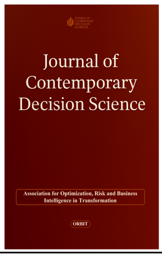
ORBIT ASSOCIATION

Journal of Contemporary Decision and Sciences

Journal homepage: www.cds-journal.org
ISSN:3104-3380



Selection of Optimal Stock for Business Investment Using Pythagorean Fuzzy Aczel-Alsina CoCoSo-Method

Amal Kumar Adak[1,*], Stefan Jovcic[2], Darko Bozanic[3], Dragan Marinkovic[4,5,6]

[1] Department of Mathematics, Ganesh Dutt College, Begusarai, India
[2] Department of Transport Management, Marketing and Logistics, University of Pardubice, Pardubice, Czech Republic
[3] Military academy, University of Defence in Belgrade, Belgrade, Serbia
[4] Department of Structural Analysis, Technische Universität Berlin, Berlin, Germany
[5] Institute of Mechanical Science, Vilnius Gediminas Technical University, Lithuania
[6] University College, Korea University, Seoul, Republic of Korea

ARTICLE INFO

Article history:
Received 16 February 2026
Received in revised form30 March 2026
Accepted 18 May 2026
Available online 22 May 2026

Keywords:
Pythagorean Fuzzy Sets; Aczel Alsina aggregation operator; Unknown weight of Criteria; CoCoSo Method; Stock investment; MCDM.

ABSTRACT

Selecting an optimal stock for business investment is a complex multi-attribute decision-making (MADM) problem often characterized by vague, inconsistent, and uncertain information. This article introduces a novel decision framework to address this challenge under Pythagorean fuzzy environments, offering greater flexibility than intuitionistic fuzzy sets. To aggregate uncertain evaluation information, Pythagorean fuzzy Aczel-Alsina weighted aggregation operators are developed, leveraging the parameterized family of Aczel-Alsina t-norms and t-conorms to capture the full range of decision-makers' risk attitudes. A modified score function is first proposed to effectively rank aggregated values. Subsequently, the combined compromise solution (CoCoSo) method is extended within the Pythagorean fuzzy Aczel-Alsina context to evaluate and rank investment alternatives under conditions of unknown attribute weights. The viability and procedural clarity of the proposed methodology are demonstrated through a numerical example involving stock selection in the financial market with ten alternatives and eight criteria. Finally, a comprehensive comparative analysis using different aggregation operators and score functions is conducted, confirming that the proposed method yields robust, consistent, and discriminative results, thereby offering a superior tool for strategic investment decision-making under complex uncertainty.

*Corresponding author.
E-mail address: amaladak1@gmail.com

1. Introduction

The selection of an optimal stock for business investment represents a quintessentially complex multi-criteria decision-making (MCDM) problem, characterized not only by conflicting financial indicators (e.g., return on equity, price-to-earnings ratio, volatility, liquidity) but also by profound uncertainty, vagueness, and often incomplete information. Traditional probabilistic models and even standard fuzzy set extensions such as intuitionistic fuzzy sets (IFS) frequently fall short in capturing scenarios where decision-makers exhibit hesitation or where membership and non-membership degrees sum to more than one. In this context, Pythagorean fuzzy sets (PFS) have emerged as a powerful generalization, offering a larger admissible space of uncertainty and thereby providing a more faithful representation of subjective judgments in high-stakes financial environments.

To address uncertainty in practical problems, Zadeh [1] introduced the concept of fuzzy sets (FS). Building on this idea, several extensions of fuzzy sets have been developed to better capture vagueness and imprecision. One such extension is the intuitionistic fuzzy set (IFS), proposed by Atanassov [2], which incorporates both membership and nonmembership degrees for each element, with the condition that the sum of membership and non-membership lie in between 0 and 1.

Yager [3] overcomes the situation when sum of membership degree and non-membership degree greater than 1 and introduced concept of Pythagorean Fuzzy Sets (PyFSs). PyFS is an extension of IFS with the condition that the square sum of the membership degree and the non-membership degree is less than or equal to 1. The concept of Pythagorean fuzzy sets (PFS) gives the larger preference domain for decision makers (DM). DMs can define their support and against the degree of membership as $\varphi = 0.78$, $\varrho = 0.52$. In this case, $0.78 + 0.52 > 1$ is not valid in IFS but squaring $0.78^2 + 0.52^2 < 1$ implies that PyFSs are more suitable than the IFSs. There are lots of research work done of PyFSs in the theoretical as well as practical areas [4, 5].

Adak et al. [6] introduced two approaches based on positive and negative ideal solutions to solve assignment problems under Pythagorean fuzzy conditions by applying a spherical distance measure and a new scoring method. Karamat et al. [7] discussed the theory of prioritized aggregation operators based on Aczel-Alsina t-norm and t-conorm for managing the theory of C-PF information, such as C-PF prioritized Aczel-Alsina averaging, C-PF prioritized Aczel-Alsina ordered averaging, C-PF prioritized Aczel-Alsina geometric, and C-PF prioritized Aczel-Alsina ordered geometric operators. Karamat et al. [8] introduced an advanced method for multiple-attribute decision-making, which employs arithmetic and geometric operations to create aggregation operators on Pythagorean fuzzy sets. Khalid et al. [9] explored the impact of behavioral biases on the investment decisions of individual investors of the Pakistan Stock Exchange, with the mediation and moderation mechanism. Sarkar et al. [10] presented a full discussion of theoretical backgrounds, methodological development, and practical implementation of MCDM within the framework of business analytics in marketing, finance and supply chain management, operations, human resource management, and strategic management. Yin et al. [11] constructed indicators to investigate the impact of ESG performance on stock price synchronicity.

The Aczel–Alsina family of t-norms and t-conorms, parameterized by $a \geq 1$, offers a flexible continuum from fully compensatory ($a \to 1$) to strictly non-compensatory ($a \to \infty$), yet existing applications naively apply a single a value globally across all criteria. Gabriel *et al.* [12] examined the applicability of the Combined Compromise Solution (CoCoSo) method (improved version of combined compromise solution), by a real case study in a university campus and compared the obtained results to other MCDMs such as Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE), Weighted Sum Method (WSM) and Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS). Adak *et al.* [13] studied on integrated Pythagorean fuzzy TOPSIS framework for supplier selection with uncertain weights. Hussain *et al.* [14] generalized the concept of pythagorean fuzzy sets (PyFSs) in the framework by utilizing the basic operations of Aczel-Alsina aggregation models. Jafari *et al.* [15] discussed the challenges of knowledge adoption in the steel supply chain and proposed CoCoSo method to address survival of the SC. Wang *et al.* [16] proposed a logistics service provider selection method based on fuzzy Aczel-Alsina power averaging operator.

Anthony et al. [17] studied on the long and short run association subsisting between stock market development (market capitalisation, value of transactions, number of deal and all share index), and Nigerian economicgrowth (RGDP) with quarterly data from 1986 to 2017. They applied the Autoregressive Distributed Lag (ARDL) model for the purpose of estimation. Arena et al.[18] identified the different factors that could influence the stock market investing of 384 Filipino tertiary students within the National Capital Region (NCR), following the theory of planned behavior, financial literacy theory, and risk perception theory. Chen [19] studied the growing influence of women in the marketplace since global human rights progress, gender equality has become a key focus, leading to greater female participation in economic activities. Haryadi et al. [20] analyzed the effect of transaction security using Blockchain technology on customers' decision to invest in the stock market that offers high security through decentralization, transparency, and immutability. This technology used for creating a safe and sustainable investment ecosystem in investment decisions and recommends better investment security standards for regulators. Ma [21] studied on the logic for specific implementation, timing of transactions, and risk management measures associated with investor expectations for the future and market sentiment. Investors prefer stocks with growing performance as they are essentially investing in the future. Segun et al. [22] investigated the relationship between stock market development and performance of the Nigerian economy. They used estimation techniques like co-integration, Granger Causality and Error Correction to check whether the exchange indices have impacted on the macroeconomic performance of Nigeria. Vijay et al. [23] investigated on factors influencing investor's perception viz., return on investment, market trend or risk, short term profitability, price of the share, dividend policy, past financial performance, company reputation, reputation of the board, current earnings of the company and expert opinion

The paper is organized as follows: Section 1 introduces the selection of the optimal business stock investment, including a comprehensive literature review, the identification of research gaps, and the contributions of the present study. Section 2 presents fundamental concepts related to Pythagorean fuzzy numbers, such as score and accuracy functions, along with Pythagorean fuzzy Aczel–Alsina weighted aggregation operators to assist readers in understanding the basic principles. Section 3 describes the CoCoSo method integrated with the Pythagorean fuzzy Aczel–Alsina aggregation operator for handling unknown criterion weights. Section 4 defines the decision-making frameworks employed in selecting the optimal stock for business investment. Section 5 discusses the results obtained and provides a comparative analysis. Finally, Section 6 concludes the paper by outlining its limitations and suggesting future research directions.

2. Theoretical Aspects

[2] An Intuitionistic fuzzy set (IFS) I in X is of the form

$$I = \{\langle \varsigma, \varphi_I(\varsigma), \varrho_I(\varsigma)\rangle : \varsigma \in X\},$$

where $\varphi_I : X \to [0,1]$, $\varrho_I : X \to [0,1]$ are membership grade (MG) and non-membership grade (NMG) with $0 \le \varphi_I(\varsigma) + \varrho_I(\varsigma) \le 1$, for all $\varsigma \in X$.

Indeterminacy $\pi_I(\varsigma) = 1 - \varphi_I(\varsigma) - \varrho_I(\varsigma)$.

[3] A Pythagorean fuzzy set P is

$$P = \{\langle \varsigma, \varphi_P(\varsigma), \varrho_P(\varsigma)\rangle | \varsigma \in X\},$$

where $\varphi_P(\varsigma) : X \to [0,1]$ and $\varrho_P(\varsigma) : X \to [0,1]$ are MG and NMG respectively with $0 \leq (\varphi_P(\varsigma))^2 + (\varrho_P(\varsigma))^2 \leq 1$.

The indeterminacy is $\varpi_P(\varsigma) = \sqrt{1 - (\varphi_P(\varsigma))^2 - (\varrho_P(\varsigma))^2}$. The order pair (φ, ϱ) denoted as pythagorean fuzzy number (PFN).

2.1 Basic Operations on PFNs

Consider three PFNs $p = \langle \varphi, \varrho \rangle$, $p_1 = \langle \varphi_1, \varrho_1 \rangle$ and $p_2 = \langle \varphi_2, \varrho_2 \rangle$. Then

(i) $\bar{p} = \langle \varrho, \varphi \rangle$

(ii) $p_1 \cup p_2 = \langle \max\{\varphi_1, \varphi_2\}, \min\{\varrho_1, \varrho_2\} \rangle$

(iii) $p_1 \cap p_2 = \langle \min\{\varphi_1, \varphi_2\}, \max\{\varrho_1, \varrho_2\} \rangle$

2.2 Comparison measure of PFNs

Different types of functions based on MG and NMG are used to rank PFNs. This is particularly important in decision-making applications, where ranking alternatives plays a critical role. The score function $s(p)$ of $p = \langle \varphi, \varrho \rangle$ is

$$s(p) = (\varphi)^2 - (\varrho)^2 \tag{1}$$

where $s(p) \in [-1, 1]$.

The accuracy function $a(p)$ of $p = \langle \varphi, \varrho \rangle$ is

$$a(p) = (\varphi)^2 + (\varrho)^2 \tag{2}$$

where $h(p) \in [0, 1]$. Let $p = \langle \varphi, \varrho \rangle$ be a pythagorean fuzzy number. The modified score function S_{ms} of p is defined as

$$S_{ms}(p) = (|\varphi^2 - \varrho^2|) + \left(\frac{e^{|\varphi^2 - \varrho^2|}}{e^{|\varphi^2 - \varrho^2|} + 1} - \frac{1}{2} \right) \pi^2. \tag{3}$$

Let us consider a PyFN, $p = (0.65, 0.39)$, then the different score values of this PyFN are $s(p) = 0.270$, $h(p) = 0.574$, $S_{ms} = 0.933$.

2.3 Aggregation Operators

A t-norm is a function $T : [0,1] \times [0,1] \to [0,1]$ if
(T1) $T(\zeta_1, 1) = \zeta_1$,
(T2) $T(\zeta_1, \zeta_2) = T(\zeta_2, \zeta_1)$,
(T3) $T(\zeta_1, T(\zeta_2, \zeta_3)) = T(T(\zeta_1, \zeta_2), \zeta_3)$,
(T4) $T(\zeta_1, \zeta_2) \leq T(\zeta_1, \zeta_3)$, whenever $\zeta_2 \leq \zeta_3$, for all $\zeta_1, \zeta_2, \zeta_3 \in [0, 1]$. Some well-known TNs are listed below.

1. Minimum TN: $T_{\min}(\zeta_1, \zeta_2) = \min(\zeta_1, \zeta_2)$;
2. Product TN: $T_{pro}(\zeta_1, \zeta_2) = \zeta_1 \zeta_2$;
3. Lukasiewicz TN: $T_{Luk}(\zeta_1, \zeta_2) = \max(\zeta_1 + \zeta_2 - 1, 0)$;
4. Drastic triangular norm:

$$T_{dra}(\zeta_1, \zeta_2) = \begin{cases} \zeta_1, & \text{if } \zeta_2 = 1 \\ \zeta_2, & \text{if } \zeta_1 = 1 \\ 0, & \text{otherwise} \end{cases}$$

5. Aczel Alsina t-norm:

$$T_A^{\phi}(\zeta_1,\zeta_2)=\begin{cases} T_{dra}(\zeta_1,\zeta_2), & \text{if } \phi=0\\ \min\{\zeta_1,\zeta_2\}, & \text{if } \phi=\infty\\ e^{((-\log\zeta_1)^{\phi}+(-\log\zeta_2)^{\phi}))^{\frac{1}{\phi}}}, & \text{otherwise}\end{cases}$$

$S:[0,1]\times[0,1]\to[0,1]$ is a t-co-norm if
(S1) $S(\zeta_1,0)=\zeta_1$,
(S2) $S(\zeta_1,\zeta_2)=S(\zeta_2,\zeta_1)$
(S3) $S(\zeta_1,S(\zeta_2,\zeta_3))=S(S(\zeta_1,\zeta_2),\zeta_3)$, for all $\zeta_1,\zeta_2,\zeta_3\in[0,1]$
(S4) If $\zeta_1\le p$ and $\zeta_2\le q$, then $S(\zeta_1,\zeta_2)\le S(p,q)$, for all $\zeta_1,\zeta_2,p,q\in[0,1]$ Some well-known TCNs are listed below.
1. Maximum TCN: $S_{\max}(\zeta_1,\zeta_2)=\max(\zeta_1,\zeta_2)$;
2. Probabilistic Sum: $S_{sum}(\zeta_1,\zeta_2)=\zeta_1+\zeta_2-\zeta_1\zeta_2$;
3. Lukasiewicz TCN: $S_{Luk}(\zeta_1,\zeta_2)=\max(\zeta_1+\zeta_2,1)$;
4. Drastic triangular conorm:

$$S_{dra}(\zeta_1,\zeta_2)=\begin{cases}\zeta_1, & \text{if } \zeta_2=0\\ \zeta_2, & \text{if } \zeta_1=0\\ 1, & \text{otherwise}\end{cases}$$

5. Aczel Alsina t-conorm:

$$T_A^{\phi}(\zeta_1,\zeta_2)=\begin{cases} S_{dra}(\zeta_1,\zeta_2), & \text{if } \phi=0\\ \max\{\zeta_1,\zeta_2\}, & \text{if } \phi=\infty\\ 1-e^{((-\log(1-\zeta_1))^{\phi}+(-\log(1-\zeta_2))^{\phi}))^{\frac{1}{\phi}}}, & \text{otherwise}\end{cases}$$

Let $P=(\varphi,\varrho)$, $P_1=(\varphi_1,\varrho_1)$ and $P_2=(\varphi_2,\varrho_2)$ be three PyFNs, $a\ge 1, b>1$. Then

(i) $P_1P_2=\left(\sqrt{1-e^{((-\log(1-\varphi_1^2))^a+(-\log(1-\varphi_2^2))^a))^{\frac{1}{a}}}},e^{-((-\log\varrho_1)^a+(-\log\varrho_2)^a))^{\frac{1}{a}}}\right)$

(ii) $P_1P_2=\left(e^{-((-\log\varphi_1)^a+(-\log\varphi_2)^a))^{\frac{1}{a}}},\sqrt{1-e^{((-\log(1-\varrho_1^2))^a+(-\log(1-\varrho_2^2))^a))^{\frac{1}{a}}}}\right)$

(iii) $bF=\left(\sqrt{1-e^{-(b(-\log(1-\varphi^2))^a)^{\frac{1}{a}}}},e^{-(b(\log\varrho)^a)^{\frac{1}{a}}}\right)$

(iv) $F^b=\left(e^{-(b(\log\varphi)^a)^{\frac{1}{a}}},\sqrt{1-e^{-(b(-\log(1-\varrho^2))^a)^{\frac{1}{a}}}}\right)$

Let $P_i=(\varphi_i,\varrho_i)$, $i=1,2,\dots,n$, is a collection of PyFSs. Suppose the weight vector $w=(w_1,w_2,\dots,w_n)^T$, of the P_i, $i=1,2,\dots,n$ follows $w_i>0$, $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. Then, Pythagorean Fuzzy Aczel-Alsina Weighted Aggregation (PyF-AA-WA) operator is a function $(F)^n\to F$ and defined as

$$PyF-AA-WA(P_1,P_2,\dots,P_n)=_{i=1}^{n}(w_iP_i)=w_1P_1w_2P_2\cdots w_nP_n \quad (4)$$

Let $P_i=(\varphi_i,\varrho_i)$, $i=1,2,\dots,n$, is a collection of PyFSs. Suppose the weight vector $w=(w_1,w_2,\dots,w_n)^T$, of the P_i, $i=1,2,\dots,n$ follows $w_i>0$, $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. Then, the accumulated value of PyFSs by PyF-AA-WA operator is defined as

$$PyF-AA-WA(P_1,P_2,\dots,P_n)=\left(\sqrt{1-e^{-(\sum_{i=1}^{n}w_i(-\log(1-\varphi_i^2))^a)^{\frac{1}{a}}}},e^{-(\sum_{i=1}^{n}w_i(-\log\varrho_i)^a)^{\frac{1}{a}}}\right) \quad (5)$$

Let us consider six PyFNs $(0.33, 0.64)$, $(0.92, 0.12)$, $(0.77, 0.42)$, $(0.83, 0.39)$, $(0.56, 0.47)$, $(0.29, 0.59)$ with associated weight $(0.19, 0.12, 0.18, 0.16, 0.21, 0.14)$ and taking $a = 4$, then the value of PyF-AA-weighted aggregation on these six PyFNs is $(0.831, 0.276)$. Let $P_i = (\varphi_i, \varrho_i)$, $i = 1, 2, \ldots, n$, is a collection of PyFSs. Suppose the weight vector $w = (w_1, w_2, \ldots, w_n)^T$, of the P_i, $i = 1, 2, \ldots, n$ follows $w_i > 0$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Then, Pythagorean Fuzzy Aczel Alsina Weighted Geometric (PyF-AA-WG) aggregation operator is a function $(F)^n \to F$ and defined as

$$PyF - AA - WG(P_1, P_2, \ldots, P_n) =_{i=1}^{n} (P_i^{w_i}) = P_1^{w_1} P_2^{w_2} \cdots P_n^{w_n} \tag{6}$$

Let $P_i = (\varphi_i, \varrho_i), i = 1, 2, \ldots, n$, is a collection of PyFSs. Suppose the weight vector $w = (w_1, w_2, \ldots, w_n)^T$, of the P_i, $i = 1, 2, \ldots, n$ follows $w_i > 0$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Then, the accumulated value of PyFSs by PyF-AA-WG aggregation operator is defined as

$$PyF - AA - WG(P_1, P_2, \ldots, P_n) = \left(e^{-(\sum_{i=1}^{n} (-\log(\varphi_i)^{w_i})^a)^{\frac{1}{a}}}, \sqrt{1 - e^{-(\sum_{i=1}^{n} (-\log(1-\varrho_i^2)^{w_i})^a)^{\frac{1}{a}}}}, \right) \tag{7}$$

Let us consider six PyFNs $(0.64, 0.24)$, $(0.75, 0.22)$, $(0.65, 0.42)$, $(0.81, 0.27)$, $(0.65, 0.37)$, $(0.49, 0.53)$ with associated weight $(0.18, 0.13, 0.19, 0.17, 0.20, 0.13)$ and taking $a = 4$, then the value of PyF-AA-weighted geometric aggregation on these six PyFNs is $(0.885, 0.219)$.

3. Proposed Methodology

The study utilizes the Pythagorean fuzzy Aczel-Alsina weighted aggregation (PyF-AA-W) operator and the Pythagorean fuzzy Aczel-Alsina weighted geometric aggregation (PyF-AA-WG) operator to aggregate optimal business stock for investment by experts under compensatory and non-compensatory decision logic respectively. Finally, the combined compromise solution (CoCoSo) method, together with three distinct cases of appraisal scores, is applied to derive robust rankings of optimal business stock for investment. This integrated framework enables a systematic comparison of different aggregation strategies and facilitates the reliable selection of the best optimal business stock for investment.

3.1 Decision Method

In this study, we consider a discrete set of investment alternatives $X = \{A_1, A_2, \ldots, A_m\}$ representing m candidate business stocks, along with a finite set of evaluation criteria $C = \{C_1, C_2, \ldots, C_n\}$. To model the imprecision and subjectivity inherent in assessing stocks, each rating is represented using a Pythagorean fuzzy number (PFN) defined as $r_{ij} = (\varphi_{ij}, \varrho_{ij})$. Here, φ_{ij} indicates the degree to which stock A_i satisfies criterion C_j, while ϱ_{ij} reflects the degree of dissatisfaction. The complete set of these PFNs forms a Pythagorean fuzzy decision matrix $D = [r_{ij}]_{m\times n}$, which serves as the foundational input for all subsequent aggregation operations.

$$D = \begin{bmatrix} \langle\varphi_{11}, \varrho_{11}\rangle & \langle\varphi_{12}, \varrho_{12}\rangle & \cdots & \langle\varphi_{1n}, \varrho_{1n}\rangle \\ \langle\varphi_{21}, \varrho_{21}\rangle & \langle\varphi_{22}, \varrho_{22}\rangle & \cdots & \langle\varphi_{2n}, \varrho_{2n}\rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle\varphi_{m1}, \varrho_{m1}\rangle & \langle\varphi_{m2}, \varrho_{m2}\rangle & \cdots & \langle\varphi_{mn}, \varrho_{mn}\rangle \end{bmatrix}.$$

Classify the criteria into two categories, one is set of benefit criteria (J_1) and other is cost criteria (J_2) categories.
Then we compute normalize the decision matrix. Replace (μ_{ij}, ν_{ij}) with its complement (ν_{ij}, μ_{ij}) in the Pythagorean fuzzy decision matrix, we obtained normalized Pythagorean fuzzy decision matrix.

3.2 Calculation of Weight

In practical multi-criteria decision-making (MCDM) contexts such as selecting the most suitable stocks for business investment, decision-makers frequently struggle to assign fixed importance levels to evaluation criteria in advance. This difficulty stems from the vague and incomplete nature of available data, differing viewpoints among experts, the lack of a standard method for weighting educational performance indicators, and the risk of biased judgments when weights are set arbitrarily. Therefore, treating criterion weights as entirely unknown offers a more practical and adaptable representation of real-world conditions. By avoiding any predefined or subjective weights, this study removes external sources of bias and assesses the strength of the proposed aggregation operators under conditions of maximal uncertainty. Embracing unknown weights thus strengthens both the real-world relevance and the analytical soundness of the suggested framework for determining optimal stock investment choices.

The weight w_j of each criteria $(j = 1, 2, \ldots, n)$ is calculated according to the Equation (8).

$$w_j = \frac{\varphi_j^2 + \pi_j^2 \left(\frac{\varphi_j^2}{\varphi_j^2 + \varrho_j^2} \right)}{\sum_{j=1}^{n} \left(\varphi_j^2 + \pi_j^2 \left(\frac{\varphi_j^2}{\varphi_j^2 + \varrho_j^2} \right) \right)}, \tag{8}$$

where φ_j and ϱ_j represent MG and NMG of the j-th criteria respectively, and π_j represents the hesitation degree of the j-th criteria and satisfy the condition $\sum_{j=1}^{n} w_j = 1$.

3.3 Aggregate the Alternatives

To aggregate the alternative, use PyF-AA-Weighted aggregation operator (5) and obtain S_i. Similarly, use PyF-AA-Weighted Geometric aggregation operator (7) and obtain P_i. Then used Equation (3) for comparison.

3.4 Appraisal Scores and Composite Score

Compute three appraisal scores for each alternative by Equations (9), (10) and (11).

$$T_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{p} (P_i + S_i)} \tag{9}$$

$$T_{ib} = \frac{S_i}{\min_i(S_i)} + \frac{P_i}{\max_i(P_i)} \tag{10}$$

$$T_{ic} = \frac{\lambda S_i + (1-\lambda) S_i}{\lambda \max_i S_i + (1-\lambda) \max_i P_i} \tag{11}$$

$\lambda \in [0, 1]$. (usually $\lambda = 0.5$).
Calculate the composite scores for each alternative by the Equation (12).

$$CC_i = (T_{ia} T_{ib} T_{ic})^{\frac{1}{3}} + \frac{T_{ia} + T_{ib} + T_{ic}}{3} \tag{12}$$

The final rank of alternatives is obtained by sorting the compromise scores CC_i in descending order, such that the alternative with the highest CC_i value is considered the best.

3.5 Algorithm for PyF-AA-CoCoSo Method

Step-I: Construct the matrix of decisions $D = (r_{ij})_{m \times n}$.
Step-II: Normalise the decision matrix.
Step-III: The weight w_j of each criteria $(j = 1, 2, \ldots, n)$ is calculated according to the Equation (8).
Step-IV: Utilize the PyF-AA-Weighted aggregation (5) to aggregate the value for each alternative and calculate S_i by the Equation (3).
Step-V: Utilize the PyF-AA-Geometric Weighted aggregation (7) to aggregate the value for each alternative and calculate P_i Equation (3).
Step-VI: Compute three appraisal scores for each alternative by Equations (9), (10) and (11).
Step-VII: Calculate the composite score for each alternative by the Equation (12).
Step-VIII: The final ranking of alternatives is obtained by sorting the compromise scores CC_i in descending order, such that the alternative with the highest CC_i value is considered the best.

4. Illustrative Example

To demonstrate the practical applicability of the proposed Pythagorean Fuzzy Aczel-Alsina CoCoSo method for business investment, a case study is conducted involving ten prominent Indian stocks $A_1, A_2, \ldots, A_{10}$. Eight selection criteria are considered, comprising five benefit criteria—Return on Equity (ROE), Earnings Per Share Growth (3-Year Average), Dividend Yield, Average Daily Volume (Liquidity), and Institutional Holding Percentage—and three cost criteria: Debt to Equity (D/E) Ratio, Price to Earnings (P/E) Ratio, and Beta (Market Risk). A panel of investment experts provides evaluations for each stock against these criteria in the form of Pythagorean fuzzy numbers to capture inherent uncertainty and hesitation. The Aczel-Alsina t-norm and t-conorm with a tunable parameter a are employed to generate aggregated fuzzy information, followed by the CoCoSo (Combined Compromise Solution) method to rank the alternatives.

4.1 Criteria focused on Business Optimal Stock Investment

C_1: **Return on Equity (ROE) (Benefit):** Return on equity measures how efficiently a company generates profits from the shareholders' invested capital. Expressed as a percentage, ROE is calculated by dividing net income by total shareholders' equity. A higher ROE indicates superior management performance and profitability. For instance, if a stock alternative A_1 reports an ROE of 18.5%, this value is considered favorable compared to a lower figure such as 10.2%, making it a benefit criterion where larger magnitudes are preferred.

C_2: **Debt to Equity (D/E) (Cost):** The debt-to-equity ratio reflects a firm's financial leverage by comparing total liabilities to shareholders' equity. This unitless ratio serves as a cost criterion because lower values imply reduced financial risk and greater stability. A numerical example shows stock A_1 with a D/E ratio of 0.65. Since this is a cost criterion, 0.65 is superior to a higher ratio like 1.20, as it suggests the company relies less on borrowed funds and faces lower bankruptcy risk.

C_3: **Earnings Per Share Growth (3-Year Average) (Benefit):** Earnings per share growth over a three-year average captures the company's historical expansion in profitability allocated per outstanding share. Measured in percentage terms, this benefit criterion rewards higher growth rates. For example, stock A_1 exhibits a 12.3% EPS growth rate, which is more attractive than a 4.5% growth rate, as it signals robust business performance and future income potential.

Fig. 1. Flowchart of the Methodology

C_4: **Price to Earnings (P/E)(Cost):** The price-to-earnings ratio represents the market price per share divided by annual earnings per share, and it is expressed as a unitless ratio. As a cost criterion, lower P/E values are preferred because they suggest that the stock may be undervalued relative to its earnings. Consider stock A_1 with a P/E of 14.2; this is more desirable than a P/E of 22.5, particularly for value-oriented investment strategies seeking favorable entry points.

C_5: **Dividend Yield (Benefit):** Dividend yield indicates the annual dividend payment per share divided by the stock's current market price, reported as a percentage. This benefit criterion is critical for income-seeking investors, as higher yields provide greater cash returns on investment. For instance, stock A_1 offers a dividend yield of 3.8%, which outperforms a yield of 1.5%, making it more suitable for generating regular treasury income.

C_6: **Beta (Market Risk) (Cost):** Beta measures a stock's systematic risk relative to the overall market, where the market itself has a beta of one. Expressed as a dimensionless unit, beta is treated as a cost criterion because lower values indicate reduced volatility and defensive characteristics. A numerical example shows stock A_1 with a beta of 0.85, which is preferable to a beta of 1.30, as it implies the stock fluctuates less than the market, offering greater stability during downturns.

C_7: **Average Daily Volume (Liquidity) (Benefit):** Average daily volume refers to the number of shares traded per day on average, reported as a raw share count. As a benefit criterion, higher volume facilitates easier entry into and exit from a position without causing significant price slippage. For example, stock A_1 has an average daily volume of 2,450,000 shares, which is substantially better than a thinly traded stock with 200,000 shares, ensuring greater marketability and lower transaction costs.

C_8: **Institutional Holding Percentage (Benefit):** Institutional holding represents the proportion of outstanding shares owned by professional entities such as mutual funds, pension funds, and insurance companies, measured as a percentage. This benefit criterion acts as a signal of governance quality and investor confidence. A numerical illustration shows stock A_1 with 62% institutional ownership, which is more favorable than 25%, as higher institutional backing often correlates with better transparency, research coverage, and long-term stability.

Fig. 2. Criteria for Optimal Stock Investment

4.2 *Pythagorean Fuzzy Decision Matrix*

The Pythagorean fuzzy decision matrix $R = [r_{ij}]_{10\times 8} = [\langle \varphi_{ij}, \varrho_{ij}\rangle]_{10\times 8}$ is constructed, where φ_{ij} and ϱ_{ij} denote the membership and non-membership degrees of alternative A_i with respect to criterion C_j. The Pythagorean fuzzy decision presents in the Table 1.

Table 1
Pythagorean Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	$\langle 0.34, 0.82\rangle$	$\langle 0.40, 0.81\rangle$	$\langle 0.62, 0.56\rangle$	$\langle 0.36, 0.84\rangle$	$\langle 0.27, 0.86\rangle$	$\langle 0.84, 0.25\rangle$	$\langle 0.71, 0.43\rangle$	$\langle 0.68, 0.44\rangle$
A_2	$\langle 0.76, 0.38\rangle$	$\langle 0.12, 0.86\rangle$	$\langle 0.46, 0.72\rangle$	$\langle 0.47, 0.76\rangle$	$\langle 0.59, 0.62\rangle$	$\langle 0.70, 0.48\rangle$	$\langle 0.51, 0.67\rangle$	$\langle 0.85, 0.21\rangle$
A_3	$\langle 0.37, 0.84\rangle$	$\langle 0.57, 0.65\rangle$	$\langle 0.72, 0.45\rangle$	$\langle 0.32, 0.77\rangle$	$\langle 0.23, 0.82\rangle$	$\langle 0.77, 0.37\rangle$	$\langle 0.91, 0.14\rangle$	$\langle 0.78, 0.36\rangle$
A_4	$\langle 0.71, 0.49\rangle$	$\langle 0.14, 0.82\rangle$	$\langle 0.55, 0.67\rangle$	$\langle 0.41, 0.70\rangle$	$\langle 0.52, 0.65\rangle$	$\langle 0.72, 0.45\rangle$	$\langle 0.58, 0.62\rangle$	$\langle 0.82, 0.23\rangle$
A_5	$\langle 0.57, 0.63\rangle$	$\langle 0.19, 0.83\rangle$	$\langle 0.37, 0.77\rangle$	$\langle 0.28, 0.80\rangle$	$\langle 0.91, 0.12\rangle$	$\langle 0.51, 0.68\rangle$	$\langle 0.42, 0.72\rangle$	$\langle 0.64, 0.52\rangle$
A_6	$\langle 0.42, 0.77\rangle$	$\langle 0.71, 0.42\rangle$	$\langle 0.67, 0.52\rangle$	$\langle 0.40, 0.78\rangle$	$\langle 0.35, 0.84\rangle$	$\langle 0.81, 0.21\rangle$	$\langle 0.34, 0.76\rangle$	$\langle 0.72, 0.39\rangle$
A_7	$\langle 0.29, 0.82\rangle$	$\langle 0.93, 0.10\rangle$	$\langle 0.88, 0.15\rangle$	$\langle 0.58, 0.63\rangle$	$\langle 0.17, 0.88\rangle$	$\langle 0.92, 0.09\rangle$	$\langle 0.87, 0.17\rangle$	$\langle 0.81, 0.29\rangle$
A_8	$\langle 0.64, 0.57\rangle$	$\langle 0.14, 0.89\rangle$	$\langle 0.52, 0.71\rangle$	$\langle 0.91, 0.11\rangle$	$\langle 0.42, 0.76\rangle$	$\langle 0.44, 0.71\rangle$	$\langle 0.22, 0.78\rangle$	$\langle 0.85, 0.23\rangle$
A_9	$\langle 0.32, 0.74\rangle$	$\langle 0.29, 0.81\rangle$	$\langle 0.41, 0.78\rangle$	$\langle 0.30, 0.81\rangle$	$\langle 0.21, 0.88\rangle$	$\langle 0.81, 0.31\rangle$	$\langle 0.34, 0.83\rangle$	$\langle 0.62, 0.54\rangle$
A_{10}	$\langle 0.91, 0.13\rangle$	$\langle 0.18, 0.86\rangle$	$\langle 0.82, 0.24\rangle$	$\langle 0.75, 0.32\rangle$	$\langle 0.38, 0.79\rangle$	$\langle 0.42, 0.81\rangle$	$\langle 0.27, 0.84\rangle$	$\langle 0.93, 0.09\rangle$

There are two types of criteria, one is benefit criteria and other is cost criteria. The set of benefit criteria $J_1 = \{C_1, C_3, C_5, C_7, C_8\}$ and set of cost criteria $J_2 = \{C_2, C_4, C_6\}$.

For cost criteria, replace (μ_{ij}, ν_{ij}) with its complement (ν_{ij}, μ_{ij}) in the decision matrix. The normalize decision matrix presents in Table 2.

Table 2
Normalized Pythagorean Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	$\langle 0.34, 0.82\rangle$	$\langle 0.81, 0.40\rangle$	$\langle 0.62, 0.56\rangle$	$\langle 0.84, 0.36\rangle$	$\langle 0.27, 0.86\rangle$	$\langle 0.25, 0.84\rangle$	$\langle 0.71, 0.43\rangle$	$\langle 0.68, 0.44\rangle$
A_2	$\langle 0.76, 0.38\rangle$	$\langle 0.86, 0.12\rangle$	$\langle 0.46, 0.72\rangle$	$\langle 0.76, 0.47\rangle$	$\langle 0.59, 0.62\rangle$	$\langle 0.48, 0.70\rangle$	$\langle 0.51, 0.67\rangle$	$\langle 0.85, 0.21\rangle$
A_3	$\langle 0.37, 0.84\rangle$	$\langle 0.65, 0.57\rangle$	$\langle 0.72, 0.45\rangle$	$\langle 0.77, 0.32\rangle$	$\langle 0.23, 0.82\rangle$	$\langle 0.37, 0.77\rangle$	$\langle 0.91, 0.14\rangle$	$\langle 0.78, 0.36\rangle$
A_4	$\langle 0.71, 0.49\rangle$	$\langle 0.82, 0.14\rangle$	$\langle 0.55, 0.67\rangle$	$\langle 0.70, 0.41\rangle$	$\langle 0.52, 0.65\rangle$	$\langle 0.45, 0.72\rangle$	$\langle 0.58, 0.62\rangle$	$\langle 0.82, 0.23\rangle$
A_5	$\langle 0.57, 0.63\rangle$	$\langle 0.83, 0.19\rangle$	$\langle 0.37, 0.77\rangle$	$\langle 0.80, 0.28\rangle$	$\langle 0.91, 0.12\rangle$	$\langle 0.68, 0.51\rangle$	$\langle 0.42, 0.72\rangle$	$\langle 0.64, 0.52\rangle$
A_6	$\langle 0.42, 0.77\rangle$	$\langle 0.42, 0.71\rangle$	$\langle 0.67, 0.52\rangle$	$\langle 0.78, 0.40\rangle$	$\langle 0.35, 0.84\rangle$	$\langle 0.21, 0.81\rangle$	$\langle 0.34, 0.76\rangle$	$\langle 0.72, 0.39\rangle$
A_7	$\langle 0.29, 0.82\rangle$	$\langle 0.10, 0.93\rangle$	$\langle 0.88, 0.15\rangle$	$\langle 0.63, 0.58\rangle$	$\langle 0.17, 0.88\rangle$	$\langle 0.09, 0.92\rangle$	$\langle 0.87, 0.17\rangle$	$\langle 0.81, 0.29\rangle$
A_8	$\langle 0.64, 0.57\rangle$	$\langle 0.89, 0.14\rangle$	$\langle 0.52, 0.71\rangle$	$\langle 0.11, 0.91\rangle$	$\langle 0.42, 0.76\rangle$	$\langle 0.71, 0.44\rangle$	$\langle 0.22, 0.78\rangle$	$\langle 0.85, 0.23\rangle$
A_9	$\langle 0.32, 0.74\rangle$	$\langle 0.81, 0.29\rangle$	$\langle 0.41, 0.78\rangle$	$\langle 0.81, 0.30\rangle$	$\langle 0.21, 0.88\rangle$	$\langle 0.81, 0.31\rangle$	$\langle 0.34, 0.83\rangle$	$\langle 0.62, 0.54\rangle$
A_{10}	$\langle 0.91, 0.13\rangle$	$\langle 0.86, 0.18\rangle$	$\langle 0.82, 0.24\rangle$	$\langle 0.32, 0.75\rangle$	$\langle 0.38, 0.79\rangle$	$\langle 0.81, 0.42\rangle$	$\langle 0.27, 0.84\rangle$	$\langle 0.93, 0.09\rangle$

Weight of the each criteria calculated by the Equation (8). The weight of the criteria are $w_1 = 0.1137$, $w_2 = 0.0702$, $w_3 = 0.1393$, $w_4 = 0.9489$, $w_5 = 0.07113$, $w_6 = 0.1805$, $w_7 = 0.1128$, $w_8 = 0.2171$.
The Pythagorean fuzzy Aczel-Alsina weighted aggregation operator given in Equation (5) is applied with the parameter $a = 3$ to aggregate the individual Pythagorean fuzzy decision matrix entries. Subsequently, the score values of the aggregated Pythagorean fuzzy numbers are computed using the score function defined in Equation (3). The resulting aggregated values and their corresponding score values are summarized in Table 3.

Table 3
PyF-AA-Weighted aggregation

Alternative	Aggregated	S_i	Rank
A_1	(0.8437, 0.3373)	2.0314	4
A_2	(0.8162, 0.2816)	1.9951	5
A_3	(0.8280, 0.2537)	2.1069	3
A_4	(0.7722, 0.2854)	1.7579	8
A_5	(0.8301, 0.2330)	2.1506	2
A_6	(0.7867, 0.3729)	1.6414	10
A_7	(0.8067, 0.2753)	1.9562	6
A_8	(0.7789, 0.3309)	1.6995	9
A_9	(0.8130, 0.2931)	1.9560	7
A_{10}	(0.8726,0.1830)	2.4492	1

Applying the Pythagorean fuzzy Aczel-Alsina weighted geometric aggregation operator (Eq. (7)) to the individual evaluations yields the aggregated Pythagorean fuzzy numbers, whose score values P_i are subsequently obtained from Equation (3). Table 4 summarizes the computed results.

Table 4
PyF-AA-Weighted Geometric aggregation

Alternative	Aggregated	P_i	Rank
A_1	(0.7500, 0.4728)	1.1673	6
A_2	(0.7562, 0.4740)	1.1952	5
A_3	(0.7497, 0.4305)	1.2954	4
A_4	(0.7048, 0.4292)	1.0773	8
A_5	(0.7831, 0.3778)	1.6106	2
A_6	(0.7115, 0.4655)	0.9992	9
A_7	(0.5712, 0.6121)	0.1678	10
A_8	(0.1230, 0.9010)	2.6650	1
A_9	(0.7441, 0.4662)	1.1584	7
A_{10}	(0.3388, 0.7378)	1.4734	3

Following the CoCoSo aggregation procedure, three distinct appraisal scores are derived for every alternative by applying Equations (9), (10), and (11), respectively. Table 5 presents the computed scores for all alternatives.

Table 5
Appraisal Scores of Alternatives

Alternative	T_{ai} (Rank)	T_{bi} (Rank)	T_{ci} (Rank)
A_1	0.0982 (5)	8.1912 (6)	0.6254 (5)
A_2	0.9799 (6)	8.3349 (5)	0.6237 (6)
A_3	0.1045 (4)	9.0002 (4)	0.6652 (4)
A_4	0.8709 (8)	7.4882 (8)	0.5543 (8)
A_5	0.1155 (3)	10.9047 (2)	0.7354 (2)
A_6	0.0811 (9)	6.9524 (9)	0.5163 (9)
A_7	0.0652 (10)	2.1917 (10)	0.4153 (10)
A_8	0.1340 (1)	16.9107 (1)	0.8534 (1)
A_9	0.0956 (7)	8.0923 (7)	0.6090 (7)
A_{10}	0.1204 (2)	9.9686 (3)	0.6705 (3)

Equation (12) is employed to derive the final composite score for each alternative, with the results presented in Table 6.

Table 6
Combine Compromise Scores of Alternatives

Alternative	CC_i (Rank)
A_1	3.7671 (6)
A_2	3.8176 (5)
A_3	4.1119 (4)
A_4	3.4222 (8)
A_5	4.8934 (2)
A_6	3.1794 (9)
A_7	1.2809 (10)
A_8	7.2121 (1)
A_9	3.7106 (7)
A_{10}	4.5170 (3)

The alternatives are ranked in descending order of their final composite scores obtained from the combined compromise solution (CoCoSo). The resulting ranking is $A_8 \succ A_5 \succ A_{10} \succ A_3 \succ A_2 \succ A_1 \succ A_9 \succ A_4 \succ A_6 \succ A_7$.

According to the proposed Pythagorean fuzzy Aczel-Alsina CoCoSo method, alternative A_8 is identified as the most suitable choice for business investment, while alternative A_7 is the least favorable.

5. Findings and Discussion

In this study, we investigated on optimal selection of stock for business investment from several alternatives and alternatives are ranked using five distinct approaches: Pythagorean fuzzy Aczel-Alsina weighted aggregation (S_i), Pythagorean fuzzy Aczel-Alsina weighted geometric aggregation (P_i), three different types of appraisal scores (Case I, II, III), and by the combine compromise score of Pythagorean fuzzy Aczel-Alsina-CoCoSo method (CC_i). The comparative analysis are presented in the Table 7 and graphically presented in the Figure 3.

Table 7
Results and discussion

Methods	Order of Stock Investment
Aczel-Alsina Weighted Aggregation	
PyF-AA-W aggregation (S_i)	$A_{10} \succ A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_7 \succ A_9 \succ A_4 \succ A_8 \succ A_6$.
PyF-AA-GW aggregation (P_i)	$A_8 \succ A_5 \succ A_{10} \succ A_3 \succ A_2 \succ A_1 \succ A_9 \succ A_4 \succ A_6 \succ A_7$.
Rank by Appraisal Scores	
Case-I (T_{ai})	$A_8 \succ A_{10} \succ A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_9 \succ A_4 \succ A_6 \succ A_7$.
Case-II (T_{bi})	$A_8 \succ A_5 \succ A_{10} \succ A_3 \succ A_2 \succ A_1 \succ A_9 \succ A_4 \succ A_6 \succ A_7$.
Case-III (T_{ic})	$A_8 \succ A_5 \succ A_{10} \succ A_3 \succ A_2 \succ A_1 \succ A_9 \succ A_4 \succ A_6 \succ A_7$.
Rank by Combine Compromise Score	
CC_i	$A_8 \succ A_5 \succ A_{10} \succ A_3 \succ A_2 \succ A_1 \succ A_9 \succ A_4 \succ A_6 \succ A_7$.

The comparative analysis revealed notable consistency among four of the five methods. Specifically, the P_i (weighted geometric aggregation), Case II (T_{bi}), Case III (T_{ic}), and the CC_i (CoCoSo) methods produced identical rankings: A8 first, followed by A_5, A_{10}, A_3, A_2, A_1, A_9, A_4, A_6, and A_7 last. This strong agreement suggests that these four approaches yield robust and interchangeable results for this decision-making problem. Case I (T_{ai}), while still similar, showed a slight variation in the upper ranks: it also placed A_8 at the top, but ranked A_{10} second and A_5 third, whereas the previous four methods ranked A5 ahead of A_{10}. From the fourth position (A_3) onward, Case I remained consistent with the other four methods. In sharp contrast, the S_i (weighted aggregation) method produced a distinctly different ordering: it ranked A_{10} as the best alternative, followed by A_5, A_3, A_1, A_2, A_7, A_9, A_4, A_8, and A_6. Notably, while A_8 was the top choice in all other methods, S_i placed it second to last (ninth position), and A6 was consistently last or near-last across all approaches. These discrepancies highlight how the choice of aggregation operator (weighted vs. weighted geometric) and the inclusion of appraisal score variations can significantly influence investment recommendations. All findings are summarized graphically in the Figure 3.

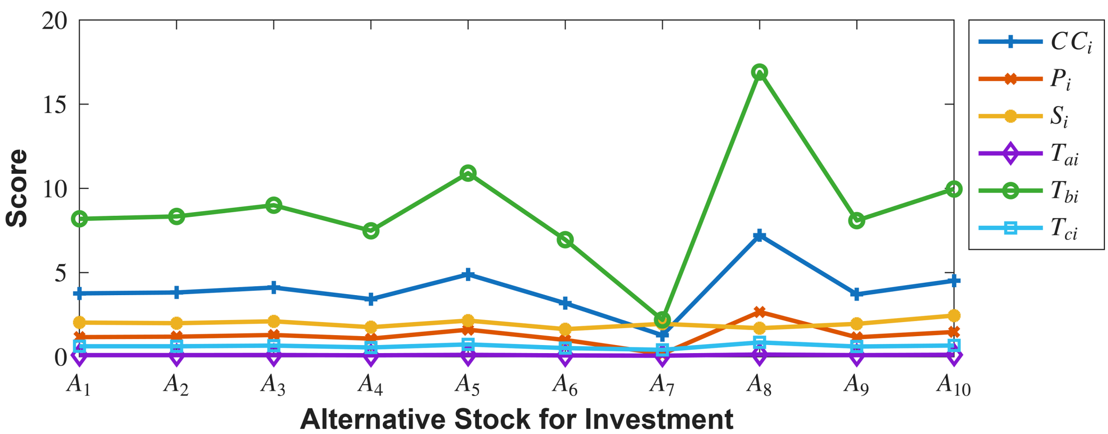


Fig. 3. Comparative Analysis

6. Conclusion and Future Work

This study proposed a novel multi-criteria decision-making framework, termed the Pythagorean Fuzzy Aczel-Alsina Combined Compromise Solution (PyF-AA-CoCoSo) method, for the selection of optimal stocks for business investment under uncertainty. The framework uniquely integrates Pythagorean fuzzy sets capable of representing membership, non-membership, and hesitation degrees simultaneously with Aczel-Alsina t-norm and t-conorm operators, which offer a flexible parameter a to adjust the aggregation intensity, within the CoCoSo ranking structure. The key findings demonstrate that the proposed method effectively handles ambiguous and incomplete information inherent in expert evaluations of financial criteria. In the illustrative case study involving ten Indian stocks ($A_1, A_2, \ldots, A_{10}$) evaluated against eight criteria (ROE, debt-to-equity, EPS growth, P/E ratio, dividend yield, beta, liquidity, and institutional holding). the method identified A_8) as the optimal investment choice while A_7 consistently ranked as the least favorable.

Several critical limitations must be acknowledged. First, the method assumes static expert preferences and stock attributes, ignoring the dynamic, time-varying nature of financial markets including volatility clustering, regime shifts, and macroeconomic shocks. Second, the framework evaluates stocks independently, failing to account for correlation structures, systemic risk (beta diversification), or portfolio-level optimization constraints such as sectoral exposure limits and risk-return trade-offs. Third, the case study relies on subjective expert evaluations rather than objective historical market data, and no empirical validation or backtesting against real-world stock performance (e.g., out-of-sample returns, Sharpe ratios) has been conducted. Fourth, the criteria selection, while financially relevant, lacks statistical validation for multicollinearity or redundancy, and the equal weighting of experts may not reflect varying domain expertise.

To address these limitations, a comprehensive future research agenda is proposed. First, a dynamic Pythagorean fuzzy decision framework will be developed by integrating the proposed CoCoSo method with time-series forecasting models such as Long Short-Term Memory (LSTM) networks and GARCH-family volatility models, enabling rolling-window re-ranking of stocks as new market data becomes available. Second, the method will be extended to multi-objective portfolio optimization by incorporating correlation matrices, Value at Risk (VaR), and Conditional Value at Risk (CVaR) constraints within a Pythagorean fuzzy environment, allowing simultaneous selection of multiple stocks with diversification benefits. Third, the criteria selection process will be enhanced using dimensionality reduction techniques such as principal component analysis (PCA) or fuzzy rough set-based feature selection to eliminate redundancy, and expert weights will be calibrated using the Bayesian Best-Worst Method (B-BWM) to reflect differential expertise. These extensions collectively aim to transform the current static, single-stock evaluation model into a dynamic, portfolio-aware, empirically validated, and behaviorally grounded decision support system suitable for real-time financial applications.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*, 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*, 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

[3] Yager, R. R. (2013). Pythagorean fuzzy subsets. *Proceedings of joint IFSA world congress and NAFIPS annual meeting*, *Edmonton*, *Canada*, 57–61.

[4] Adak, A. K., & Kumar, D. (2022). Some properties of pythagorean fuzzy ideals of Γ-near-rings. *Palestine Journal of Mathematics*, *11(4)*, 336–346.

[5] Adak, A. K., & Kumar, D. (2023). Spherical distance measurement method for solving mcdm problems under pythagorean fuzzy environment. *Journal of fuzzy extension and applications*, *4(4)*, 28–39. https://doi.org/10.22105/jfea.2022.351677.1224
[6] Adak, A., Pamucar, D., & Ali, W. (2025). Solving pythagorean fuzzy assignment problems in management: A framework based on spherical distance measures. *Management Science Advances*, *3(1)*, 84–95. https://doi.org/10.31181/msa31202636
[7] Karamat, T., & Sarfraz, M. (2025). Applied multi-attribute decision-making with complex pyth agorean fuzzy data based on prioritized aczel–alsina aggregation operators: A case for a software company. *Applied Research Advances*, *1(1)*, 14–27. https://doi.org/10.65069/ara1120254
[8] Karamat, T., & Sarfraz, M. (2026). A pythagorean fuzzy multiple-attribute decision-making model for smart governance research applications. *Applied Research Advances*, *2(1)*, 141–161. https://doi.org/10.65069/ara2120266
[9] Khalid, I., & Riaz, L. (2025). Behavioral biases and investment management decisions: The mediating role of risk perception and the moderating role of financial literacy. *Management Science Advances*, *3(1)*, 1–19. https://doi.org/10.31181/msa31202629
[10] Sarkar, A., & Goswami, S. S. (2026). A review of the application of mcdm methods in business analytics. *Applied Decision Analytics*, *2(1)*, 150–180. https://doi.org/10.66972/ada21202614
[11] Yin, Y., Li, Z., Li, K., Wang, T., & Cui, S. (2026). Research on the role of esg performance in capital market pricing efficiency: An analysis based on stock price synchronicity. *International Journal of Economic Sciences*, *15(1)*, 19–40. https://doi.org/10.31181/ijes1512026218
[12] Gabriel Rasoanaivo, R., Yazdani, M., Zaraté, P., & Fateh, A. (2024). Combined compromise for ideal solution (cocofiso): A multi-criteria decision-making based on the cocoso method algorithm. *Expert Systems with Applications*, *251*. https://doi.org/10.1016/j.eswa.2024.124079
[13] Adak, A. K., Pamucar, D., & Simic, V. (2026). An integrated pythagorean fuzzy topsis framework for supplier selection with uncertain attribute weights. *Recent Advances in Computational and Applied Mathematics: Modelling, Methods, and Applications*, *44(2)*, 1–14. https://doi.org/10.5269/bspm.81585
[14] Hussain, A., Ullah, K., Mubasher, M., Senapati, T., & Moslem, S. (2023). Interval-valued pytha gorean fuzzy information aggregation based on aczel-alsina operations and their application in multiple attribute decision making. *IEEE Access*, *11*, 34575–34594. https://doi.org/10.1109/ACCESS.2023.3244612
[15] Jafari, M., & Naghdi Khanachah, S. (2024). Integrated knowledge management in the supply chain: Assessment of knowledge adoption solutions through a comprehensive cocoso method under uncertainty. *Journal of Industrial Information Integration*, *39*. https://doi.org/10.1016/j.jii.2024.100581
[16] Wang, P., Zhu, B., Yan, K., Zhang, Z., Ali, Z., & Pamucar, D. (2025). Power aggregation operators based on aczel-alsina t-norm and t-conorm for intuitionistic hesitant fuzzy information and their application to logistics service provider selection. *Artificial Intelligence Review*, *58(7)*. https://doi.org/10.1007/s10462-025-11155-4
[17] Adaramola, A. O., & Popoola, M. F. (2019). Long and short run relationship between stock market development and economic growth in nigeria. *Journal of Economics and Behavioral Studies*, *11(5(J))*, 45–53. https://doi.org/10.22610/jebs.v11i5(j).2965
[18] Arena, C. M. R., Batac, A. A. S., Religioso, A. M. A., Magbata, E. V. S., & Mandigma, M. B. S. (2023). Influences on the stockmarket investing of tertiary students in the national capital region, philippines. *Review of Integrative Business and Economics Research*, *12(2)*, 148–166.
[19] Chen, Q. (n.d.). A study of the impact of the rise of “her power.

[20] Haryadi, R., & Princes, E. (2025). Enhancing stock market investment decisions through block chain transaction security: A study on investor intentions. *Journal of Theoretical and Applied Information Technology*, *103(8)*, 3385–3415.

[21] Ma, N. (2024). Dual-factor investment strategy: Market trends and performance growth. *Advances in Economics, Management and Political Sciences*, *73(1)*, 136–143. https://doi.org/10.54254/2754-1169/73/20231597

[22] Segun, I. B. (2021). The relationship between stockmarket development and the performance of the nigerian economy. *International Journal of Multidisciplinary Research and Analysis*, *04(06)*. https://doi.org/10.47191/ijmra/v4-i6-07

[23] Vijay, S., & Rao, D. C. B. (2018). A study on investors perception towards sharemarket in sharekhan ltd. *International Journal of Trend in Scientific Research and Development*, *2(6)*, 103–111. https://doi.org/10.31142/ijtsrd18384