}\right| + |(\mu_\sigma^u)^{\varDelta} - (\mu_k^u)^{\varDelta}| + \left|\left(v_e^{\mathcal{G}}\right)^{\varDelta} - \left(v_\sigma^{\mathcal{G}}\right)^{\varDelta}\right| + |(v_\sigma^u)^{\varDelta} - (v_k^u)^{\varDelta}| + \left|\left(\pi_e^{\mathcal{G}}\right)^{\varDelta} - \left(\pi_\sigma^{\mathcal{G}}\right)^{\varDelta}\right| + |(\pi_\sigma^u)^{\varDelta} - (\pi_k^u)^{\varDelta}|\Big)$.

Step 4: Calculate the degree of weighted support:

$$A(\Delta) = \sum_{\substack{e=\mathbb{1} \\ e\neq\mathbb{j}}}^{\psi} E_e\, supp(\Delta_e, \Delta_{\mathbb{j}}), E = (E_{\mathbb{1}}, E_2, \dots, E_\psi), E_e > 0 \text{ and } \sum_{\substack{e=\mathbb{1} \\ e\neq\mathbb{j}}}^{\psi} E_e = \mathbb{1}$$

Step 5: Investigate the degree of support:

$$\text{Where } \emptyset = \frac{E_e(\mathbb{1} + A(\Delta_e))}{\sum_{\substack{e=\mathbb{1} \\ e\neq\mathbb{j}}}^{\psi} E_e(\mathbb{1} + A(\Delta_e))}$$

Step 6: The provided information is aggregated by using the IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG operators.

Step 7: The score values of each alternative are carefully examined to identify the best-ranked option.

Step 8: A ranking procedure is then performed to reorder all alternatives according to their assessment values.

4.3 Experimental Case Study

Artificial intelligence (AI) is transforming numerous sectors, particularly healthcare, by enhancing diagnostic accuracy and improving treatment planning. AI-based systems can accurately identify abnormalities and disease indicators in medical images such as MRIs and X-rays, enabling healthcare professionals to diagnose patients more efficiently and reliably. In addition, AI-driven chatbots and virtual healthcare assistants are increasingly being used to provide personalized patient care, answer medical inquiries, and offer mental health support. By integrating AI technologies into healthcare systems, medical providers can improve patient outcomes, streamline operational processes, and ultimately save lives.

4.3.1 Applications of AI in daily life

Artificial intelligence has become an essential part of everyday life and is widely applied across various domains. In video games, AI is used to create intelligent virtual opponents, dynamic gaming environments, and adaptive gameplay based on player behavior. Financial institutions employ AI algorithms to detect fraudulent activities by analyzing transaction patterns, identifying anomalies, and reporting suspicious operations in real time.

In the healthcare sector, AI systems analyze medical data such as MRIs, X-rays, and patient records to assist physicians in diagnosing diseases, identifying trends, and recommending treatment plans. AI-powered image recognition systems are capable of recognizing objects, individuals, locations, and activities in images and videos, making them valuable in areas such as security surveillance, facial recognition, and medical imaging.

Self-driving vehicles utilize AI technologies to perceive their environment, make driving decisions, and navigate safely without human intervention. Furthermore, advances in natural language processing (NLP) enable machines to understand, interpret, and generate human language. Applications of NLP include sentiment analysis, chatbots, and language translation systems.

Many companies, such as Amazon, Spotify, and Netflix, use AI algorithms to analyze user preferences and behavior in order to provide personalized recommendations for products, music, and movies. Similarly, virtual assistants such as Siri, Alexa, and Google Assistant rely on AI algorithms to understand voice commands, perform tasks, and provide users with relevant information.

In this study, five AI-based alternatives and four evaluation criteria are considered for the proposed MADM framework. The alternatives are defined as follows:

A_1: *Smart Home Systems* – AI-driven systems that manage household devices such as lighting, thermostats, and security cameras.

A_2: *Financial Forecasting* – AI algorithms that analyze financial data to predict stock prices, investment opportunities, and market trends.

A_3: *Language Translation* – AI-based translation systems that facilitate communication between people speaking different languages.

A_4: *Content Creation* – AI tools that generate text, images, and music, assisting creators in producing high-quality content efficiently.

A_5: *Supply Chain Optimization* – AI technologies that improve supply chain efficiency through inventory management, demand forecasting, and logistics optimization.

These alternatives represent different aspects of AI technology and demonstrate its diverse applications and capabilities. Machine learning enables systems to learn from data, neural networks simulate the architecture of the human brain, NLP facilitates communication between humans and machines, robotics integrates intelligent physical capabilities, and computer vision allows machines to interpret visual information.

The alternatives are evaluated according to the following criteria:

G_1: *Machine Learning* – The ability of AI systems to improve performance automatically through learning from data without explicit programming.

G_2: *Natural Language Processing (NLP)* – The capability of AI systems to understand, interpret, and generate human language for effective human–machine interaction.

G_3: *Computer Vision* – The ability of AI systems to process and analyze visual information from images and videos.

G_4: *Algorithmic Decision-Making* – The capability of AI algorithms to optimize decisions based on predefined rules, objectives, and input data.

The decision-maker evaluates the alternatives according to these predefined criteria (Table 1). The corresponding weight vector assigned to the criteria is given as follows: $(0.10,\ 0.25,\ 0.35,\ 0.30)$, Table 2.

Using the proposed MADM framework and aggregation operators, the decision-maker assesses the suitability and effectiveness of the considered AI alternatives under the interval-valued T-spherical fuzzy environment.

Table 1
Details about the AI suppliers

	G_1	G_2	G_3	G_4
A_1	$\begin{pmatrix}[0.28, 0.35],\\ [0.47, 0.55],\\ [0.55, 0.56]\end{pmatrix}$	$\begin{pmatrix}[0.44, 0.51],\\ [0.26, 0.33],\\ [0.24, 0.33]\end{pmatrix}$	$\begin{pmatrix}[0.15, 0.28],\\ [0.44, 0.55],\\ [0.61, 0.66]\end{pmatrix}$	$\begin{pmatrix}[0.46, 0.47],\\ [0.33, 0.46],\\ [0.15, 0.17]\end{pmatrix}$
A_2	$\begin{pmatrix}[0.34, 0.88],\\ [0.22, 0.34],\\ [0.11, 0.1\mathbb{1}5]\end{pmatrix}$	$\begin{pmatrix}[0.12, 0.15],\\ [0.17, 0.27],\\ [0.45, 0.77]\end{pmatrix}$	$\begin{pmatrix}[0.26, 0.33],\\ [0.48, 0.49],\\ [0.15, 0.17]\end{pmatrix}$	$\begin{pmatrix}[0.23, 0.32],\\ [0.21, 0.47],\\ [0.14, 0.19]\end{pmatrix}$
A_3	$\begin{pmatrix}[0.55, 0.6\mathbb{1}1],\\ [0.33, 0.35],\\ [0.25, 0.35]\end{pmatrix}$	$\begin{pmatrix}[0.37, 0.42],\\ [0.15, 0.18],\\ [0.23, 0.24]\end{pmatrix}$	$\begin{pmatrix}[0.49, 0.59],\\ [0.16, 0.18],\\ [0.22, 0.23]\end{pmatrix}$	$\begin{pmatrix}[0.45, 0.49],\\ [0.47, 0.56],\\ [0.11, 0.15]\end{pmatrix}$
A_4	$\begin{pmatrix}[0.11, 0.15],\\ [0.45, 0.55],\\ [0.44, 0.47]\end{pmatrix}$	$\begin{pmatrix}[0.16, 0.19],\\ [0.44, 0.52],\\ [0.12, 0.17]\end{pmatrix}$	$\begin{pmatrix}[0.13, 0.18],\\ [0.79, 0.82],\\ [0.19, 0.21]\end{pmatrix}$	$\begin{pmatrix}[0.15, 0.17],\\ [0.45, 0.56],\\ [0.18, 0.44]\end{pmatrix}$
A_5	$\begin{pmatrix}[0.21, 0.24],\\ [0.37, 0.41],\\ [0.46, 0.57]\end{pmatrix}$	$\begin{pmatrix}[0.53, 0.57],\\ [0.45, 0.48],\\ [0.14, 0.22]\end{pmatrix}$	$\begin{pmatrix}[0.45, 0.48],\\ [0.17, 0.23],\\ [0.47, 0.52]\end{pmatrix}$	$\begin{pmatrix}[0.36, 0.38],\\ [0.15, 0.19],\\ [0.12, 0.29]\end{pmatrix}$

Table 2
Shows computed degree of unknown weights

Alternatives	$IVT-SFSWPA$	$IVT-SFSWPG$
A_1	$\begin{pmatrix}[0.3734, 0.4228],\\ [0.3916, 0.4867],\\ [0.4630, 0.4954]\end{pmatrix}$	$\begin{pmatrix}[0.3705, 0.4201],\\ [0.3935, 0.4897],\\ [0.4732, 0.5075]\end{pmatrix}$
A_2	$\begin{pmatrix}[0.2624, 0.6103],\\ [0.3169, 0.3625],\\ [0.2928, 0.4766]\end{pmatrix}$	$\begin{pmatrix}[0.2616, 0.5533],\\ [0.3214, 0.3654],\\ [0.2970, 0.5136]\end{pmatrix}$
A_3	$\begin{pmatrix}[0.4744, 0.5389],\\ [0.3328, 0.3842],\\ [0.2144, 0.2506]\end{pmatrix}$	$\begin{pmatrix}[0.4725, 0.5357],\\ [0.3365, 0.3921],\\ [0.2145, 0.2623]\end{pmatrix}$
A_4	$\begin{pmatrix}[0.1394, 0.1739],\\ [0.5766, 0.6390],\\ [0.2885, 0.3164]\end{pmatrix}$	$\begin{pmatrix}[0.1394, 0.1739],\\ [0.5984, 0.6557],\\ [0.2920, 0.3204]\end{pmatrix}$
A_5	$\begin{pmatrix}[0.4223, 0.4531],\\ [0.3356, 0.3680],\\ [0.3681, 0.4434]\end{pmatrix}$	$\begin{pmatrix}[0.4185, 0.4482],\\ [0.3384, 0.3713],\\ [0.3724, 0.4502]\end{pmatrix}$

Table 2
Continued

Alternatives	$IVT-SFSWPWA$	$IVT-SFSWPWG$
A_1	$\begin{pmatrix}[0.3734, 0.4228],\\ [0.3916, 0.4867],\\ [0.4630, 0.4559]\end{pmatrix}$	$\begin{pmatrix}[0.3705, 0.4201],\\ [0.3935, 0.4897],\\ [0.4732, 0.5075]\end{pmatrix}$
A_2	$\begin{pmatrix}[0.2624, 0.6103],\\ [0.3169, 0.3625],\\ [0.2928, 0.4766]\end{pmatrix}$	$\begin{pmatrix}[0.2616, 0.5533],\\ [0.3214, 0.3654],\\ [0.2970, 0.5136]\end{pmatrix}$
A_3	$\begin{pmatrix}[0.4744, 0.5389],\\ [0.3328, 0.3842],\\ [0.2144, 0.2615]\end{pmatrix}$	$\begin{pmatrix}[0.4725, 0.5357],\\ [0.3365, 0.3921],\\ [0.2145, 0.2623]\end{pmatrix}$
A_4	$\begin{pmatrix}[0.1394, 0.1739],\\ [0.5766, 0.6390],\\ [0.2099, 0.3164]\end{pmatrix}$	$\begin{pmatrix}[0.1394, 0.1739],\\ [0.5984, 0.6557],\\ [0.2920, 0.3204]\end{pmatrix}$
A_5	$\begin{pmatrix}[0.4223, 0.4531],\\ [0.3356, 0.3680],\\ [0.3681, 0.4434]\end{pmatrix}$	$\begin{pmatrix}[0.4185, 0.4482],\\ [0.3384, 0.3713],\\ [0.3724, 0.4502]\end{pmatrix}$

Step 2: The collected data are classified into benefit-type and cost-type attributes. If the given information contains multiple types of attributes, the original decision matrix must be transformed into a normalized decision matrix. Otherwise, this normalization procedure is not required.

Step 3: Calculate the support degree by using the following expression:

$$supp(\Delta_{e\sigma}, \Delta_{\sigma k}) = \mathbb{1} - D(\Delta_{e\sigma}, \Delta_{\sigma k})$$

And $D(\Delta_e, \Delta_{\mathbb{j}}) = \frac{\mathbb{1}}{6}\left(\left|\left(\mu_e^{\mathcal{G}}\right)^{\Delta} - \left(\mu_k^{\mathcal{G}}\right)^{\Delta}\right| + |(\mu_\sigma^u)^{\Delta} - (\mu_k^u)^{\Delta}| + \left|\left(v_e^{\mathcal{G}}\right)^{\Delta} - \left(v_\sigma^{\mathcal{G}}\right)^{\Delta}\right| + |(v_\sigma^u)^{\Delta} - (v_k^u)^{\Delta}| + \left|\left(\pi_e^{\mathcal{G}}\right)^{\Delta} - \left(\pi_\sigma^{\mathcal{G}}\right)^{\Delta}\right| + |(\pi_\sigma^u)^{\Delta} - (\pi_k^u)^{\Delta}|\right)$.

Step 4: Calculate the degree of weighted support:

$A(\Delta) = \sum_{\substack{e=\mathbb{1}\\ e\neq \mathbb{j}}}^{\psi} E_e\, supp(\Delta_e, \Delta_{\mathbb{j}}), E = (E_{\mathbb{1}}, E_2, \dots, E_\psi), E_e > 0$ And $\sum_{\substack{e=\mathbb{1}\\ e\neq \mathbb{j}}}^{\psi} E_e = \mathbb{1}$

Step 5: Examine the degree of support:

Where $\emptyset = \frac{E_e(\mathbb{1}+A(\Delta_e))}{\sum_{\substack{e=\mathbb{1}\\ e\neq \mathbb{j}}}^{\psi} E_e(\mathbb{1}+A(\Delta_e))}$

Step 6: The provided data are aggregated using the IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG operators, Table 3.

Table 3
Covered score values corresponding to each alternative

Alternatives	$IVT-SFSWPA$	$IVT-SFSWPG$	$IVT-SFSWPWA$	$IVT-SFSWPWG$
A_1	0.5111	0.4945	0.5772	0.5700
A_2	0.6418	0.6012	0.6775	0.6479
A_3	0.7276	0.7202	0.7140	0.7110
A_4	0.3994	0.382$\mathbb{1}$	0.4999	0.4846
A_5	0.6202	0.6071	0.6479	0.6435

Step 7: To identify the most suitable alternative, the score values of all alternatives are carefully examined and compared, Table 4.

Table 4
Ranking of score values

Operator	Ranking
$\mathbf{IVT-SFSWPA}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\mathbf{IVT-SFSWPG}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\mathbf{IVT-SFSWPWA}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\mathbf{IVT-SFSWPWG}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$

It is clear that A_3 is the best alternative among all alternatives, as it has the highest score value across all the considered operators, Figure 2.

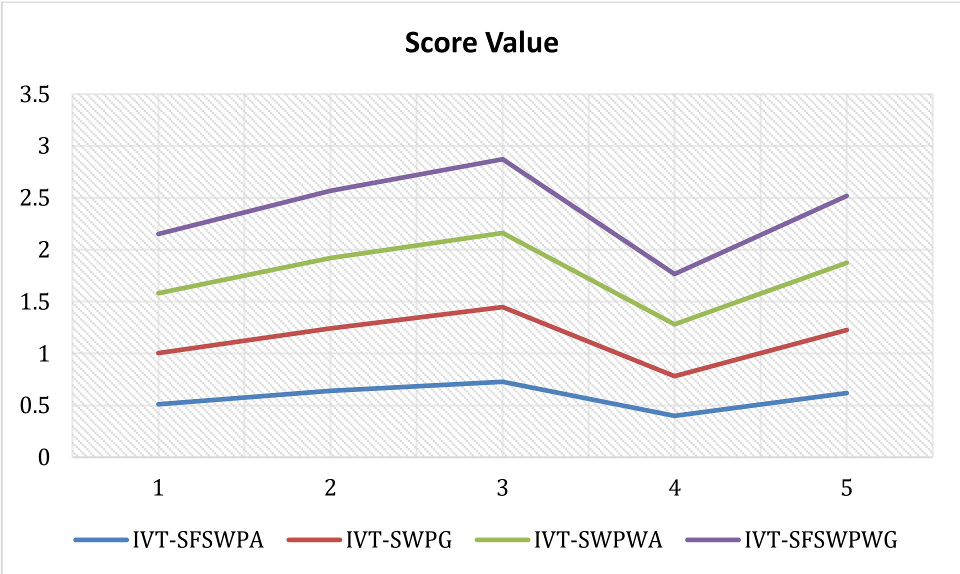

Fig. 2. Results of alternatives

4.3. Influence Study

This analysis examines the effect of varying the parameter ϕ from 1 to 100 on the ranking results of the alternatives. The obtained results indicate that, despite changes in the parameter values, the ranking order of the alternatives remains unchanged for all tested cases. In particular, the alternative A_3consistently retains the highest ranking under different parameter settings. This demonstrates the stability, robustness, and reliability of the proposed operators and decision-making framework. Furthermore, the consistency of the ranking results confirms that the proposed approach is not significantly affected by variations in parameter values, Tables 5 and 6.

Table 5
Results obtained by IVT-SFSWPWA

Parametric value	Ranking and ordering
$\wp = \mathbf{1}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{5}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{20}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{35}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{55}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{70}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{80}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{90}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\wp = \mathbf{100}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$

Table 6
Results obtained by IVT-SFSWPWG

Parametric values	Ranking and ordering
$\boldsymbol{p} = \mathbf{1}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{5}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{20}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{35}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{55}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{70}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{80}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{90}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
$\boldsymbol{p} = \mathbf{100}$	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$

Clearly, A_3 is the most suitable alternative among all the considered alternatives, since it possesses the highest score value under all the proposed aggregation operators. Therefore, the obtained results indicate that Language Translation (A_3) is the optimal and most effective AI alternative among the evaluated options.

5. Conclusions

In this paper, we first introduced the fundamental operational laws of interval-valued T-spherical fuzzy sets (IVT-SFSs) based on the Sugeno–Weber (SW) t-norms and t-conorms (TCNs). Subsequently, several novel aggregation operators, namely IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG, were developed by integrating the SW aggregation framework within the IVT-SFS environment. The proposed aggregation operators were thoroughly investigated with respect to their important mathematical properties, including boundedness, idempotency, and monotonicity.

Furthermore, the applicability of the proposed IVT-SFSWPA, IVT-SFSWPG, IVT-SFSWPWA, and IVT-SFSWPWG operators was demonstrated through a multi-attribute decision-making (MADM) framework. A practical case study on prioritizing and evaluating artificial intelligence alternatives was conducted to validate the effectiveness of the proposed methodology. The influence of varying parameter values was also analyzed to examine the stability and robustness of the proposed operators.

The most appropriate AI alternative was selected by considering several key criteria and characteristics within the developed decision-making framework. In addition, a comparative analysis with existing aggregation approaches was performed, which confirmed the superiority, consistency, and reliability of the proposed methods. The obtained results and sensitivity analysis demonstrate that the developed aggregation operators provide an effective and flexible tool for solving complex MADM problems under interval-valued T-spherical fuzzy environments.

Acknowledgement
This research was not funded by any grant.

Conflicts of Interest
The authors declare no conflicts of interest.

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